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Coupled-Channel Analyses on $Ti^{46,48,50} + Sn^{124}$ Heav-ion Fusion Reactions

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Abstract: Heavy-ion fusion near the Coulomb barrier attract experimental and theoretical interest. The collisons are typically characterized by the presence of many open reaction channels. In the energies around the Coulomb barrier, the processes are elastic scattering, inelastic excitations and fusion operations of one or two nuclei. The fusion process is defined as the effect of one-dimensional barrier penetration model, taking scattering potential as the sum of Coulomb and proximity potential. We have performed heay-ion fusion reactions with coupled-channels (CC) calculations. CC formalism is carried out under barrier energy in heavy- ion fusion reactions. In this work fusion cross sections have been calculated and analyzed in detail for the three systems $Ti^{46,48,50} + Sn^{124}$ in the framework of CC approach (using the codes CCFULL[16], CCFUS [17] and CCDEF [18]) . Calculated results are compared with experimental data, including excitation of the projectile and target to the lowest $2^{+}$and 3^{-} states and with the datas computed from 'nrv'. CCDEF, CCFULL and 'nrv' explains the fusion reactions of heavy-ions very well. There is a good agreement between the calculated results with the experimental and nrv results [19].

Keywords: Heavy-ion reactions, coupled-channel calculations, sub-barrier fusion.

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Özet: Coulomb bariyeri yakınındaki ağır iyon füzyonu, deneysel ve teorik ilgi çekmektedir. Çarpışmalar tipik olarak birçok açık reaksiyon kanalı varlığı ile karakterize edilir. Coulomb bariyerinin etrafındaki enerjilerde, süreç, bir veya iki çekirdeğin elastik saçılma, inelastik uyarımlar ve füzyon operasyonlarıdır. Füzyon süreci, saçılma potansiyelini Coulomb ve yakınlık potansiyelinin toplamı olarak alan tek boyutlu bariyer penetrasyon modelinin etkisi olarak tanımlanır. Çiftlenmiş kanallar (CC) hesaplamaları ile ağır iyon füzyon reaksiyonları gerçekleştirdik. CC formalizmi, ağır füzyon reaksiyonlarında bariyer enerjisi altında gerçekleştirilir. Buçalışmada, füzyon tesir kesitleri CC hesaplamaları ile (CCFULL [16], CCFUS [17] ve CCDEF [18] kodları kullanılarak) $Ti^{46,48,50} + Sn^{124}$sistemleri için detaylı olarak incelenmiştir. Hesaplanan sonuçlar, deneysel veri ve 'nrv' de hesaplanan mermi ve hedef için 2^{+}ve 3^{-} uyarılmalarını içeren verilerle karşılaştırılmıştır. Hesaplanan Sonuçlar ile deneysel ve 'nrv' [19] sonuçları arasında iyi bir uyum mecvuttur.

Anahtar Kelimeler: Ağır-iyon reaksiyonları, çiftlenmiş kanal hesaplamaları, sub-bariyer füzyon.

1. INTRODUCTION

Heavy-ion collisions in low energy range at, above and below the Coulomb get attracts both experimentalists and theorists. It has been a very rich variety of phenomena for many years in Nuclear Physics. The fusion of two nuclei at very low energy is an example of tunnelling phenomena in nuclear physics. These reactions are not only of central important for stellar

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energy production and nucleosynthesis, but they also provide new insights into reaction dynamics and nuclear structure.

To analyze heavy-ion fusion cross sections above the Coulomb barrier, the inter-nuclear interaction, a combination of the repulsive, long range Coulomb, centrifugal potentials and the attractive, short range nuclear potential plays a major role.

The total potential attains a maximum value at a distance beyond the touching configuration where the repulsive Coulomb force and the attractive nuclear forces balance each other and when the energy of relative motion overcomes this potential barrier, the nuclei gets captured and fused.

Many different phenomenas take place in heavy-ion collisions depending on the bombarding energy, the impact parameter, the mass number of the target and projectile. We discuss here fusion reactions at energies near and below the Coulomb barrier. The fusion cross section in heavy-ion collisions at energies somewhat above the Coulomb barrier can be well accounted for by a simple potential model, which explicitly deals with only the relative distance between the projectile and target or a model supplemented by a friction [1]. After comprehensive theoretical as well as experimental studies, it is pretty good now set that the large enhancement of the fusion cross section is caused by the coupling of the relative motion between the colliding nuclei to other degrees of freedom, [10,11]. They are called channel-coupling effects. [1]

The aim of the present work to investigate the effectt of the coupled-channels caalculations using codes CCFULL, CCFUS, CCDEF and 'nrv'. The fusion excitation functions for the fusion of $Ti^{46,48,50}$ on Sn^{124} have been calculated using onde dimensional barrier penetration model, taking scattering potential as the sum of Coulomb and proximity potential and the calculated values of $Ti^{46,48,50}$ on Sn^{124} are compared with experimental data from 'nrv'[19]. Reduced reaction cross sections for the fusion have been also described.

