Cumhuriyet Science Journal
CSJ

e-ISSN: 2587-246X
ISSN: 2587-2680

Cumhuriyet Sci. J., Vol.38-4, Supplement (2017) 77-85

Relationship Between Fuzzy Soft Topological Spaces and (X,τ_e) Parameter Spaces

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Received: 29.09.2017; Accepted: 22.11.2017 *http://dx.doi.org/10.17776/csj.340541*

Abstract: In this paper, the relation between fuzzy soft topological spaces and (X,τ_e) parameter spaces is introduced. After defining the parametrical property of fuzzy soft sets and we give some examples.

Keywords: Fuzzy Soft Set, Fuzzy Soft Topology, Parameter Spaces [2000] 03E72, 54C05, 54D30

Bulanık Esnek Topolojik Uzaylar ve (X,τ_e) Parametre Uzayları Arasındakı İlişkiler

Özet: Bu makalede bulanık esnek topolojik uzaylar ile (X,τ_e) parametre uzayları arasındaki ilişkilere giriş yapıldı. Bulanık esnek kümelerde parametrik özellik tanımlandı ve örnekler verildi.

Anahtar Kelimeler: Bulanık esnek küme, bulanık esnek topoloji, parametre uzayları [2000] 03E72, 54C05, 54D30

INTRODUCTION

The notion of the fuzzy soft set, which is the combination of fuzzy sets and soft sets, was introduced by Maji et. Al.[16] in 2001. In 2011, Tanay and Kandemir [22] defined the topological structure of fuzzy soft sets. In this work Tanay and Kandemir gave basic topological definition such as neighborhood of a fuzzy soft set, interior fuzzy soft set, fuzzy soft basis and fuzzy soft subspace topology. Afterwards, a lot of researches studied this theory in several area of mathematics such as topology in [1, 3, 23, 8, 9], algebraic structures in [11, 2] and decision making in [10, 14, 20]. On the other hand, Varol and Aygun [3] gave an example of (X,τ_e) parameter spaces notion in 2012.

In this study, the relation between concepts on fuzzy soft topological spaces and concepts of parameter spaces is introduced. Then parametrical property of concept for fuzzy soft topological spaces which is similiar to topological and hereditical property on classical topological spaces is defined and its examples are given.

1. Preliminaries:

Definition 1 *[21] Let* $A \subseteq E$. *A fuzzy soft set* f_A *over universe* X *is mapping from the parameter set* E *to* I^X, *i.e.* $f_A : E \to I^X$, *where* $f_A(e) \neq 0_X$ *if* $e \in A \subset E$ *and* $f_A(e) = 0_X$ *if* $e \notin A$, *where* 0_X *denotes empty fuzzy set on* X.

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Definition 2 *[21] Let $FS(X,E)$ denote the family of all fuzzy soft sets on X. If $f_A, g_B \in FS(X,E)$, then some basic set operations for fuzzy soft sets are given by Roy and Samanta as follows:*

(1) The fuzzy soft set $f_\varnothing \in FS(X,E)$ is called null fuzzy soft set if $f_\varnothing(e) = 0_X$ for all $e \in E$ and denoted by $\tilde{0}_E$.

(2) The fuzzy soft set $f_E \in FS(X,E)$ is called universal fuzzy soft set if $f_E(e) = 1_X$ for all $e \in E$ and denoted by $\tilde{1}_E$.

(3) f_A is called a fuzzy soft subset of g_B if $f_A(e) \le g_B(e)$ for all $e \in E$ and denoted by $f_A \tilde{\subseteq} g_B$.

(4) f_A and g_B are said to be equal if $f_A \tilde{\subseteq} g_B$ and $g_B \tilde{\subseteq} f_A$ and denoted by $f_A = g_B$.

(5) The union of f_A and g_B is also a fuzzy soft set h_C, defined by $h_C(e) = f_A(e) \vee g_B(e)$ for all $e \in E$, where $C = A \cup B$. Here, we write $h_C = f_A \tilde{\cup} g_B$.

(6) The intersection of f_A and g_B is also a fuzzy soft set h_C, defined by $h_C(e) = f_A(e) \wedge g_B(e)$ for all $e \in E$, where $C = A \cap B$. Here, we write $h_C = f_A \tilde{\cap} g_B$.

Definition 3 *[22] Let $f_A \in FS(X,E)$. The complement of f_A, denoted by f_A^c, is a fuzzy soft set defined by $f_A^c(e) = 1 - f_A(e)$ for every $e \in E$.*

Let us call f_A^c to be fuzzy soft complement function of f_A. Clearly $(f_A^c)^c = f_A$, $(\tilde{1}_E)^c = \tilde{0}_E$ and $(\tilde{0}_E)^c = \tilde{1}_E$.

