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Discriminating between the Lognormal and Weibull Distributions under Progressive Censoring

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Abstract. In this paper, the ratio of maximized likelihood and Minimized Kullback-Leibler Divergence methods are discussed for discrimination between log-normal and Weibull distributions. The progressive Type-II right censored sample is considered in the study. The probability of correct selections is simulated and compared to investigate the performance of the procedures for different censoring schemes and parameter settings.

Keywords: Discrimination, Log-normal distribution, Power analysis, Simulation, Progressive type-II right censoring.

İlerleyen Tür Sansür Altında Lognormal ve Weibull Dağılımlarının Ayrımı

Özet. Bu çalışmada, log-normal ve weibull dağılımları arasında ayırım için en çok olabilirlik oran ve Kullback-Leibler uzaklık metotları tartışılmıştır. Çalışmada, ilerleyen tür sansürlü veri durumu ele alınmıştır. Doğru seçim oranları hesaplanmış ve farklı parametre ve sansür şemaları altında testlerin performansları karşılaştırılmıştır.

Anahtar Kelimeler: Ayırım, Log-normal dağılım, Güç analizi, Simülasyon, İlerleyen tür sansürleme.

1. INTRODUCTION

A discrimination procedure focus on making suitable selection from two or more distributions based sample. In other words, discrimination procedure tries to get decision on which distribution is more effective to modeling the data. A lot of papers in the literature on discrimination two or three distributions. Most of them are based on Kullback-Leibler Divergence (KLD) and ratio of maximized log-likelihood (RML). There are a lot of works in this area. Some of them are Alzaid & Sultan [1], Kundu & Manglick [2], Bromideh and Valizadeh [3], Dey and Kundu [4], Dey and Kundu [5], Kundu [6], Kantam et al. [7], Ngom, et al. [8], Ravikumar and Kantam, [9], Qaffou and Zoglat, [10] and Algamal [11].

In this study, we consider on discrimination between log-normal and Weibull distributions. The probability density function (pdf) of log-normal and Weibull distribution are given, respectively, by

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$$f_{\theta_1}(x) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\left\{ \frac{-1}{2} \left(\frac{\log(x) - \mu}{\sigma} \right)^2 \right\} I_{(0,\infty)}(x)$$

and

$$g_{\theta_2}(x) = \alpha x^{\alpha-1} \beta^{-\alpha} \exp\left\{ -\left(\frac{x}{\beta} \right)^{\alpha} \right\} I_{(0,\infty)}(x)$$

where $I_A(x)$ is an indicator function on set A and $\theta_1 = (\mu, \sigma)'$ and $\theta_2 = (\alpha, \beta)'$ are distribution parameter vectors.

Some papers related the discrimination between log-normal and Weibull distributions are Quesenberry & Kent [12], Dumonceaux & Antle [13], Pasha et al. [14], Dey & Kundu [4,5], Bromideh [15], Raqab, et al. [16] and Elsherpieny et al [17]. Quesenberry & Kent [12], proposed selection statistic that is essentially the value of the density function of a scale transformation maximal invariant. They considered include the exponential, gamma, Weibull, and lognormal. Note that this method works only complete sample case. Dumonceaux & Antle [13] used the difference of the RML, in discriminating between the Weibull or Log-Normal distribution based on complete sample. Kundu & Manglick [18] obtained the asymptotic distribution of the discrimination statistic RML and determined the probability of correct selection (PCS) by using asymptotic distribution in this discrimination process. Dey and Kundu [19] extended the Kundu & Manglick [18]'s results to Type-II censored sample case. Pasha et al. [14] used RML and most powerful invariant for discriminating these distributions based on complete sample. Kim & Yum [20] extended to Pasha et al. [14]'s results to Type-I and Type-II censored sample cases. Dey & Kundu [4, 5] used the RML, in discriminating between the Weibull, Generalized Exponential Distributions or Log-Normal distribution based on complete and Type-I censored sample. They obtained the asymptotic distribution of the discrimination statistic and determined the PCS by using asymptotic distribution in this discrimination process. Bromideh [15] examined the use the KLD in discriminating either the Weibull or Log-Normal distribution based on complete sample. Raqab, et al. [16] used the RML, in discriminating between the Weibull, Log-logistic or Log-Normal distribution based on doubly censored sample. Elsherpieny et al. [17] considered test based RML and Ratio Minimized Kullback-Leibler Divergence RMKLD for discrimination between Gamma and Log-logistic Distributions based on progressive Type-II right censored data. The model of progressive Type-II right censoring is of importance in the field of reliability and life testing.