2. COUPLED CHANNELS FORMALISM FOR HEAVY-ION FUSION REACTIONS

The coupled-channels calculatons are not only important for theoretical point of wiev, it is also necessary for obtaining new data. The starting point to figure fusion reactions at energies below and slightly above the barrier where the coupling effects are the strongest and the number of channels coupled are workable. At energies high above the barrier so many channels become involved that it is no longer possible to treat them individually. For simplicity we consider only coupling to inelastic channels, ignoring rearrangement processes such as transfer.

For a brief description of the coupled-channels formalism, the total Hamiltonian of the system can be written as;

$$\boldsymbol{H} = -\frac{\hbar^2}{2\mu}\nabla^2 + \boldsymbol{h}(\xi) + \boldsymbol{V_0}(\boldsymbol{r}) + \boldsymbol{V_{coup}}(\boldsymbol{r},\xi) \quad (1)$$

Where $h(\xi)$ is the internal Hamiltonian for the target nucleus here ξ stands for the internal dynamical variables. $V_{coup}(r,\xi)$ is the coupling term, introduces the coupling betweeen the relative motion and the internal degrees of freedom. $V_0(r)$ is the bare potential in the relative distance r. For the total wave function $\psi(r,\xi)$ the Schrödinger equation;

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \boldsymbol{h}(\xi) + \boldsymbol{V_0}(\boldsymbol{r}) + \boldsymbol{V_{coup}}(\boldsymbol{r},\xi)\right)\boldsymbol{\psi}(\boldsymbol{r},\xi) = \boldsymbol{E}\boldsymbol{\psi}(\boldsymbol{r},\xi) \quad (2)$$

The internal eigenstates $\phi_b(\xi)$ with eigenvalues can be presented as

$$(\xi)\boldsymbol{\phi_b}(\xi) = \epsilon_b\boldsymbol{\phi_b}(\xi) \quad (3)$$

and also matrix elements are

$$V_{bc}(r) = \int d\xi \phi_b^* \ (\xi)\, V_{coup}(r,\xi)\phi_c(\xi)\ . \qquad (4)$$

The coupling matrix elements V_{bc} will consist of Coulomb and nuclear components. In terms of the internal eigenstates the total wave function $\psi(r,\xi)$ is

$$\psi(r,\xi) = \sum_b \varphi_b(r)\, \phi_b(\xi) \qquad (5)$$

And finally for the channels wave function coupled-channels equations can be defined from the Schrödinger equation

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + V_{bb}(r) + \epsilon_b - E\right)\varphi_b(r) = \sum_{c(c\neq b)} V_{bc}(r)\, \varphi_c(r) \qquad (6)$$

This system of coupled Schrödinger equations should be modified according to the boundary condition since the solution of the event channel consists of an input and an output wave, while all other channels contain only outgoing waves. The outgoing waves's coefficients determine the cross-sections of the various reactions. In practical applications imaginary potentials are introduced into the coupled-channels equations to account for the bulk of the flux which is lost from the direct channels to the compound reaction. Such a model is useful for calculating direct reactions. The fusion cross section can also be calculated from the difference between the total flux lost from the entrance channel and the flux which appears in the direct reaction channels [6,7]. However, this approach is not suited to study the channel coupling effects on barrier penetration since imaginary potentials suppress the coupling which acts as the nuclei inter-penetrate. The artificial limitations resulting from using strongly absorbing imaginary potentials in the coupled equations can be removed by imposing an ingoing boundary condition (IWBC) to account for the flux which leads to fusion.

The IWBC form as propesed by Rawitscher [5] is

$$\varphi_b(r) \propto \frac{1}{\sqrt{k_b(r)}} exp\left[-i\int_{R_b}^{r} k_b(r')d\,r'\right]. \qquad (7)$$

k_b is the asymptotic wave number in the channel band,

R_b is the starting point of the integration inside the Coulomb barrier. The coupled equations solved under ingoing-wave boundary conditions provide a more realistic framework for describing fusion reactions. Within the coupled-channels formalism one determines the total reaction cross section and the total cross section in the excited channels. This difference is equal to the ingoing flux at R_b and is equated with the fusion cross section. The IWBC is applied at some point inside the barrier to obtain the S-matrices. The fusion cross section is then obtained as

$$\sigma = \frac{\pi}{k^2}\sum_l (2l+1)\left(1 - \sum_l |S_l(l)|^2\right) \qquad (8)$$

here the parameter l defines different scattering channels [7,20].