Definition 4 *[13] Let $FS(X,E)$ and $FS(Y,K)$ be the families of all fuzzy soft sets over X and Y, respectively. Let $u: X \to Y$ and $p: E \to K$ be two functions. Then f_{up} is called a fuzzy soft mapping from X to Y and denoted by $f_{up}: FS(X,E) \to FS(Y,K)$.*

(1) Let $f_A \in FS(X,E)$ then the image of f_A under the fuzzy soft mapping f_{up} is the fuzzy soft set over Y and defined by $f_{up}(f_A)$, where

$$f_{up}(f_A)(k)(y) = \begin{cases} \bigvee_{x \in u^{-1}(y)} \left(\bigvee_{e \in p^{-1}(k)} f_A(e) \right)(x) & \text{, if } u^{-1}(y) \neq \varnothing \text{ and } p^{-1}(k) \neq \varnothing; \\ 0_Y & \text{, otherwise.} \end{cases}$$

(2) Let $g_B \in FS(Y,K)$ then the preimage of g_B under the fuzzy soft mapping f_{up} is the fuzzy soft set over X and defined by $f_{up}^{-1}(g_B)$, where

$$f_{up}^{-1}(g_B)(e)(x) = \begin{cases} g_B(p(e))(u(x)) & \text{, for } p(e) \in B; \\ 0_X & \text{, otherwise.} \end{cases}$$

If u and p are injective, then the fuzzy soft mapping f_{up} is said to be injective. If u and p are surjective, then the fuzzy soft mapping f_{up} is said to be surjective. The fuzzy soft mapping f_{up} is called constant if u and p are constant.

Definition 5 *[1] The fuzzy soft set* $f_A \in FS(X,E)$ *is called fuzzy soft point if* $A=\{e\} \subseteq E$ *and* $f_A(e)$ *is a fuzzy point in* X*, i.e. there exists* $x \in X$ *such that* $f_A(e)(x)=\alpha$ $(0<\alpha \le 1)$ *and* $f_A(e)(y)=0$ *for all* $y \in X-\{x\}$*. We denote this fuzzy soft point* $f_A = e_x^\alpha = \{(e, x_\alpha)\}$.

Definition 6 *[1] Let* e_x^α*,* $f_A \in FS(X,E)$*. We say that* $e_x^\alpha \tilde{\in} f_A$*, read as,* e_x^α *belongs to the fuzzy soft set* f_A *if for the element* $e \in A$*,* $\alpha \le f_A(e)(x)$.

Evidently, every fuzzy soft set f_A can be expresssed as the union of all the fuzzy soft points which belong to f_A.

Definition 7 *[1] Let* $f_A, g_B \in FS(X,E)$*.* f_A *is said to be soft quasi-coincident with* g_B *and denoted by* $f_A q g_B$ *if there exist* $e \in E$ *and* $x \in X$ *such that* $f_A(e)(x)+g_B(e)(x)>1$.

If f_A is not soft quasi-coincident with g_B, then we write $f_A \bar{q} g_B$.

Definition 8 *(see [22, 21]) A fuzzy soft topological space is a pair* (X,τ) *where* X *is a nonempty set and* τ *is a family of fuzzy soft sets over* X *satisfying the following properties:*

(1) $\tilde{0}_E, \tilde{1}_E \in \tau$

(2) If f_A, $g_B \in \tau$, then $f_A \tilde{\cap} g_B \in \tau$

(3) If $f_{A_i} \in \tau \;\; \forall i \in J$, then $\tilde{\bigcup}_{i \in J} f_{A_i} \in \tau$.

Then τ is called a topology of fuzzy soft sets on X. Every member of τ is called fuzzy soft open g_B is called fuzzy soft closed in (X,τ) if $(g_B)^c \in \tau$.

Definition 9 *[1] Let* (X,τ) *be a fuzzy soft topological space and* $f_A \in FS(X;E)$*. The fuzzy soft closure of* f_A *denoted by* $\overline{f_A}$ *is the intersection of all fuzzy soft closed supersets of* f_A

Definition 10 *[1] Let* (X,τ) *be a fuzzy soft topological space and* $f_A \in FS(X;E)$*. The fuzzy soft interior of* f_A *denoted by* f_A° *is the union of all fuzzy soft open subsets of* f_A.

Definition 11 *[1] A fuzzy soft set* f_A *in* $FS(X,E)$ *is called Q-neighborhood (briefly, Q-nbd) of* g_B *if and only if there exists a fuzzy soft open set* h_C *in* τ *such that* $g_B q h_C \tilde{\subseteq} f_A$*. All the Q-nbds of fuzzy soft point of* e_x^α *are shown as* $N_q(e_x^\alpha)$.