Table 1. The papers related to discrimination between lognormal and Weibull distribution

<table>
<tr><th></th><th colspan="5">Type of Data Schemes</th></tr>
<tr><th>Discrimination and test statistics</th><th>Complete Data</th><th>Type-I Censored</th><th>Type-II Censored</th><th>Doubly Censored</th><th>Progressively Type-II Right Censored</th></tr>
<tr><td>Kullback-Leibler (KLD)</td><td>Bromideh (2012)</td><td></td><td></td><td></td><td rowspan="2">Elsherpieny et al. (2017)</td></tr>
<tr><td rowspan="3">Ratio of the Maximized Likelihood (RML)</td><td>Kundu & Manglick (2004)</td><td>Dey & Kundu (2009)</td><td>Dey & Kundu (2012)</td><td>Raqab, et al. (2018)</td></tr>
<tr><td>Dumonceaux & Antle (1973)</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Pasha et al. (2006)</td><td colspan="2">Kim & Yum (2008)</td><td></td><td></td></tr>
<tr><td>Scale Invariant Test (SI)</td><td>Quesenberry& Kent (1982)</td><td></td><td></td><td></td><td></td></tr>
</table>

All the papers except for Elsherpieny et al. [17], consider complete or Type-I and Type-II censored sample. In this work, we consider discrimination under progressive Type-II right censored schemes. Progressive Type-II right censoring scheme is explained as follows: Let n identical units are subject to a lifetime test. r_i surviving units are randomly withdrawn from the test, $1 \le i \le m$ as soon as i-th failure is occured. Hence, if m failures are observed then $r_1 + \cdots + r_m$ units are progressively Type-II right censored; Thus, $n = m + r_1 + \cdots + r_m$. Let $X^{\mathbf{r}}_{1:m:n} < X^{\mathbf{r}}_{2:m:n} < \cdots < X^{\mathbf{r}}_{m:m:n}$ be the progressively Type-II right censored failure times, where $\mathbf{r} = (r_1, \ldots, r_m)$ denotes the censoring scheme for the life test. As a special case if $\mathbf{r} = (0, \ldots, 0)$, ordinary order statistics are obtained[21]. If $\mathbf{r} = (0, \ldots, 0, m)$, the progressive Type-II right censoring becomes type-II censoring. For more details please see [22,23,24].

In this paper, the discrimination methods are given in Section 2. In Section 3, PCS are simulated by Monte Carlo methods and results are discussed. Finally, a numerical example is provided to illustrate the methodology.

2. RULES OF DISCRIMINATION

Let $X^{\mathbf{r}}_{1:m:n} < X^{\mathbf{r}}_{2:m:n} < \cdots < X^{\mathbf{r}}_{m:m:n}$ are progressive Type-II right censored sample from log-normal (μ, σ) distribution. Then log-likelihood function [26] is given by

$$\begin{aligned} L_{LN}(\theta_1) \propto & -m\log(\sigma) - \sum_{i=1}^{m}\log(x_i) - \sum_{i=1}^{m}\log\left(\phi\left(\frac{x_{(i)}-\mu}{\sigma}\right)\right) \\ & + \sum_{i=1}^{m}(r_i+1)\left(1-\Phi\left(\frac{x_{(i)}-\mu}{\sigma}\right)\right), \end{aligned} \tag{1}$$

where ϕ and Φ denotes the pdf and cdf of a standard normal distribution. Hence, ML estimate (it is denoted by $\hat{\boldsymbol{\theta}}_1 = (\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\sigma}})$) of θ_1 can be obtained numerically which maximize the likelihood function (1).

