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Scattering of Neutrons on Fluctuations of the Density of the Thin Films

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Abstract. The cross section for scattering neutron on the density of fluctuations of the thin films is obtained in the framework of the quantum theory of multiple scattering in the quasielastic approximation. Inhomogeneity can be caused by dynamic density fluctuations, and be statistical in nature. Fluctuations in the density of the scattering material cause neutron scattering wave. The probability of a collision between a neutron and an atomic nucleus depends on the number of neutrons and on their velocity. The formulas have been obtained under the assumption that the imaginary part of the optical potential is a local operator. It was determined that the scattering in density fluctuations does not contribute to the attenuation of the coherent neutron wave. In the approximation of a thin target the solution of the equation for the total scattering amplitude is identical to the expression obtained in the usual eikonal approximation and differs significantly, at least functionally, from the solution for the case of a thick target. There have been detailed investigations of the reflection and refraction of neutron waves in matter, and the details of their dispersion law have been studied. The results are hown also, that the total cross section for scattering by the complete target becomes universal and does not depend on cross section for scattering by one nucleus.

Keywords: 25.40-Ep

1. INTRODUCTION

Nuclear processes with neutrons make up the most-studied field of physics of nuclear interactions. It is well known that neutron scattering can provide a rigorous testing ground for nuclear wave functions. This is due mainly to the critical dependence of the cross section on the coherent interference of terms with different principal quantum number for the c.m. motion of the neutron. Neutrons undergo extremely weak electromagnetic interactions, therefore pass through matter largely unimpeded, only interacting with atomic nuclei. Therefore, the atomic characteristics of the medium do not play any role in the spread of neutrons in matter. This is purely a nuclear process. Because of its neutrality the neutron has a great penetrating power and it could be detected indirectly by ionization measurements of recoiling nuclei, i.e. by collision of neutrons in passage through the matter with an atomic nucleus. The intensity of these microscopic processes ultimately determines all the macroscopic properties of matter, such as slowing, diffusion, absorption, and etc.

2. PASSAGE OF THE NEUTRONS THROUGH A MATTER

In fact collisional processes are described quantitatively in terms of cross sections and to study them one needs the quantum mechanics. One can distinguish between elastic and nonelastic collisions depending on whether or not translational momentum and energy are conserved. Among the many processes of the interaction of neutrons with nuclei, which occur multiple scattering effects most important is the elastic scattering. Hence in its passage through the matter the neutron is deflected from its path because of the internal field of the nucleus. The struck nucleus recoils and acquires energy to produce ions which can be detected by a ionization chamber connected to an amplifier and oscillograph. The probability of a collision between a neutron and an atomic nucleus depends on the number of neutrons and on their velocity.

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When neutron are scattered by matter, the process can change the momentum and energy of the neutrons. In a crystalline substance atoms are arranged in an orderly manner in space. Neutron waves add up the point of observation in accordance with the laws of interference if the phase difference between the scattered waves is constant (coherent scattering), we can observe the pattern of alternating in the space diffraction minima and maxima. If the order in the arrangement of atoms is broken, scattering will not be coherent.

Let a stationary flux of neutrons be incident on a target. Our problem is to find the amplitude for scattering of the particles by nuclei matter, therefore, the cross section and refractive indices for neutron.

Refractive indices for neutron is close to unity and difficult to measure its. If the plate has a thickness *d* and refractive index *n* the neutron wave undergoes a phase shift and exits in the form of [1]

$$\psi_k(z>d)=e^{ik(z-d)+nkd}, \quad (1)$$

here k momentum of neutron after scattering.

Now suppose that there are two different types of randomly distributed scattering centers scattering lengths a_1 and a_2. Let there be a first type centers N_1 and N_2 - the second type, with the relative concentrations. The centers of the two types may be two isotopes of the same element, or more important for us to vary only the spin orientation. In such an analysis the relation (1) should be replaced by

$$|\psi_k|^2=b^2\left|a_1\sum_{i=1}^{N_1}\frac{\exp[ik(z_i+d_i)]}{d_i}+a_2\sum_{I=1}^{N_2}\frac{\exp[ik(z_i+d_i)]}{d_i}\right|^2 . \quad (2)$$

When disclosed the square of the ratio (2), the sums of the squares of each contribute to the coherent and incoherent parts, while the mixed terms contribute only to the coherent strength.

The movement of neutrons in matter is fully described by frequency for each wave vector. The resultant field of the scattered neutrons is a superposition of waves scattered by nuclei at all times prior to this moment t .