Definition 12 *[22] A fuzzy soft set* g_B *in a fuzzy soft topological space* (X,τ) *is called a fuzzy soft neighborhood (briefly: nbd) of the fuzzy soft set* f_A *if there exists a fuzzy soft open set* h_C *such that* $f_A \tilde{\subseteq} h_C \tilde{\subseteq} g_B$.

Definition 13 *[1] Let* (X,τ_1) *and* (Y,τ_2) *be two fuzzy soft topological spaces. A fuzzy soft mapping* $f_{up}:(X,\tau_1) \to (Y,\tau_2)$ *is called fuzzy soft continuous if* $f_{up}^{-1}(g_B) \in \tau_1$ *for all* $g_B \in \tau_2$.

Example 1 *[3] Let* (X,τ) *be a fuzzy soft topological space. Then the families* $\tau_e = \{f_A(e) : f_A \in \tau\}$ *are fuzzy topologies on* X *for all* $e \in E$.

Throughout this study, without the loss of generality parameter spaces is used for (X,τ_e) fuzzy topological spaces.

Definition 14 *[22] Let* (X,τ) *be a fuzzy soft topological space and* B *be a subfamily of* τ*. If every element of* τ *can be written as a arbitrary* *fuzzy soft union of some elements of* B, *then* B *is called a fuzzy soft basis for fuzzy soft topology* τ.

Theorem 1 *Let* (X,τ) *be a fuzzy soft topological space and* B *be a base of* τ. *Then* $B_e=\{f_A(e): f_A\in B\}$ *is a base of* τ_e *for* $e\in E$.

Proof. Let B be a base for τ and $f_A(e)\in\tau_e$. Then $f_A\in\tau$. Since B is the basis τ, there exist a $B^{'}\subset B$ such as

$$f_A=\tilde{\bigcup}_{g_B\in B^{'}} g_B$$

. If we choose $B_e^{'}=\{f_A(e): f_A\in B^{'}\}\subset B_e$, this gives

$$f_A(e)=\left(\tilde{\bigcup}_{g_B\in B^{'}} g_B\right)(e)$$

$$=\bigvee_{g_B\in B^{'}} g_B(e)=\bigvee_{g_B(e)\in B_e^{'}} g_B(e)$$

. That shows B_e is a basis τ_e.

The converse of this theorem does not usually hold.

Example 2 *Let* $E=\{e_1,e_2,e_3\}$ *be a set of parameters and* $X=\{x_1,x_2,x_3\}$ *be a initial universe. If*

$$f_{A_1}=\{(e_1,\{x_1^{0,2},x_2^{0,4},x_3^{0,7}\}),(e_2,\{x_1^{0,5},x_2^{0,7},x_3^{1}\}),(e_3,1_X)\}$$

$$f_{A_2}=\{(e_1,\{x_1^{0,4},x_2^{0,2},x_3^{0,6}\}),(e_2,\{x_1^{0,3},x_3^{0,6}\}),(e_3,\{x_1^{0,5},x_2^{0,4},x_3^{0,6}\}\}$$

$$f_{A_3}=\{(e_1,\{x_1^{0,2},x_2^{0,2},x_3^{0,6}\}),(e_2,\{x_1^{0,3},x_3^{0,6}\}),(e_3,\{x_1^{0,5},x_2^{0,4},x_3^{0,6}\})\}$$

$$f_{A_4}=\{(e_1,\{x_1^{0,4},x_2^{0,4},x_3^{0,7}\}),(e_2,\{x_1^{0,5},x_2^{0,7},x_3^{1}\}),(e_3,1_X)\}$$

,the family, $\tau=\{\tilde{0}_E,\tilde{1}_E,f_{A_1},f_{A_2},f_{A_3},f_{A_4}\}$, is a fuzzy soft topological space. Then

$$\tau_{e_1}=\{0_X,1_X,\{x_1^{0,2},x_2^{0,4},x_3^{0,7}\},\{x_1^{0,4},x_2^{0,2},x_3^{0,6}\},\{x_1^{0,2},x_2^{0,2},x_3^{0,6}\},\{x_1^{0,4},x_2^{0,4},x_3^{0,7}\}\}$$

$$\tau_{e_2}=\{0_X,1_X,\{x_1^{0,5},x_2^{0,7},x_3^{1}\},\{x_1^{0,3},x_3^{0,6}\}\}$$

