Subdivision of the Spectra for the Generalized Difference Operator $\Delta_{a,b}$ on the Sequence Space ℓ_p $(1 < p < \infty)$

Nuh Durna[1]*

[1]Cumhuriyet University, Faculty of Science, Department of Mathematics, Sivas, Turkey, +90 346 2191010, ndurna@cumhuriyet.edu.tr
*İletişimden sorumlu yazar / Corresponding author

Recieved: 28th August 2016
Accepted: 4th December 2016
DOI: 10.18466/cbayarfbe.319876

Abstract

El-Shabrawy has introduced the generalized difference operator denoted by $\Delta_{a,b}$. Let nonzero real numbers sequences (a_k) and (b_k) be convergent sequences such that $\lim_{k\to\infty} a_k = a > 0$, $\lim_{k\to\infty} b_k = b$, $|b| = a$ and $\sup_k a_k \leq a$, $b_k^2 \leq a_k^2$, for all $k \in \mathbb{N}$. The generalized difference operator $\Delta_{a,b}$ is $\Delta_{a,b}x = \Delta_{a,b}(x_n) = (a_n x_n + b_{n-1}x_{n-1})_{n=0}^{\infty}$ with $x_{-1} = b_{-1} = 0$. In this paper we study the approximate point spectrum, the defect spectrum and the compression spectrum of the operator $\Delta_{a,b}$ on the p-absolutely summable sequences space ℓ_p.

Keywords — Generalized difference operator, approximate point spectrum, defect spectrum, compression spectrum.

1 Introduction

In functional analysis spectral theory has many applications in mathematics. In mathematics and physics we can see these applications in matrix theory, function theory, complex analysis, differential and integral equations, control theory and quantum physics.

Quite recently, many authors have studied several types of spectra, both for one or many commuting operators, with important applications, for example the approximate point spectrum, defect spectrum, compression spectrum, essential spectrum, etc.

The spectrum and fine spectrum of linear operators defined by some particular summable matrices over the sequence spaces have been studied by several authors. We give some basic study in the existing literature concerned with the spectrum and the fine spectrum. In [1], Gonzalez has investigated the fine spectrum of the Césaro operator on the sequence space ℓ_p for $1 < p < \infty$. Also, Wenger [2] has studied the fine spectrum of the Hölder operator over c, and in [3] these results have been generalized to the weighted mean methods by Rhoades. In [4], Reade exemined the spectrum of the Césaro operator over the sequence space c_0. The spectrum of the Rhaly operators on the certain sequence spaces is studied by Yildirim [5,6,7] and the fine spectrum of the Rhaly operators on the certain sequence spaces is studied by Yildirim [8,9]. Quite recently, several authors have investigated spectral divisions of generalized differance matrix. For example, the spectrum and fine spectrum of the generalized lower triangle double-band matrix Δ_v have been worked by Akhmedov and El-Shabrawy, [10,11] over the sequence spaces c, c_0 and ℓ_p where $1 < p < \infty$. In [12], Kayaduman and Furkan have studied the fine spectrum of the difference operator Δ on the sequence spaces ℓ_1 and bv. Also in [13], Altay and Başar have studied it on c_0 and c.

The above-mentioned articles, concerned with the decomposition of spectrum which defined by Goldberg. However, in [14] Durna and Yildirim have investigated subdivision of the spectra for factorable matrices on c_0 and in [15] Basar et al. have investigated subdivisions of the spectra for genarilized difference operator over some sequence spaces.

1.1 Preliminaries, Background and Notation

Let $L : X \to Y$ be a bounded linear operator where X and Y are Banach spaces. The range of L, denoted by $R(L)$, is

$$R(L) = \{ y \in Y : y = Lx,\ x \in X \}.$$

Let $B(X)$ be the set of all bounded linear operators on X into X. Let X be a Banach space and $L \in B(X)$. In this case the adjoint operator L^* of L is a bounded linear operator on the dual space X^* of X and it is defined by $(L^* f)(x) = f(Lx)$ for all $f \in X^*$ and $x \in X$.