Let $X^{\mathbf{r}}_{1:m:n} < X^{\mathbf{r}}_{2:m:n} < \cdots < X^{\mathbf{r}}_{m:m:n}$ are progressive Type-II right censored sample from Weibull (α, β) distribution. Then the log-likelihood function (see [27]) is given by

$$L_W(\theta_2) \propto m\log(\alpha) - m\alpha\log(\beta) + (\alpha-1)\sum_{i=1}^{m}\log(x_i) - \sum_{i=1}^{m}(r_i+1)\left(\frac{x_i}{\beta}\right)^{\alpha}. \tag{2}$$

Hence, maximum likelihood (ML) estimate of θ_2 (it is denoted by $\hat{\boldsymbol{\theta}}_2 = (\hat{\alpha}, \hat{\beta})$) can be obtained numerically which maximize the likelihood function (2).

One of the rules of discrimination is ratio of the maximized likelihood (RML). The ratio of maximized likelihood is defined as follows

$$RML = L_{LN}(\hat{\boldsymbol{\theta}}_1) - L_W(\hat{\boldsymbol{\theta}}_2)$$

where $L_{LN}(\theta_1)$ and $L_W(\theta_2)$ are defined by (1) and (2), respectively and $\hat{\boldsymbol{\theta}}_1$ and $\hat{\boldsymbol{\theta}}_2$ are ML estimates of θ_1 and θ_2. If the $RML > 0$ then log-normal distribution is selected for the modeling data otherwise Weibull distribution is selected against log-normal distribution.

Second one is based on Kullback-Leibler divergence. The KLD is a non-symmetric measure of the difference (dissimilarity) between two probability distributions f_{θ_1} and g_{θ_2}. Kullback-Leibler divergence between models is defined by

$$D(f_{\theta_1}, g_{\theta_2}) = \int_0^\infty f_{\theta_1}(x)\log\left(\frac{f_{\theta_1}(x)}{g_{\theta_2}(x)}\right)dx$$
$$= \int_0^\infty f_{\theta_1}(x)\log(f_{\theta_1}(x))dx - \int_0^\infty f_{\theta_1}(x)\log(g_{\theta_2}(x))dx.$$

It is noted that the $D(f_{\theta_1}, g_{\theta_2})$ can also be written by

$$D(f_{\theta_1}, g_{\theta_2}) = -H(f_{\theta_1}) - \int_0^\infty f_{\theta_1}(x)\log(g_{\theta_2}(x))dx$$

where $H(f_{\theta_1})$ is Shannon's entropy of f_{θ_1} defined as

$$H(f_{\theta_1}) = -\int_0^\infty f_{\theta_1}(x)\log(f_{\theta_1}(x))dx.$$

It is well known that $D(f_{\theta_1}, g_{\theta_2}) \geq 0$ and the equality holds if and only if $f_{\theta_1}(x) = g_{\theta_2}(x)$, almost surely [28], [29]. Furthermore, $D(f_{\theta_1}, g_{\theta_2})$ can be considered to serve as a measure of disparity between f_{θ_1} and g_{θ_2}.

$D(f_{\theta_1}, g_{\theta_2})$ denotes the "information lost when g_{θ_2} is used to approximate f_{θ_1}. Namely, KLD is a measure of inefficiency of assuming that the distribution of population is g_{θ_2} when the underlying distribution is f_{θ_1}. The smaller $D(f_{\theta_1}, g_{\theta_2})$ means that f_{θ_1} is selected and large values of $D(f_{\theta_1}, g_{\theta_2})$ favor g_{θ_2} [15].

Let f_{θ_1} and g_{θ_2} are probability density functions of log-normal and Weibull distribution respectively. Then $D(f_{\theta_1}, g_{\theta_2})$ and $D(g_{\theta_2}, f_{\theta_1})$ are given by

$$D(f_{\theta_1}, g_{\theta_2}) = \int_0^\infty f_{\theta_1}(x)\log\left(\frac{f_{\theta_1}(x)}{g_{\theta_2}(x)}\right)dx$$
$$= -1/2 - 1/2\log(2) - 1/2\log(\pi) - \log(\sigma) - \mu\alpha$$
$$+\alpha\log(\beta) - \log(\alpha) + \beta^{-\alpha}\exp(1/2\alpha(2\mu + \alpha\sigma^2))$$