$$\tau_{e_3}=\{0_X,1_X,\{x_1^{0,5},x_2^{0,4},x_3^{0,6}\}\}.$$

Moreover, if

$$g_{B_1}=\{(e_1,\{x_1^{0,4},x_2^{0,4},x_3^{0,7}\}),(e_2,\{x_1^{0,3},x_3^{0,6}\}),(e_3,\{x_1^{0,5},x_2^{0,4},x_3^{0,6}\})\}$$

$$g_{B_2}=\{(e_1,\{x_1^{0,2},x_2^{0,2},x_3^{0,6}\}),(e_2,\{x_1^{0,5},x_2^{0,7},x_3^{1}\})\}$$

$$g_{B_3}=\{(e_1,\{x_1^{0,4},x_2^{0,2},x_3^{0,6}\}),(e_2,\{x_1^{0,5},x_2^{0,7},x_3^{1}\})\}$$

$$g_{B_4}=\{(e_1,\{x_1^{0,2},x_2^{0,4},x_3^{0,7}\}),(e_2,\{x_1^{0,3},x_3^{0,6}\})\},$$

although, for the fuzzy topologies τ_{e_1},τ_{e_2} and τ_{e_3} the families,

$$B_{e_1}=\{\{x_1^{0,2},x_2^{0,4},x_3^{0,7}\},\{x_1^{0,4},x_2^{0,2},x_3^{0,6}\},\{x_1^{0,2},x_2^{0,2},x_3^{0,6}\},\{x_1^{0,4},x_2^{0,4},x_3^{0,7}\}\}$$

$$B_{e_2}=\{\{x_1^{0,5},x_2^{0,7},x_3^{1}\},\{x_1^{0,3},x_3^{0,6}\}\}$$

$$B_{e_3}=\{\{x_1^{0,5},x_2^{0,4},x_3^{0,6}\}\}$$

are basis, $B=\{g_{B_1},g_{B_2},g_{B_3},g_{B_4}\}$ is not a base of τ.

Theorem 2 *Let* (X,τ) *be a fuzzy soft topological space.* $e_x^{\alpha}\,\tilde{\in}\,\overline{f_A}\Leftrightarrow x_\alpha\in\overline{f_A(e)}$

Proof. $(\Rightarrow:)$ Let $e_x^{\alpha}\,\tilde{\in}\,\overline{f_A}$. Suppose that $x_\alpha\notin\overline{f_A(e)}$, then for $\exists T\in N_q(x_\alpha)$, $T\bar{q}f_A(e)$. Since $T\in N_q(x_\alpha)$, then there exist $K\in\tau_e$ such that $x_\alpha qK\le T$. Hence $\alpha+K(x)>1$, $K(x)\le T(x)$ for all $x\in X$ and $T(x)+f_A(e)(x)\le 1$. On the other hand, since

$K \in \tau_e$, then there exist $g_B \in \tau$ such that $g_B(e) = K$. Therefore we get $\alpha + g_B(e)(x) > 1$, $g_B(e)(x) \le T(x)$ for all $x \in X$ and $g_B(e)(x) + f_A(e)(x) \le 1$. Consequently, $g_B \in N_q(e_x^\alpha)$ and $g_B \overline{q} f_A$ obtained which is a contradiction.

($\Leftarrow$) Let $x_\alpha \in \overline{f_A(e)}$. Suppose that $e_x^\alpha \tilde{\notin} \overline{f_A}$, then $g_B \overline{q} f_A$ for $\exists g_B \in N_q(e_x^\alpha)$. Hence, $e_x^\alpha q h_C \tilde{\subseteq} g_B$ and $g_B(e)(x) + f_A(e)(x) \le 1$ for all $e \in E$ and $x \in X$. Then $\alpha + h_C(e)(x) > 1$ and $h_c(e)(x) \le g_B(e)(x)$ for all $e \in E$ and $x \in X$. Since $h_C(e) \in \tau_e$, $h_C(e) \in N_q(x_\alpha)$ and $h_C(e)(x) + f_A(e)(x) \le 1$ for all $x \in X$. Consequently, $x_\alpha \notin \overline{f_A(e)}$ is obtained which is a contradiction.