In this paper we call as X be a Banach space over $\mathbb{C}$ and $L \in B(X)$. The resolvent set of L is denoted by

$$\rho(L) = \{\lambda \in \mathbb{C}: \lambda I - L \text{ bijection}\}. \tag{1.1}$$

The spectrum of L is defined by

$$\sigma(L) := \mathbb{C} - \rho(L) \tag{1.2}$$

where $\mathbb{C}$ is compleks plain. The inverse operator

$$R(\lambda; L) := (\lambda I - L)^{-1} \quad (\lambda \in \rho(L)) \tag{1.3}$$

is always bounded from closed graph theorem. Also it is usually called resolvent operator of L at λ.

Recall that a number $\lambda \in \mathbb{C}$ is called eigenvalue of L if the equation

$$Lx = \lambda x \tag{1.4}$$

has a nontrivial solution $x \in X$. Any such x is then called eigenvector, and the set of all eigenvectors is a subspace of X called eigenspace.

Throughout the text, we will call the set of eigenvalues

$$\sigma_p(L) = \{\lambda \in \mathbb{C}: Lx = \lambda x \text{ for some } x \neq 0 \}. \tag{1.5}$$

We say that $\lambda \in \mathbb{C}$ belongs to the continuous spectrum $\sigma_c(L)$ of L if the resolvent operator (1.3) is defined on a dense subspace of X and is unbounded. Furthermore, we say that $\lambda \in \mathbb{C}$ belongs to the residual spectrum $\sigma_r(L)$ of L if the resolvent operator (1.3) exists, but its domain of definition (i.e. the range $R(\lambda I - L)$ of $\lambda I - L$) is not dense in X; in this case $R(\lambda; L)$ may be bounded or unbounded. Together with the point spectrum (1.5), these two subspectra form a disjoint subdivision

$$\sigma(L) = \sigma_p(L) \cup \sigma_c(L) \cup \sigma_r(L). \tag{1.6}$$

of the spectrum of L.

The sequence $(x_n) \subset X$ a Weyl sequence for L if $\|x_n\| = 1$ and $\|Lx_n\| \to 1$ as $n \to \infty$.

The approximate point spectrum of L is defined by

$$\sigma_{ap}(L) = \{\lambda \in \mathbb{C}: \lambda I - L \text{ has a Weyl sequence } \}. \tag{1.7}$$

Also, the set

$$\sigma_\delta(L) = \{\lambda \in \sigma(L): \lambda I - L \text{ is not surjective}\} \tag{1.8}$$

is called defect spectrum of L.

The two subspectra (1.7) and (1.8) form a (not necessarily disjoint) subdivision

$$\sigma(L) = \sigma_{ap}(L) \cup \sigma_\delta(L) \tag{1.9}$$

of the spectrum. Finally, the compression spectrum of L defined by,

$$\sigma_{co}(L) = \{\lambda \in \mathbb{C}: \overline{R(\lambda I - L)} \neq X\}. \tag{1.10}$$

Hence we have another (not necessarily disjoint) decomposition of spectrum:

$$\sigma(L) = \sigma_{ap}(L) \cup \sigma_{co}(L). \tag{1.11}$$

Clearly, $\sigma_p(L) \subseteq \sigma_{ap}(L)$ and $\sigma_{co}(L) \subseteq \sigma_\delta(L)$. Also, we obtain that

$$\sigma_r(L) = \sigma_{co}(L) \setminus \sigma_p(L) \tag{1.12}$$

and

$$\sigma_c(L) = \sigma(L) \setminus \left[\sigma_p(L) \cup \sigma_{co}(L) \right] \tag{1.13}$$

from these subspectras and (1.6).

The following Proposition is quite useful for calculating the decomposition of the spectrum of a bounded linear operatör.

Proposition 1 ([16], Proposition 1.3). Let X be a Banach space over $\mathbb{C}$, $L \in B(X)$ and its adjoint $L^* \in B(X^*)$. Then we have the following relations for their spectrum and subspectrum;

(a) $\sigma(L^*) = \sigma(L)$,

(b) $\sigma_c(L^*) \subseteq \sigma_{ap}(L)$,

(c) $\sigma_{ap}(L^*) = \sigma_\delta(L)$,

(d) $\sigma_\delta(L^*) = \sigma_{ap}(L)$,

(e) $\sigma_p(L^*) = \sigma_{co}(L)$,

(f) $\sigma_{co}(L^*) \supseteq \sigma_p(L)$,

(g) $\sigma(L) = \sigma_{ap}(L) \cup \sigma_p(L^*) = \sigma_p(L) \cup \sigma_{ap}(L^*)$.