and

$$D\left(g_{\theta_2}, f_{\theta_1}\right) = \int_0^\infty g_{\theta_2}(x) \log\left(\frac{g_{\theta_2}(x)}{f_{\theta_1}(x)}\right) dx$$
$$= (\alpha \log(\alpha) - \alpha\gamma - \alpha \log(\beta) + \gamma - \alpha)/\alpha - (-1/12(6\alpha^2 \log(\beta)^2$$
$$+12\alpha^2 \log(\beta)\sigma^2 - 12\alpha^2 \log(\beta)\mu - 12\alpha \log(\beta)\gamma + 12\log(\sigma)\alpha^2\sigma^2$$
$$+6\log(\pi)\alpha^2\sigma^2 + 6\log(2)\alpha^2\sigma^2 + 6\mu^2\alpha^2 - 12\alpha\gamma\sigma^2 + 12\alpha\mu\gamma + 6\gamma^2 + \pi^2)/\alpha^2/\sigma^2).$$

$D\left(f_{\theta_1}, g_{\theta_2}\right)$ and $D\left(g_{\theta_2}, f_{\theta_1}\right)$ were given by Bromideh [15] but they cannot read clearly in their paper. Therefore, these equations are obtained using by Maple. Second method for discrimination is the ratio of Minimized Kullback-Leibler Divergence $(RMKLD)$ rule (Elsherpieny et al., [17]) which is defined by

$$RMKLD = \log\left(\frac{D\left(f_{\hat{\theta}_1}, g_{\hat{\theta}_2}\right)}{D\left(g_{\hat{\theta}_2}, f_{\hat{\theta}_1}\right)}\right)$$

If $RMKLD < 0$, then we select the log-normal distribution for modeling data otherwise we select the Weibull distribution for modeling data.

3. SIMULATION STUDY

In this section, the PCS of RML and RMKLD methods are obtained and compared for different censoring schemes. The censoring schemes used in simulation are given in Table 2. Probabilities of correct selection of rules are simulated and given in Table 3-4.

Table 2. The censoring schemes used in simulation

Scheme	m	$\mathbf{r} = (r_1, \ldots, r_m)$
1	10	(5,9*0)
2	10	(9*0,5)
3	10	(5*1,5*0)
4	10	(5*0,5*1)
5	10	(4*0,5,5*0)
6	13	(2,12*0)
7	13	(12*0,2)
8	13	(2*1,11*0)
9	13	(11*0,2*1)
10	13	(4*0,2,8*0)
11	15	(15*0)
12	30	(15,29*0)
13	30	(29*0,15)
14	30	(15*1,15*0)
15	30	(15*0,15*1)
16	30	(14*0,15,15*0)
17	40	(5,39*0)
18	40	(39*0,5)
19	40	(5*1,35*0)
20	40	(35*0,5*1)
21	40	(19*0,5,20*0)
22	45	(45*0)

Let us consider the data come from log-normal distribution. From Fig. 1 and Fig. 2 the PCS of the RML and RMKLD are similar in general but the PCS of RML and KLD is slightly better than the PCS of other for some schemes. The selection of parameter values does not affect to the PCS so much.

Secondly, the PCS of the RML and RMKLD are better when the censoring is made at the beginning of the life test.

Now let us consider the data come from Weibull distribution. From Fig. 3 and Fig. 4 the PCS of RMKLD is better than the power of RML for all schemes. Secondly, the PCS of the KLD are better when the censoring is made at the end of the life test. The PCS of the RML are better when the censoring is made at the beginning of the life test.