Theorem 3 *Let (X,τ) and (Y,υ) be fuzzy soft topological spaces. Then $u_p : FS(X,E) \to FS(Y,K)$ is fuzzy soft continuous if and only if $u : (X,\tau_e) \to (Y,\upsilon_{p(e)})$ are fuzzy continuous for all $e \in E$.*

Proof. Let $u_p : FS(X,E) \to FS(Y,K)$ is a fuzzy soft continuous and $G \in \upsilon_{p(e)}$ for $e \in E$. Then there exists $g_B \in \upsilon$ such that $G = g_B(p(e))$ .Moreover

$u^{-1}(G)(x) = G(u(x)) = g_B(p(e))(u(x))$

$= u_p^{-1}(g_B)(e)(x)$ for all $x \in X$. Then it is clear that $u^{-1}(G)$ and $u_p^{-1}(g_B)(e)$ are same fuzzy sets. Since u_p fuzzy soft continuous, then $u_p^{-1}(g_B) \in \tau$. Hence $u_p^{-1}(g_B)(e) = u^{-1}(G) \in \tau_e$.Therefore $u : (X,\tau_e) \to (Y,\upsilon_{p(e)})$ are fuzzy continuous for all $e \in E$.

Conversely, let $u : (X,\tau_e) \to (Y,\upsilon_{p(e)})$ be fuzzy continuous for all $e \in E$ and $g_B \in \upsilon$. Since $g_B(p(e)) \in \upsilon_{p(e)}$ and $u : (X,\tau_e) \to (Y,\upsilon_{p(e)})$ are fuzzy continuous for all $e \in E$, $u^{-1}(g_B(p(e))) \in \tau_e$.Hence $u^{-1}(g_B) = \{(e, u^{-1}(g_B(p(e))) : e \in E\} \in \tau$. Then u_p is fuzzy soft continuous.

Example 3 *Let $E = \{e_1, e_2\}$ and $K = \{k_1, k_2, k_3\}$ be parameter set $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3\}$ be universal sets and $u_p : FS(X,E) \to FS(Y,K)$ be fuzzy soft function, where $u : X \to Y$, $p : E \to K$ are functions such that $u(x_1) = y_1, u(x_2) = y_3$, $u(x_3) = y_1$ and $p(e_1) = k_2$, $p(e_2) = k_1$. If we take $f_A = \{(e_1, \{x_1^{0,5}, x_2^{0,8}, x_3^{0,5}\}), (e_2, \{x_1^{0,4}, x_2^{0,7}, x_3^{0,4}\})\}$ and $g_B = \{(k_1, \{y_1^{0,4}, y_2^{0,5}, y_3^{0,7}\}), (k_2, \{y_1^{0,5}, y_3^{0,8}\}), (k_3, \{y_1^1, y_2^{0,2}, y_3^{0,6}\})\}$, then $\tau = \{f_\varnothing, f_E, f_A\}$ and $V = \{f_\varnothing, f_K, g_B\}$, (X,τ) and (Y,υ) are two topological spaces. And also $u_p^{-1}(g_B)$ is a h_C fuzzy soft set on X and*

$$h_C(e_1)(x_1) = g_B(p(e_1))(u(x_1)) = g_B(k_2)(y_1) = 0{,}5$$
$$h_C(e_1)(x_2) = g_B(p(e_1))(u(x_2)) = g_B(k_2)(y_3) = 0{,}8$$
$$h_C(e_1)(x_3) = g_B(p(e_1))(u(x_3)) = g_B(k_2)(y_1) = 0{,}5$$
$$h_C(e_2)(x_1) = g_B(p(e_2))(u(x_1)) = g_B(k_1)(y_1) = 0{,}4$$
$$h_C(e_2)(x_2) = g_B(p(e_2))(u(x_2)) = g_B(k_1)(y_3) = 0{,}7$$
$$h_C(e_2)(x_3) = g_B(p(e_2))(u(x_3)) = g_B(k_1)(y_1) = 0{,}4$$

.Then we have $h_C = u_p^{-1}(g_B) = \{(e_1, \{x_1^{0,6}, x_2^{0,9}, x_3^{0,6}\})\ (e_2, \{x_1^{0,4}, x_2^{0,7}, x_3^{0,4}\}), (e_3, \{x_2^{0,1}\})\} = f_A$. This shows that u_p is fuzzy soft continuous. On the other hand, since $f_A(e_1) = \{x_1^{0,5}, x_2^{0,8}, x_3^{0,5}\}$, $f_A(e_2) = \{x_1^{0,4}, x_2^{0,7}, x_3^{0,4}\}$, $g_B(k_1) = \{y_1^{0,4}, y_2^{0,5}, y_3^{0,7}\}$,

$g_B(k_2)=\{y_1^{0,5}, y_3^{0,8}\}$ and

$g_B(k_3)=\{y_1^{1}, y_2^{0,2}, y_3^{0,6}\}$

, then the families of fuzzy soft sets

$\tau_{e_1}=\{0_E, 1_E, f_A(e_1)\}$,

$\tau_{e_2}=\{0_E, 1_E, f_A(e_2)\}$,

$V_{k_1}=\{0_K, 1_K, g_B(k_1)\}$,

$V_{k_2}=\{0_K, 1_K, g_B(k_2)\}$,

$V_{k_3}=\{0_K, 1_K, g_B(k_3)\}$

are topological spaces. Since $u^{-1}(g_B(k_1))=f_A(e_2)$ and $u^{-1}(g_B(k_2))=f_A(e_1)$, then the functions $u:(X,\tau_{e_1})\to(Y,\upsilon_{k_2})$ and, $u:(X,\tau_{e_2})\to(Y,\upsilon_{k_1})$ are fuzzy continuous.