1.2. Goldberg's Classification of Spectrum

Let $T \in B(X)$, then there are three possibilities for

$R(T)$, the range of T :

(I) $R(T) = X$,

(II) $\overline{R(T)} = X$, but $R(T) \neq X$,

(III) $\overline{R(T)} \neq X$.

and three possibilities for T^{-1}:

(1) T^{-1} exists and continuous,

(2) T^{-1} exists but discontinuous,

(3) T^{-1} does not exist.

If these possibilities are combined in all possible ways, nine different states are created. These are labelled by: $I_1, I_2, I_3, II_1, II_2, II_3, III_1, III_2, III_3$. If an operator has state I_3 for example, then $R(T) = X$, and T^{-1} does not exist ([17]).

Let $\lambda \in \mathbb{C}$ if $T = \lambda I - L \in I_1$ or $T = \lambda I - L \in II_1$ then $\lambda \in \rho(L, X)$. All scalar values of λ not in $\rho(L, X)$ contain the $\sigma(L,X)$. This classification of $\sigma(L, X)$ gives rise to the fine spectrum of L .This means that, we can find disjoint subsets of $\sigma(L, X)$:

$I_2\sigma(L, X) = \emptyset$,

$I_3\sigma(L, X)$, $II_2\sigma(L, X)$, $II_3\sigma(L, X)$, $III_1\sigma(L, X)$, $III_2\sigma(L, X)$, $III_3\sigma(L, X)$. Instance, if we have state, III_2 for $T = \lambda I - L$ than we can say that $\lambda \in III_2\sigma(L, X)$.

By the definitions given above we can write following table:

Table 1.1

		1	2	3
		$R(\lambda; L)$ exits and is bounded	$R(\lambda; L)$ exits and is unbounded	$R(\lambda; L)$ does not exits
I	$R(\lambda I - L) = X$	$\lambda \in \rho(L)$ $\lambda \in \rho(L)$	–	$\lambda \in \sigma_p(L)$ $\lambda \in \sigma_{ap}(L)$
II	$\overline{R(\lambda I - L)} = X$	$\lambda \in \rho(L)$	$\lambda \in \sigma_c(L)$ $\lambda \in \sigma_{ap}(L)$ $\lambda \in \sigma_\delta(L)$	$\lambda \in \sigma_p(L)$ $\lambda \in \sigma_{ap}(L)$ $\lambda \in \sigma_\delta(L)$
III	$\overline{R(\lambda I - L)} \neq X$	$\lambda \in \sigma_r(L)$ $\lambda \in \sigma_\delta(L)$ $\lambda \in \sigma_{co}(L)$	$\lambda \in \sigma_r(L)$ $\lambda \in \sigma_{ap}(L)$ $\lambda \in \sigma_\delta(L)$ $\lambda \in \sigma_{co}(L)$	$\lambda \in \sigma_p(L)$ $\lambda \in \sigma_{ap}(L)$ $\lambda \in \sigma_\delta(L)$ $\lambda \in \sigma_{co}(L)$

2. The fine spectrum of the operator $\boldsymbol{\Delta_{a,b}}$ on $\boldsymbol{\ell_p}$ $(\mathbf{1} < p < \infty)$.

The generalized difference operator $\Delta_{a,b}$ has been defined by El-Shabrawy [18]. In [18] El-Shabrawy considers the $\Delta_{a,b}$ which is represented by the lower triangular double band matrix

$$\Delta_{a,b} = \begin{pmatrix} a_0 & 0 & 0 & 0 & \cdots \\ b_0 & a_1 & 0 & 0 & \cdots \\ 0 & b_1 & a_2 & 0 & \cdots \\ 0 & 0 & b_2 & a_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{2.1}$$

We suppose that here and hereafter that the nonzero real numbers sequences (a_k) and (b_k) are convergent sequences such that

$$\lim_{k\to\infty} a_k = a > 0, \tag{2.2}$$

$$\lim_{k\to\infty} b_k = b, \quad |b| = a \tag{2.3}$$

and

$$\sup_k a_k \le a,\ b_k^2 \le a_k^2, \text{ for all } k \in \mathbb{N}. \tag{2.4}$$

Note that, if (a_k) and (b_k) is a constant sequence, say

$a_k = r \neq 0$ and $b_k = s \neq 0$ for all $k \in \mathbb{N}$, then the operator $\Delta_{a,b}$ is reduced to the operator $B(r,s)$ and the results for the subdivisions of the spectra for generalized difference operator $\Delta_{a,b}$ over c_0, c, ℓ_p and bv_p have been studied in [15].