Table 3. Probability of Correct Selection of RML and RMKLD rule when the data come from log-normal distribution

	RML			**RMKLD**		
	(μ=0.5,σ=1)	(μ=1,σ=1)	(μ=2,σ=1)	(μ=0.5,σ=1)	(μ=1,σ=1)	(μ=2,σ=1)
Scheme1	0.6763	0.6764	0.6802	0.7004	0.7012	0.6999
Scheme2	0.6565	0.6572	0.6568	0.5906	0.5909	0.5853
Scheme3	0.6721	0.6826	0.6737	0.6883	0.6922	0.6914
Scheme4	0.6421	0.6416	0.6384	0.6391	0.6461	0.6437
Scheme5	0.6831	0.6791	0.6820	0.6770	0.6881	0.6887
Scheme6	0.7019	0.7102	0.7116	0.7192	0.7249	0.7129
Scheme7	0.6960	0.6850	0.6920	0.6596	0.6559	0.6509
Scheme8	0.7092	0.7037	0.7054	0.7107	0.7165	0.7165
Scheme9	0.6931	0.7007	0.6950	0.6583	0.6649	0.6555
Scheme10	0.7127	0.7058	0.7054	0.7086	0.7093	0.7109
Scheme11	0.7318	0.7349	0.7235	0.7277	0.7187	0.7282
Scheme12	0.8496	0.8469	0.8499	0.8601	0.8583	0.8526
Scheme13	0.7763	0.7693	0.7654	0.7221	0.7241	0.7236
Scheme14	0.8473	0.8523	0.8486	0.8457	0.8467	0.8536
Scheme15	0.7771	0.7859	0.7868	0.7874	0.7857	0.7837
Scheme16	0.8448	0.8467	0.8378	0.8448	0.8503	0.8503
Scheme17	0.8766	0.8743	0.8790	0.8860	0.8808	0.7618
Scheme18	0.8398	0.8411	0.8371	0.8209	0.8194	0.7616
Scheme19	0.8781	0.8776	0.8772	0.8783	0.8847	0.7660
Scheme20	0.8533	0.8381	0.8394	0.8242	0.8335	0.7665
Scheme21	0.8764	0.8800	0.8798	0.8776	0.8819	0.7816
Scheme22	0.8857	0.8918	0.8859	0.8897	0.8832	0.8121

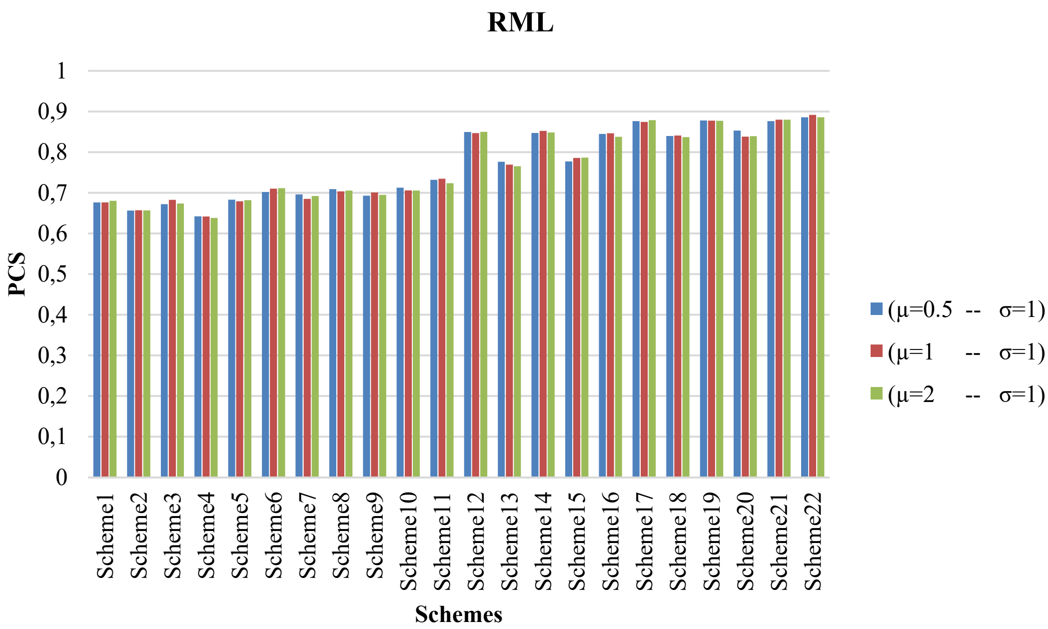

Figure 1. Probability of Correct Selection of RML rule when the data come from log-normal distribution