3. THE FUSION CROSS SECTION

At energies above the barrier and at higher energies Wong's barrier penetration model has been usually used for the fusion reactions, [14]. The results with this formula match well with the experimental result properly. Wong inverted-wave oscillator potentiates various obstacles for different partial waves of height E and frequency ω_l monitoring the Works [21-23]. Using the probability for the absorption partial wave given by Hill–Wheeler formula [19], for energy E_l, Wong arrived at the total cross section for the fusion of two nuclei by quantum mechanical penetration of simple one-dimensional potential barrier as[24]

$$\sigma = \frac{\pi}{k^2}\sum_l \frac{2l+1}{1+\exp[2\pi(E_l - E)/\hbar\omega_l]} \quad (9)$$

here ω_l is is the curvature of the inverted parabola.

In the region $l=0$ if we make some differences in parameters, we can get the reeaction cross section as Wong[5]

$$\sigma = \frac{R_B^2\hbar\omega_0}{2E}\ln\left\{1+\exp\left[\frac{2\pi(E-E_B}{\hbar\omega_0}\right]\right\}$$

The Reduced Reaction Cross Section

The coupled-channels method is quite useful in treating collective excitations enhanced in nuclear scattering. To make a comparision between excitation functions which have differences in reaction mechanism; such as different projectiles on the same target nucleus; the procedure is to eliminate the geometrical factors concerning different systems by reducing the cross section and the centre of mass energy. The reducing process is mainly occurs with the division of the cross section by πR_0^2, here R_0 is the barrier radius and division of energy by Coulomb barrier E_0. [4,9]

4. RESULTS AND DISCUSSIONS

The results of coupled-channels calculations are performed by using CCFULL, CCDEF, CCFUS and nrv codes and compared with the experimental data. In all coupled-channels codes calculations the lines represent calculations the vibrational couplings in projectile and target. The depth parameter V_0 and the surface diffuseness parameter a_0 (of the Wood-Saxon potentials), radius parameter r_0 have been computed and the values are shown in Table1.

In codes we include the lowest states for all the reactions, that is the 2^+ (quadrupole) and 3^- (octupole) states. The deformation parameters and excitation energy values for 3^- states are given in Table2 [19].

For $Ti^{46}+Sn^{124}$ reaction $V_0 = 85$, $a_0 =$ 0,64fm and $r_0 = 1{,}14$fm are the parameters.

For $Ti^{48}+Sn^{124}$ reaction $V_0 = 100$, $a_0 =$ 0,64fm and $r_0 = 1{,}1$fm

And for the reaction $Ti^{50}+Sn^{124}$ $V_0 = 90$, $a_0 = 0{,}63$fm and $r_0 = 1{,}15$fm.

We investigate here the roles of parameteres and which parameters are more effective for the reactions using different codes. The results of coupled-channels calculations are compared with the experimental data. The computed cross sections with codes show good agreement with 'nrv' and experimental data.

Table 1. List of depth parameter $\boldsymbol{V_0}$ and surface diffuseness parameter $\boldsymbol{a_0}$ for $\boldsymbol{Ti^{46,48,50}+Sn^{124}}$.

Reaction	$V_0(MeV)$	$a_0(fm)$
$Ti^{46}+Sn^{124}$	85	0,64
$Ti^{48}+Sn^{124}$	100	0,68
$Ti^{50}+Sn^{124}$	90	0,63

Table 2. Deformation parameters and excitation energies of $\boldsymbol{3^-}$ states of $\boldsymbol{Ti^{46}}$, $\boldsymbol{Ti^{48}}$, $\boldsymbol{Ti^{50}}$ projectiles[19].

Target	Deformation Parameter (β_3)	Excitation energy (E_3)
Ti^{46}	0,142	3,06
Ti^{48}	0,197	3,359
Ti^{50}	0,106	4,41

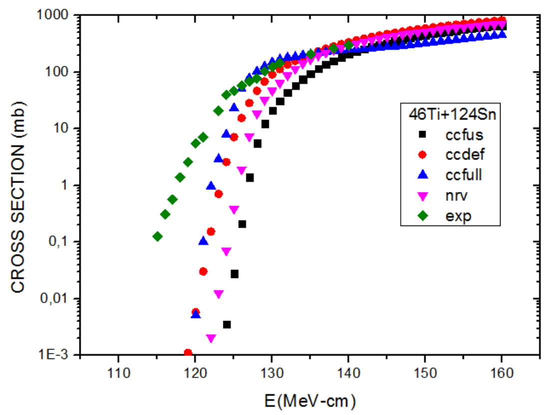

Figure 1. Fusion excitation functions for the reaction of $\boldsymbol{Ti^{46}+Sn^{124}}$ with CCFUS, CCDEF, CCFULL codes, the comparison with 'nrv' and experimental data.