2. Subdivision of the spectrum of $\boldsymbol{\Delta_{a,b}}$ on $\boldsymbol{\ell_p}$ $(\mathbf{1} < p < \infty)$.

Let $T:\ell_p \to \ell_p$ $(1<p<\infty)$ be a bounded linear operator with matrix A. Then the adjoint operator $T^*:\ell_p^* \to \ell_p^*$ is defined by the transpoze of the matrix A. We notice that the dual space ℓ_p^* of ℓ_p is isomorphic to ℓ_q with $p^{-1}+q^{-1}=1$.

In [18], El-Shabrawy has investigated the fine spectrum of the operator $\Delta_{a,b}$ on the sequence space ℓ_p $(1<p<\infty)$. In this subsection we summarize the main results.

Theorem 1 *([18], Theorem 3.2) Let*

$$D=\{\lambda\in\mathbb{C}:|\lambda-a|\le a\}$$

and

$$E=\{a_k:k\in\mathbb{N},\ |a_k-a|>a\}.$$

Then $\sigma(\Delta_{a,b},\ell_p)=D\cup E$.

Theorem 2 *([18], Theorem 3.3)* $\sigma_p(\Delta_{a,b},\ell_p)=E$.

Theorem 3 *([18], Theorem 3.4)*

$$\sigma_p(\Delta_{a,b}^*,\ell_p^*)=\{\lambda\in\mathbb{C}:|\lambda-a|<a\}\cup E.$$

Theorem 4 *([18], Theorem 3.6)*

$$\sigma_r(\Delta_{a,b},\ell_p)=\{\lambda\in\mathbb{C}:|\lambda-a|<a\}.$$

Theorem 5 *([18], Theorem 3.7)*

$$\sigma_c(\Delta_{a,b},\ell_p)=\{\lambda\in\mathbb{C}:|\lambda-a|=a\}.$$

Lemma 1 *([17], Theorem II 3.1). The adjoint operator T^* is surjective $\Leftrightarrow T$ has a bounded inverse..*

Lemma 2 *([17], Theorem II 3.7). A linear operator T has a dense range $\Leftrightarrow$ the adjoint operator T^* is 1-1.*

Lemma 3 *If $\lim_{k\to\infty} a_k=a\neq 1$ for all $k\in\mathbb{N}$ where $a_k\neq 0$ for all $k\in\mathbb{N}$, then the product $\prod_k a_k$ is divergent.*

Theorem 6 $III_1\sigma(\Delta_{a,b},\ell_p)=\{a_k:k\in\mathbb{N},\ |a_k-a|<a\}$.

Proof We investigate whether the operator $(\lambda I-\Delta_{a,b})^*=\lambda I-\Delta_{a,b}^*$ is surjective or not. Does there exist $x\in\ell_q$ for all $y\in\ell_q$ such that $(\lambda I-\Delta_{a,b}^*)x=y$?

If for all $y\in\ell_q$, $(\lambda I-\Delta_{a,b}^*)x=y$, then we get

$$\begin{array}{lcccc}(\lambda-a_0)x_0 & - & b_0x_1 & = & y_0\\ (\lambda-a_1)x_1 & - & b_1x_2 & = & y_1\\ (\lambda-a_2)x_1 & - & b_2x_3 & = & y_2\\ & & & \vdots & \\ (\lambda-a_n)x_n & + & b_nx_{n+1} & = & y_n\\ & & & \vdots & \end{array}.$$

Thus we have

$$\begin{aligned} x_1 &= \frac{\lambda-a_0}{b_0}x_0-\frac{1}{b_0}y_1\\ x_2 &= \frac{\lambda-a_1}{b_1}x_1-\frac{1}{b_1}y_1\\ &= \frac{\lambda-a_0}{b_0}\frac{\lambda-a_1}{b_1}x_0-\frac{1}{b_0}\frac{\lambda-a_1}{b_1}y_0-\frac{1}{b_1}y_1\\ x_3 &= \frac{\lambda-a_2}{b_2}x_2-\frac{1}{b_2}y_2\\ &= \frac{\lambda-a_0}{b_0}\frac{\lambda-a_1}{b_1}\frac{\lambda-a_2}{b_2}x_0-\frac{1}{b_0}\frac{\lambda-a_1}{b_1}\frac{\lambda-a_2}{b_2}y_0.\\ &\quad -\frac{1}{b_1}\frac{\lambda-a_2}{b_2}y_1-\frac{1}{b_2}y_2\\ &\quad\ldots \end{aligned}$$