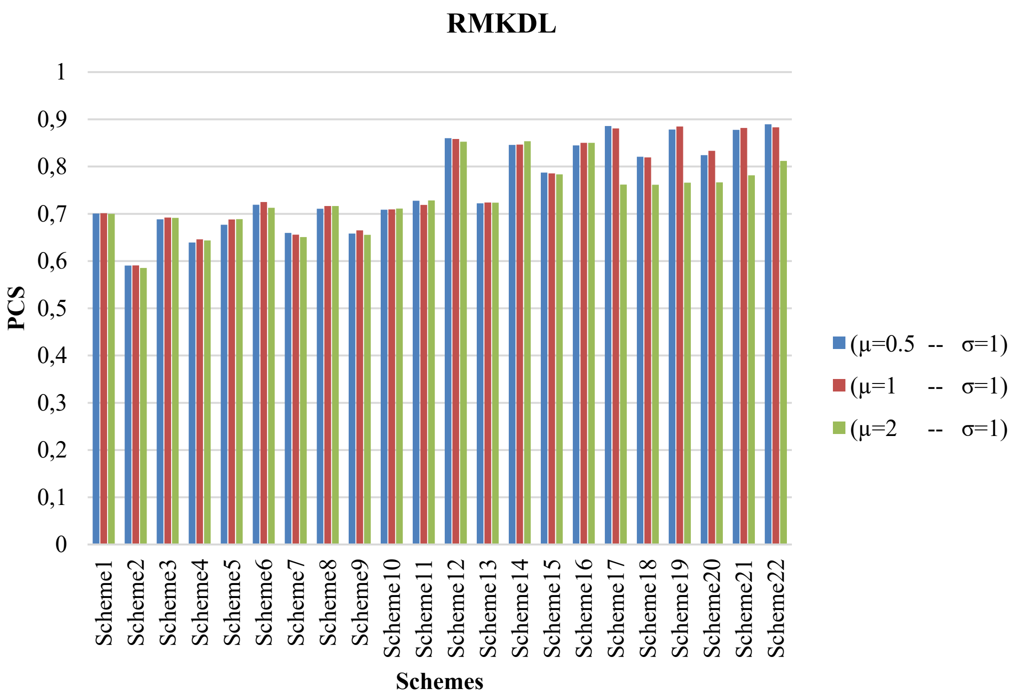

Figure 2. Probability of Correct Selection of RMKLD rule when the data come from log-normal distribution

Table 4. Probability of Correct Selection of RML and RMKLD rule when the data come from Weibull distribution

	RML			RMKLD		
	(α=1.8,β=1.5)	(α=2,β=1.5)	(α=5,β=1.5)	(α=1.8,β=1.5)	(α=2,β=1.5)	(α=5,β=1.5)
Scheme1	0.7004	0.7012	0.6999	0.8074	0.8202	0.8130
Scheme2	0.5906	0.5909	0.5853	0.9990	0.9988	0.9992
Scheme3	0.6883	0.6922	0.6914	0.8115	0.8188	0.8170
Scheme4	0.6391	0.6461	0.6437	0.9636	0.9678	0.9650
Scheme5	0.6770	0.6881	0.6887	0.8229	0.8323	0.8259
Scheme6	0.7192	0.7249	0.7129	0.7604	0.7644	0.7609
Scheme7	0.6596	0.6559	0.6509	0.9183	0.9152	0.9184
Scheme8	0.7107	0.7165	0.7165	0.7694	0.7631	0.7681
Scheme9	0.6583	0.6649	0.6555	0.9094	0.9113	0.9028
Scheme10	0.7086	0.7093	0.7109	0.7738	0.7644	0.7741
Scheme11	0.7277	0.7187	0.7282	0.7350	0.7345	0.7292
Scheme12	0.8601	0.8583	0.8526	0.9286	0.9224	0.9266
Scheme13	0.7221	0.7241	0.7236	1.0000	1.0000	1.0000
Scheme14	0.8457	0.8467	0.8536	0.9409	0.9417	0.9403
Scheme15	0.7874	0.7857	0.7837	0.9970	0.9965	0.9974
Scheme16	0.8448	0.8503	0.8503	0.9503	0.9520	0.9535
Scheme17	0.8860	0.8808	0.7618	0.8954	0.8961	0.8991
Scheme18	0.8209	0.8194	0.7616	0.9838	0.9866	0.9865
Scheme19	0.8783	0.8847	0.7660	0.8992	0.8979	0.8957
Scheme20	0.8242	0.8335	0.7665	0.9816	0.9788	0.9798
Scheme21	0.8776	0.8819	0.7816	0.9186	0.9123	0.9127
Scheme22	0.8897	0.8832	0.8121	0.8776	0.8797	0.8818