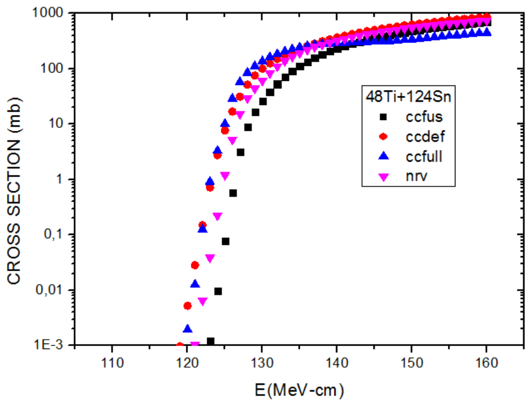

Figure 2. For the reaction $Ti^{48} + Sn^{124}$, the comparison of the coupled-channels calculations with CCFUS, CCDEF, CCFULL and 'nrv' data.

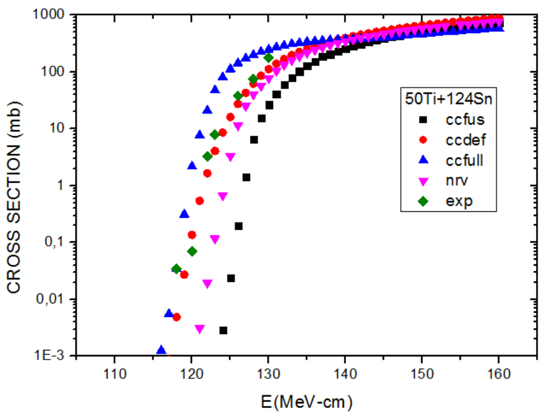

Figure 3. Fusion excitation functions with coupled-channels code CCFUS, CCDEF, CCFULL for the reaction Ti^{50} projectile on Sn^{124} target, figure shows the comparison of the codes's calculations with experimental and 'nrv' data.

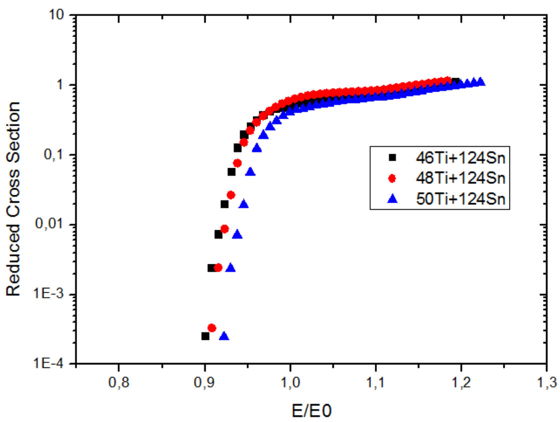

Figure 4. Reduced reaction cross sections for $Ti^{46,48,50} + Sn^{124}$ reactions.

It has been found that the most utilizable codes are CCFULL and CCDEF. CCFULL is a code that's for coupled-channels calculations with all order couplings for heavy-ion fusion reactions. It takes into account the effects of nonlinear couplings to all orders. And for CCDEF it can be noticed that, the difference from the codes using the coupled-channels method is that the projectile and the target nucleus are deformed. If parameters are meticulously calculated, the best fit to the experimental result is achieved with these codes. CCFUS needs less parameter according to others. So when we look at the graphics we see that it is out of the scale.

5. CONCLUSIONS

We have investigated the effect of coupled-channels for heavy-ion fusion reactions of the $Ti^{46,48,50} + Sn^{124}$ reactions using coupled-channels codes CCFULL, CCDEF and CCFUS. The codes use the scattering potential as the sum of Coulomb and proximity potentials. Calculated results are compared with experimental data, including excitation of the projectile and target to the lowest 2+ and 3− states and with the datas computed from 'nrv'. CCDEF, CCFULL and 'nrv' explains the fusion reactions of heavy-ions very well, while using the scattering potential as WOODS- SAXON volume potential with Akyuz-Winther parameters. It was observed that AW potential parameters are able to reproduce the experimentally observed fusion cross sections reasonably well for these systems. There is a good agreement between the calculated results with the experimental and nrv results.

We concluded that; below the barrier larger deformations corresponds to large sub barrier enhancement of fusion cross section. It can be added that the fusion process is as a tunelling process below the barrier.

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