Hence

$$x_n=x_0\prod_{k=0}^{n-1}\frac{\lambda-a_k}{b_k}+\sum_{k=1}^{n-1}\frac{y_k}{b_{k-1}}\prod_{i=k}^{n-1}\frac{\lambda-a_i}{b_i}\frac{\lambda-a_1}{b_1}+\frac{y_{n-1}}{b_{n-1}},\ n\ge 2. \tag{2.5}$$

Now, we must show that $x\in\ell_q$. That is, is the series $\sum_{n=1}^{\infty}|x_n|^q$ convergent? Since

$$\lim_{n\to\infty}x_n=x_0\prod_{k=0}^{\infty}\frac{\lambda-a_k}{b_k}+\sum_{k=1}^{\infty}\frac{y_k}{b_{k-1}}\prod_{i=k}^{\infty}\frac{\lambda-a_i}{b_i}\frac{\lambda-a_1}{b_1}+\lim_{n\to\infty}\frac{y_{n-1}}{b_{n-1}},$$

if for all $k\in\mathbb{N}$, $\lambda\neq a_k$ then the limit of the general term of the infinite product $\prod_{k=0}^{\infty}\frac{\lambda-a_k}{b_k}$ is $\lim_{n\to\infty}\left|\frac{\lambda-a_n}{b_n}\right|=\left|\frac{\lambda-a}{b}\right|$. If $\lambda\in\sigma_r(\Delta_{a,b},\ell_p)$, then we get $\left|\frac{\lambda-a}{b}\right|<1$ from Theorem 4. Thus, the infinite product $\prod_{k=0}^{\infty}\frac{\lambda-a_k}{b_k}$ is divergent from Lemma 3. This means that $\lim_{n\to\infty}|x_n|^q\neq 0$. Thus for $\lambda\in\sigma_r(\Delta_{a,b},\ell_p)$ and for all $k\in\mathbb{N}$, $\lambda\neq a_k$ implies $x\notin\ell_q$. In this case, $\lambda I-\Delta_{a,b}^*$ is not surjective and so $\lambda I-\Delta_{a,b}$ has not bounded inverse from Lemma 1.

Now let we assume that $\lambda\in\sigma_r(\Delta_{a,b},\ell_p)$ and for some $k_0\in\mathbb{N}$, $\lambda=a_{k_0}$. In this case, $x_n=\frac{y_{n-1}}{b_{n-1}}$, since the products are zero on the right hand of 2.5). Since for $n\in\mathbb{N}$, $b_n\neq 0$, from (2.3), we have $\lim_{n\to\infty}\frac{1}{|b_n|}=\frac{1}{a}$. Since every convergent sequence is bounded, there exists $m>0$such that $\frac{1}{|b_n|}\le m$ for all $n\in\mathbb{N}$. Therefore we have

$$\sum_{n=2}^{\infty}|x_n|^q < m^q \sum_{n=1}^{\infty}|y_n|^q \le m^q \|y\|_{\ell_q}^q.$$

That is, the operator $(\lambda I - \Delta_{a,b})^*$ is surjective if and only if $\in \{a_k : k \in \mathbb{N}, |a_k - a| < a\}$. Hence $\lambda I - \Delta_{a,b}$ has bounded inverse from Lemma 1.

Corollary 1

$$III_2\sigma(\Delta_{a,b}, \ell_p) = \{\lambda \in \mathbb{C} : |\lambda - a| < a\} - \{a_k : k \in \mathbb{N}, |a_k - a| < a\}.$$

Proof This is clear from Theorem 4 and Theorem 6, since $III_2\sigma(\Delta_{a,b}, \ell_p) = \sigma_r(\Delta_{a,b}, \ell_p) \setminus III_1\sigma(\Delta_{a,b}, \ell_p)$.

Theorem 7 $III_3\sigma(\Delta_{a,b}, \ell_p) = E$.