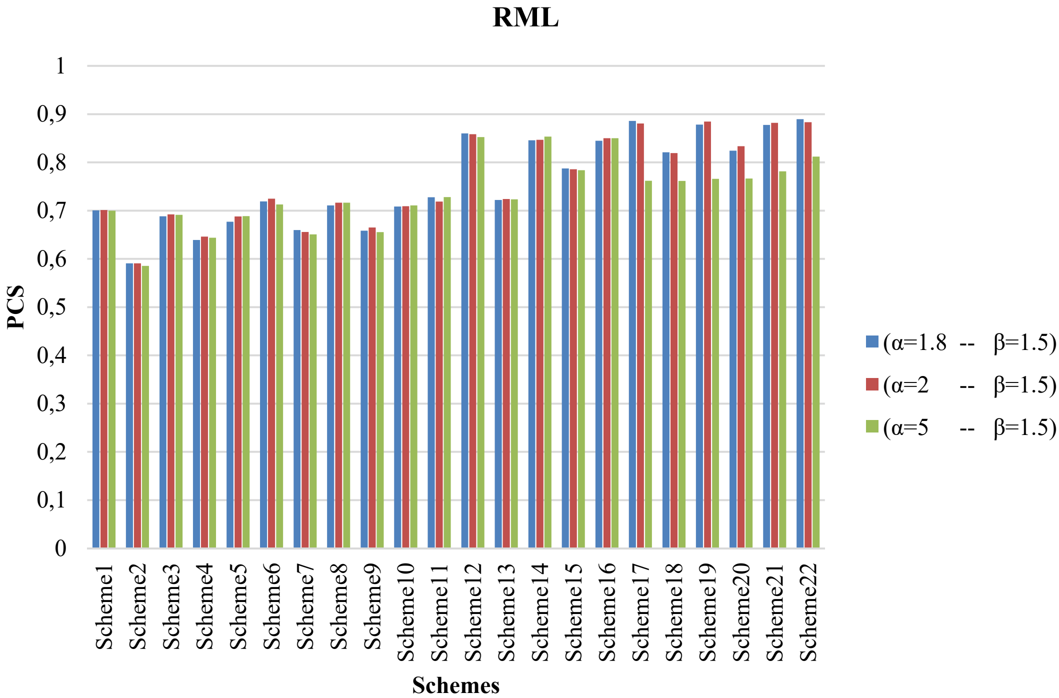

Figure 3. Probability of Correct Selection of RML rule when the data come from Weibull distribution