Definition 15 *[7] A fuzzy topological space (X,τ) is compact iff each open cover has a finite subcover.*

Definition 16 *[19] A fuzzy soft topological space (X,τ) is said to be fuzzy soft compact if each cover of $\tilde{1}_E$ by fuzzy soft open sets over X has a finite subcover.*

Theorem 4 *Let (X,τ) be a fuzzy soft topological space. If (X,τ) is fuzzy soft compact, then (X,τ_e) fuzzy compact for all $e\in E$.*

Proof. Let (X,τ) be a fuzzy soft topological space and $U_{e_i}=\{f_{A_i}(e_i): f_{A_i}\in\tau\}$ be a open cover of τ_{e_i} for all $e_i\in E$. Then $\vee_{i\in I} f_{A_i}(e_i)=X$. Therefore, the family $=\{f_{A_i}\}_{i\in I}$ is a fuzzy soft open cover of $\tilde{1}_E$. Since (X,τ) is fuzzy soft compact, there exist $U^*=\{f_{A_i}\}_{i=1}^n$ finite subcover of U. Hence, this $U_{e_i}^*=\{f_{A_i}(e_i)\}_{i=1}^n$ family, which is constructed by some of f_{A_i}, is a finite fuzzy cover of X for all $e_i\in E$. Therefore (X,τ_{e_i}) is fuzzy compact for all $e_i\in E$.

The converse of this theorem does not usually hold.

Definition 17 *Let (X,τ) be a fuzzy soft topological space. A property of (X,τ) is said to be parametrical if for all $e\in E$, we have that parameter spaces (X,τ_e) also has that property.*

Corollary 1 *Fuzzy soft compactness is a parametrical property.*

Example 4 *Let $X=\{x_1,x_2,x_3\}$ be a finite initial universe, $E=\{e_1,e_2,...\}$ be an infinite set of parameters and $f_{A_k}=\{(e_1,\{x_1^{\frac{1}{k}}, x_2^{\frac{1}{k}}, x_3^{\frac{1}{k}}\}),(e_2,\{x_1^{\frac{1}{k-1}}, x_2^{\frac{1}{k-1}}, x_3^{\frac{1}{k-1}}),...,(e_k,\{x_1^1,x_2^1,x_3^1\})\}$. Then the family $\tau=\{f_{A_k}:k=1,2,...,n\}\cup\{\tilde{0}_E,\tilde{1}_E\}$ is a fuzzy soft topological space on X. Therefore, the family $U=\{f_{A_k}:k=1,2,...,n\}$ is a open cover of $\tilde{1}_E$ but this cover does not have any finite cover. So, (X,τ) is not a compact fuzzy soft topological space. On the other hand the families $\tau_{e_i}=\{F_{A_k}(e_i):k=1,2,...,n\}\cup\{1_X,0_X\}=\{\{x_1^1,x_2^1,x_3^1\}\ \{x_1^{\frac{1}{2}}, x_2^{\frac{1}{2}}, x_3^{\frac{1}{2}}\},...\}\cup\{0_X\}$ are compact on X.*

Theorem 5 *Let X be any initial universe and E be a finite set of parameters. Then (X,τ_{e_i}) is compact for all $e_i\in E$ if and only if (X,τ) is compact.*

Proof. ($\Rightarrow$:) Let (X,τ_{e_i}) be compact for all $e_i\in E$ .and $U=\{f_{A_k}: f_{A_k}\in\tau, k\in I\}$ be a open cover of (X,τ). Then the families

$U_{e_i} = \{ f_{A_k}(e_i) : f_{A_k} \in U \}$ are fuzzy open covers of (X, τ_{e_i}) for all $e_i \in E$. Since (X, τ_{e_i}) spaces are compact for all $e_i \in E$, there exist finite subcovers of U_{e_i} like $U^*_{e_i}$. That is $1_X = \vee_{finite} f_{A_{k_i}}(e_i)$. Therefore for all $e_i \in E$, there exist $f_{A_{k_i}}$ finite number of sets and the family $\{ f_{A_{k_i}} \}$, which covers $\tilde{1}_E$, is also finite. Consequently, (X, τ) is compact.