Proof Let us find $\ker(\lambda I - \Delta^*_{a,b})$. If $(\lambda I - \Delta^*_{a,b})x = 0$, then we have

$$\begin{array}{rcrcl} (\lambda - a_0)x_0 & - & b_0 x_1 & = & 0 \\ (\lambda - a_1)x_1 & - & b_1 x_2 & = & 0 \\ (\lambda - a_2)x_1 & - & b_2 x_3 & = & 0 \\ & & & \vdots & \\ (\lambda - a_n)x_n & + & b_n x_{n+1} & = & 0 \\ & & & \vdots & \end{array}.$$

Hence we get

$$x_n = x_0 \prod_{k=0}^{n-1} \frac{\lambda - a_k}{b_k}, \; n \ge 1.$$

Since for $\lambda \in E$, $\lambda = a_k$ and $|a_k - a| > a$, we obtain that

$$\begin{aligned} \ker(\lambda I - \Delta^*_{a,b}) &= \left\{ (x_0, 0, 0, \cdots), \left(x_0, \frac{\lambda - a_0}{b_0} x_0, 0, 0, \cdots \right), \right. \\ &\quad \left. \left(x_0, \frac{\lambda - a_0}{b_0} x_0, \frac{\lambda - a_0}{b_0} \frac{\lambda - a_1}{b_1} x_0, 0, 0, \cdots \right) \right\} \\ &\ne \{(0, 0, 0, \cdots)\}. \end{aligned}$$

This means that if $\lambda \in E$, then $\lambda I - \Delta^*_{a,b}$ is not one to one. Thus for $\lambda \in E$, $\lambda I - \Delta_{a,b}$ does not have dense range from Lemma 2. Therefore we get $III_3\sigma(\Delta_{a,b}, \ell_p) = E$.

Corollary 2 $I_3\sigma(\Delta_{a,b}, \ell_p) = II_3\sigma(\Delta_{a,b}, \ell_p) = \varnothing$.

Proof This is clear from Theorem 7 since from Table 1.1,

$$\begin{aligned} \sigma_p(\Delta_{a,b}, \ell_p) &= I_3\sigma(\Delta_{a,b}, \ell_p) \cup II_3\sigma(\Delta_{a,b}, \ell_p) \\ &\quad \cup III_3\sigma(\Delta_{a,b}, \ell_p) \\ &= E \end{aligned}$$

and

$$I_3\sigma(\Delta_{a,b}, \ell_p) \cap II_3\sigma(\Delta_{a,b}, \ell_p) \cap III_3\sigma(\Delta_{a,b}, \ell_p) = \varnothing.$$

Theorem 8

(a) $\sigma_{ap}(\Delta_{a,b}, \ell_p) = (D \cup E) - \{a_k : k \in \mathbb{N}, |a_k - a| < a\}$,
(b) $\sigma_\delta(\Delta_{a,b}, \ell_p) = D \cup E$,
(c) $\sigma_{co}(\Delta_{a,b}, \ell_p) = \{\lambda \in \mathbb{C} : |\lambda - a| < a\} \cup E$.

Proof (a) This is clear from Theorem 1 and Theorem 6.
(b) This is clear from Theorem 7 and Corollary 2, since from Table 1.1,

$$\sigma_\delta(\Delta_{a,b}, \ell_p) = \sigma_\delta(\Delta_{a,b}, \ell_p) \setminus I_3\sigma(\Delta_{a,b}, \ell_p).$$

(c) This is clear from Theorem 4 and Theorem 7, since from Table 1.1,

$$\begin{aligned} \sigma_{co}(\Delta_{a,b}, \ell_p) &= III_1\sigma(\Delta_{a,b}, \ell_p) \cup III_2\sigma(\Delta_{a,b}, \ell_p) \\ &\quad \cup III_3\sigma(\Delta_{a,b}, \ell_p) \\ &= \sigma_r(\Delta_{a,b}, \ell_p) \cup III_3\sigma(\Delta_{a,b}, \ell_p). \end{aligned}$$

Corollary 3 (a) $\sigma_{ap}(\Delta^*_{a,b}, \ell_q) = D \cup E$,
(b) $\sigma_\delta(\Delta^*_{a,b}, \ell_q) = (D \cup E) - \{a_k : k \in \mathbb{N}, |a_k - a| < a\}$.