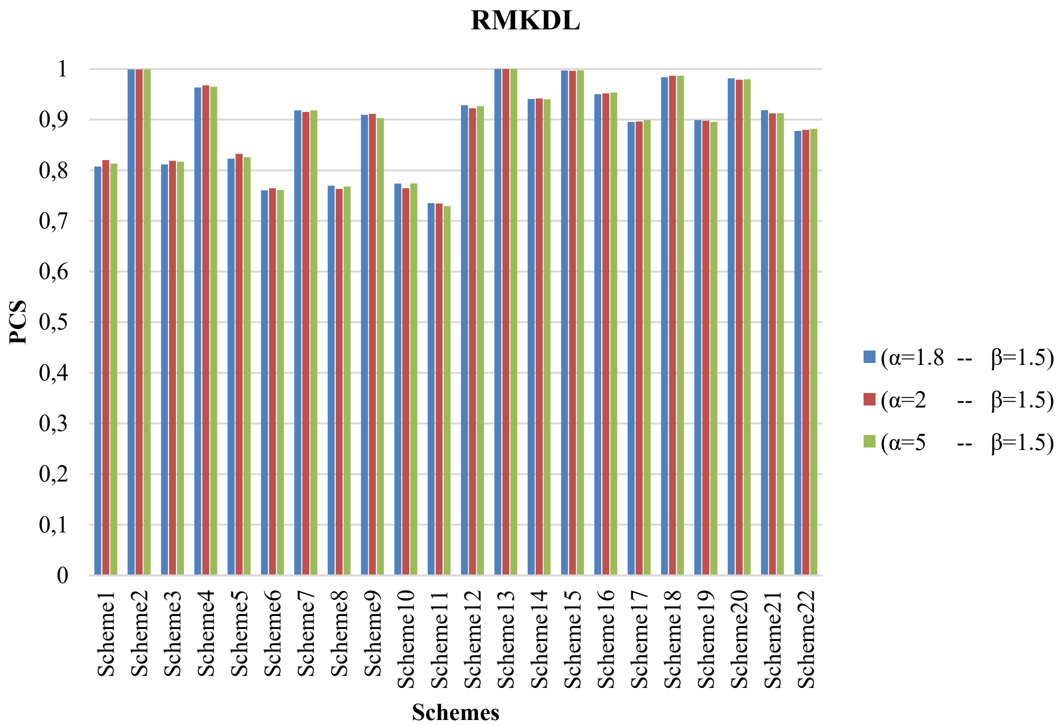

Figure 4. Probability of Correct Selection of RMKLD rule when the data come from Weibull distribution

4. Numerical Example

4.1. First Example

Let us consider the real data which is given by [30]. This data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the lifetime tests. The progressively Type-II right censored data are obtained from complete data and it is given by

17.88 28.92 33.00 41.52 42.12 45.60 48.80 51.84 51.96 54.12 55.56 67.80 68.44 68.64 68.88 84.12 93.12 98.64 105.12 105.84 127.92 128.04 173.40 with $r=(5,13*0)$ and $m=18$.

Discrimination procedure is performed to get decision whether the data come from a Weibull or a Log-Normal. Using R code with **nlm** command (it uses Newton type algorithm), ML estimates of lognormal parameters are obtained by $\hat{\mu}=4.3079$, $\hat{\sigma}=0.5886$, ML estimates of Weibull parameters are obtained by $\hat{\alpha}=2.1122$, $\hat{\beta}=95.3497$. Test statistics are calculated as RML=0.3321 and $D\left(f_{\hat{\theta}_1}, g_{\hat{\theta}_2}\right)=$ **0.1688** and $D\left(g_{\hat{\theta}_2}, f_{\hat{\theta}_1}\right)=$ **0.0924**.

Since the **RML=0.3321>0** then lognormal distribution is selected for modeling this real data.
On the other hand, since the **RMKLD=0.6028>0** then Weibull distribution is selected for modeling this real data.

4.2. Second Example

Let us consider well-known data in reliability theory. This data was analyzed by many authors included in [31] and [27]. The progressive Type-II right censored data is given by
0.19 0.78 0.96 1.31 2.78 4.85 6.50 7.35 with $r=$ (0, 0, 3, 0, 3, 0, 0, 5) and $m=8$.
Discrimination procedure is performed to get decision whether the data come from a Weibull or a Log-Normal. Using R code with **nlm** command (it uses Newton type algorithm), ML estimates of lognormal parameters are obtained by $\hat{\mu}=1.8821$, $\hat{\sigma}=1.6152$, ML estimates of Weibull parameters are obtained by $\hat{\alpha}=0.9745$, $\hat{\beta}=9.2253$. Test statistics are calculated as **RML=-0.1519** and $D\left(f_{\hat{\theta}_1}, g_{\hat{\theta}_2}\right)=$ **0.9369** and $D\left(g_{\hat{\theta}_2}, f_{\hat{\theta}_1}\right)=$ **0.1395**

Since the **RML=-0.1519<0** then Weibull distribution is selected for modeling this real data.
Since the **RMKLD=1.9042>0** then Weibull distribution is selected for modeling this real data.

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