($\Leftarrow$) Obvious from Theorem 4.

Definition 18 *[12] A fuzzy topological space X is said to be disconnected if* $1_X = A \vee B$, *where* A *and* B *are non-empty open fuzzy sets in* X *such that* $A \wedge B = 0_X$.

Definition 19 *[15] Let* (X, τ) *be a fuzzy soft topological space.* (X, τ) *is disconnected if and only if there exist fuzzy soft open sets* f_A *and* g_B *which are not* $\tilde{0}_E$ *such that* $\tilde{1}_E = f_A \tilde{\cup} g_B$ *and* $f_A \tilde{\cap} g_B = \tilde{0}_E$.

Theorem 6 *Let* (X, τ) *be a fuzzy soft topological space and* (X, τ) *is disconnected. Then* (X, τ_e) *is disconnected for all* $e \in E$.

Proof. Let (X, τ) be disconnected. Then there exist fuzzy soft open sets f_A and g_B such that $\tilde{1}_E = f_A \tilde{\cup} g_B$ *and* $f_A \tilde{\cap} g_B = \tilde{0}_E$. Therefore, $1_X = f_A(e) \vee G_B(e)$ and $f_A(e) \wedge g_B(e) = 0_X$ for all $e \in E$. Moreover, $f_A(e)$ and $g_B(e)$ are also open sets on (X, τ_e). Consequently, (X, τ_e) is disconnected for all $e \in E$.

The converse of this theorem does not usually hold.

Theorem 7 *Let* (X, τ) *be a fuzzy soft topological space. If* (X, τ_e) *are disconnect for all* $e \in E$ *same* f_A *and* g_B *then* (X, τ) *is a disconnect.*

Corollary 2 *Fuzzy soft disconnectedness is a parametrical property.*

Example 5 *Let* $X = \{x_1, x_2, x_3\}$ *be a initial universe,* $E = \{e_1, e_2\}$ *be a set of parameters and*

$f_{A_1} = \{(e_1, \{x_1^1, x_2^1\}), (e_2, \{x_2^1, x_3^1\})\}$,

$f_{A_2} = \{(e_1, \{x_3^1\}), (e_2, \{x_1^{0,5}, x_2^{0,7}, x_3^{0,4}\})\}$,

$f_{A_3} = \{(e_1, \{x_1^{0,4}, x_2^{0,7}\}), (e_2, \{x_1^1\})\}$,

$f_{A_4} = \{(e_2, \{x_2^{0,7}, x_3^{0,4}\})\}$,

$f_{A_5} = \{(e_1, \{x_1^{0,4}, x_2^{0,7}\})\}$,

$f_{A_6} = \{(e_2, \{x_1^{0,5}\})\}$,

$f_{A_7} = \{(e_1, 1_X), (e_2, \{x_1^{0,5}, x_2^1, x_3^1\})\}$,

$f_{A8} = \{(e_1, \{x_1^1, x_2^1\}), (e_2, 1_X)\}$,

$f_{A_9} = \{(e_1, \{x_1^{0,4}, x_2^{0,7}, x_3^1\}), (e_2, \{x_1^1, x_2^{0,7}, x_3^{0,4}\})\}$.

Then the family $\tau = \{ f_{A_1}, f_{A_2}, f_{A_3}, f_{A_4}, f_{A_5}, f_{A_6}, f_{A_7}, f_{A_8}, f_{A_9}, \tilde{0}_E, \tilde{1}_E \}$ is a fuzzy soft topological space on X. The families

$$\tau_{e_1} = \{\{x_1^1, x_2^1\}, \{x_3^1\}, \{x_1^{0,4}, x_2^{0,7}\}, \{x_1^{0,4}, x_2^{0,7}, x_3^1\}, 1_X, 0_X\},$$

$$\tau_{e_2} = \{\{x_2^1, x_3^1\}, \{x_1^{0,5}, x_2^{0,7}, x_3^{0,4}\}, \{x_1^1\}, \{x_2^{0,7}, x_3^{0,4}\}, \{x_1^{0,5}\}, \{x_1^{0,5}, x_2^1, x_3^1\}, \{x_1^1, x_2^{0,7}, x_3^{0,4}\}, 1_X, 0_X\}$$

are fuzzy topological spaces. Although τ_{e_1} and τ_{e_2} are disconnceted, (X, τ) is a connected fuzzy soft topological space.