Proof This is clear from Theorem 8 and Proposition 1 (c) and (d).

3. Conclusion

In 2011, Durna and Yildirim [14] calculated apporoximate point spectrum, defect spectrum and compression spectrum of factorable matrices on c_0 using the relationship between spectral divisions of operators easily. After this study, these spectral parameters are taken into account by the authors while the fine division of spectrum was found. So far the studies, aporoximate point spectrum, defect spectrum and compression spectrum of operators were calculated using fine division of spectrum. Generally, it is obtained that operator has dense image or bounded inverse using the injectivity and surjectivity of its adjoint while the fine division of operator was investigated. However, it may not be possible to find adjoint operator. Even if adjoint operator is found, it may not be possible to examine character of obtained series while injectivity and surjectivity of operator were investigated. For example, on ℓ_∞, it is not possible to mention the adjoint of operator in the usual sense since ℓ_∞ does not have the Schauder basis in the usual sense. Therefore we will first calculate apporoximate point spectrum, defect spectrum and compression spectrum of operator with the help of the relationship between spectral division of operator and spectral division of its adjoint.

5 References

[1] Gonzalez, M. The fine spectrum of the Cesaro operator in ℓ_p, $(1 < p < \infty)$. Archiv der Mathematik. 1985; 44, 355-358.

[2] Wenger, R.B. The fine spectra of Hölder summability operators. Indian Journal of Pure and Applied Mathematics. 1975; 6, 695-712.

[3] Rhoades, B.E. The fine spectra for weighted mean operators. Pacific Journal of Mathematics. 1983; 104, 263-267.

[4] Reade, J.B. On the spectrum of the Cesaro operator. Bulletin of the London Mathematical Society. 1985; 17, 263-267.

[5] Yildirim, M. On the spectrum and fine spectrum of the compact Rhaly operators. Indian Journal of Pure and Applied Mathematics. 1996; 27, 779-784.

[6] Yildirim, M. On the spectrum of the Rhaly operators on c_0 and c. Indian Journal of Pure and Applied Mathematics. 1998; 29, 1301-1309.

[7] Yildirim, M. On the spectrum of the Rhaly operators on ℓ_p. Indian Journal of Pure and Applied Mathematics. 2001; 32, 191-198.

[8] Yildirim, M: The Fine Spectra of the Rhaly Operators on c_0. Turkish Journal of Mathematics. 2002; 26(3), 273-282.

[9] Yildirim, M. On the spectrum and fine spectrum of the Rhaly operators. Indian Journal of Pure and Applied Mathematics. 2003; 32, 1443-1452.

[10] Akhmedov, A.M.; El-Shabrawy, S.R. The spectrum of the generalized lower triangle double-band matrix Δ_a over the sequence space c. Al-Azhar Univ. Eng. J., JAUES (speacial issue). 2010; 5(9), 54-60.

[11] Akhmedov, A.M.; El-Shabrawy, S.R. On the fine spectrum of theoperator Δ_v over the sequence space c and ℓ_p, $(1 < p < \infty)$. Applied Mathematics and Information Sciences. 2011; 5(3), 635-654.

[12] Kayaduman, K.; Furkan, H. On the fine spectrum of the difference operator Δ over the sequence spaces ℓ_1 and bv. International Mathematical Forum. 2006; 24(1), 1153-1160.

[13] Altay, B.; Başar, F. On the fine spectrum of the difference operator on c_0 and c. Information Science. 2004; 168, 217-224.

[14] Durna, N.; Yildirim, M. Subdivision of the spectra for factorable matrices on c_0. GUJ Science. 2011; 24(1), 45-49.

[15] Başar, F.; Durna, N.; Yildirim, M. Subdivisions of the spectra for genarilized difference operator over certain sequence spaces. Thai Journal of Mathematics. 2011; 9 (2), 285-295.

[16] Appell, J.; Pascale, E.D.; Vignoli, A. Nonlinear Spectral Theory Walter de Gruyter Berlin New York, 2004.

[17] Goldberg, S. Unbounded Linear Operators. McGraw Hill, New York, 1966.

[18] El-Shabrawy, S.R. On the fine spectrum of the generalized difference operator Δ_{ab} over the sequence space ℓ_p, $(1 < p < \infty)$. Applied Mathematics and Information Sciences. 2012; 6(1), 111-118.