Proof. Let $f_A, g_B \in \tau$ and for all $e \in E$, $f_A(e) \vee g_B(e) = 1_X$ and $f_A(e) \wedge g_B(e) = 0_X$. Then we have $\tilde{1}_E = f_A \tilde{\cup} g_B$ *and* $f_A \tilde{\cap} g_B = \tilde{0}_E$. Therefore, (X, τ) is a disconnect fuzzy topological space.

CONCLUSION *In the present work, we have continued to study of fuzzy soft topological spaces. We introduced the relation between concepts of fuzzy soft topological spaces and concepts of parameter spaces and parametrical properties. We hope that the findings in this paper will help researcher and literature.*

Acknowledgement *The author is gratefull for financial support from the Research Fund of Cumhuriyet University under grand no: F-533*

REFERENCES

[1]. Atmaca S. and Zorlutuna İ. *On fuzzy soft topological spaces*, Ann. Fuzzy Math. Inform. 5 (2), 377-386, 2013

[2]. Aygünoglu A. and Aygün H. *Introduction to fuzzy soft groups*, Comput. Math. Appl. 58, 1279-1286, 2009.

[3]. Varol B. P. and Aygün H. *Fuzzy soft topology*, Hacet. J. Math. Stat. 41 (3), 407-419, 2012.

[4]. Çagman N., Çıtak F. and Enginoglu S. *FP-soft set theory and its applications*, Ann. Fuzzy Math. Inform. 2 (2), 219-226, 2011.

[5]. Çagman N. and Deli İ. *Products of FP-soft sets and their applications*, Hacet. J. Math. Stat. 41 (3), 365-374, 2012.

[6]. Çagman N. and Deli İ. *Means of FP-soft sets and their applications*, Hacet. J. Math. Stat. 41 (5), 615-625, 2012.

[7]. Chang C. L. *Fuzzy topological spaces*, J. Math. Anal. Appl. 24, 182-190, 1968.

[8]. Gunduz C. and Bayramov S. *Some results on fuzzy soft topological spaces*, Mathematical problems in engineering, ID 855308, (2013)

[9]. Gunduz C. and Bayramov S. *Soft locally compact spaces and paramcompact spaces* Journal of mathematics and system science, 3, 122-130, (2013)

[10]. Feng F., Jun Y.B., Liu X. and Li L.F. *An adjustable approach to fuzzy soft set based decision making*, J. Comput. Appl. Math. 234, 10-20, 2010.

[11]. Inan E. and Öztürk M. A. *Fuzzy soft rings and fuzzy soft ideals*, Neural Comput. and Applic DOI 10.1007/s00521-011-0550-5.

[12]. Raja Sethupathy K. S., Lakshmivarahanc S., *Connectedness in Fuzzy Topology*, Kybernetika volume 13 (1977), Number 3

[13]. Kharal B. Ahmad, Mappings on Fuzzy Soft Classes, Hindawi Publising Corporation Advances in Fuzzy Systems, Article ID 407890 (2009) 6 pages.

[14]. Kong Z., Gao L.Q. and Wang L.F. *Comment on a fuzzy soft set theoretic approach to decision making problems*, J. Comput. Appl. Math. 223, 540-542, 2009.

[15]. Mahanta J., and Das P. K., *Results on fuzzy soft topological spaces*, arXiv:1203.0634v1,2012.

[16]. Maji P. K., Biswas R. and Roy A. R. *Fuzzy soft sets*, Journal of Fuzzy Mathematics, 203 (2), 589-602, 2001.

[17]. Ming P. B. and Ming L. Y. *Fuzzy topology I.Neighbourhood structure of a fuzzy point and Moore-Smith convergence*, J. Math. Anal. Appl. 76, 571-599, 1980.

[18]. Molodtsov D. *Soft set theory-First results*, Comput. Math. Appl. 37 (4/5), 19-31, 1999.

[19]. Pradip K.G., Ramkrishna P. C. and Madhumangal P. *Compact and Semicompact fuzzy soft topological spaces*, J. Math. Comput. Sci. 4 (2014), No. 2, 425-445.

[20]. Roy A.R. and Maji P.K. *A fuzzy soft set theoretic approach to decision making problems*, J. Comput. Appl. Math. 203, 412-418, 2007.

[21]. Roy S., Samanta T. K., A note on fuzzy soft topological spaces, Annals of Fuzzy Mathematics and Informatics, 2011

[22]. Tanay B. and Kandemir M. B. *Topological structures of fuzzy soft sets*, Comput. Math. Appl. 61, 412-418, 2011.

[23]. Simsekler T., Yüksel S. *Fuzzy soft topological spaces*, Ann. Fuzzy Math. Inform. 5 (1), 87-96, 2013.

[24]. Zadeh L. A. *Fuzzy sets*, Inform. and Control 8, 338-353, 1965.