After solving the Schrödinger equation and Fourier - transform in some great period T we obtain the amplitude [2]

$$\begin{aligned} f(r,\omega')=\frac{i}{T}\left(\frac{m}{2\pi\hbar}\right)^{1/2}\int_0^\infty\frac{dt}{t^{3/2}}\int dt_0 e^{-i\omega t_0}e^{i\omega' t} \\ x\sum_n d_n\int dr\delta[r-r_n] \end{aligned}, \quad (3)$$

where $\hbar\omega'$ - the neutron energy after scattering, r_n - the distance from the center to the point where there is the neutron wave.

In fact collisional processes are described quantitatively in terms of cross sections and to study them one need the quantum mechanics. One can distinguish between elastic and nonelastic collisions depending on whether or not translational momentum and energy are conserved. In this section we recall the main steps of the quantum collision theory in the case of two elastic interacting particles. The corresponding section is associated with this amplitude follows. The number of neutrons passing through the area $r'd\Omega$

, there is the ratio incident flux to the scattered flux. Then double differential scattering cross section is equal to

$$\frac{d^2\sigma}{d\Omega dE} = \frac{v}{v_0} \cdot \frac{1}{2\pi\hbar} \int d\tau e^{-i\omega\tau} \sum_{i,j} b_i^* b_j e^{iq(r_i - r_j)}, \tag{4}$$

where $\tau = t - t_0$, $\omega = \omega_0 - \omega'$, v velocity neutron after scattering, v_0 initial velocity of neutron and $q = k_0 - k$.

It is useful to distinguish from the expression (4) the part that depends on the average values b corresponding to different sets of identical atoms. This part of the section is called a section of coherent scattering. This cross section is obtained by summing (4) over all final states f and averaging over all initial states i. In this way,

$$\frac{d^2\sigma^{\text{ког.}}}{d\Omega dE} = \frac{k}{k_0} \cdot \bar{b}^2 \sum_{\substack{m,n,\\ i,j,}} < i \left| e^{-iqr_m} \right| f >< f \left| e^{iqr_n} \right| i > . \tag{5}$$

Part of the section, which remains after the separation of the coherent, is called incoherent scattering cross-section:

$$\frac{d^2\sigma^{\text{неког.}}}{d\Omega dE} = \frac{k}{k_0} \cdot \left(\overline{b^2} - \bar{b}^2 \right) \sum_{\substack{n,\\ i,j,}} < i \left| e^{-iqr_n} \right| f > . \tag{6}$$

$$x < f \left| e^{iqr_n} \right| i >$$

Interference effects present in the incoherent scattering cross section depends only on the average length of the scattering nuclei. The amplitude of the incoherent scattering is of the order of several units at 10^{-5}. Thus, after the "intersection" of about 10^4 perfectly parallel atomic planes of the crystal beam incident neutrons will be very much weakened. In this case, the contribution to the reflected intensity from crystal planes of said internal thickness will not be as great as from outside, close to its surface.

Consider the effect of the inhomogeneity of the crystal with the volume V on the distribution of coherent neutron wave. Inhomogeneity can be caused by dynamic density fluctuations, and be statistical in nature. Fluctuations in the density of the scattering material cause neutron scattering wave.

We define the scattering cross-section of the fluctuation of the N localized impurity - scatterers. The discussion applies to the case where there is only one isotope of the element. In those cases where the neutron wavelength is large compared to the size of the impurity following model can be used [3]

$$\delta\eta(r) = \sum_{i=1}^{N} \delta\eta_0 (r - R_i), \tag{7}$$

where $\delta\eta$ - random density fluctuations, R_i the radius vector of the centre of gravity of i impurity, and $\delta\eta_0$ describes the action of one of the scattering centre.

Neutron scattering wave can be taken into account by choosing the real part of the optical potential in the form of:

$$\delta U_R = (2\pi\hbar^2 / \mu) < \delta\eta > \mathrm{Re}\, f(0)\,. \tag{8}$$

Here brackets <...> denote averaging over the distribution of static states of the scattering system. Averaging over configurations of the scattering nuclei, ie, at the equilibrium position is an independent operation only in case of the crystal.

Similar to (6), can select the imaginary part of the optical potential

$$\delta U_I = (2\pi\hbar^2 / \mu) < \delta\eta > \mathrm{Im}\, f(0)\,. \tag{9}$$

The imaginary part of the potential is models the inelastic processes related to elastic scattering and determines the weakening of the coherent wave in the entrance channel. In general, optical potential is nonlocal and depends of neutron's energy. For thermal neutron the dependence of energy and nonlocalness of the optical potential is very little effect on the propagation of a neutron wave in the crystal.

According to the optical theorem

$$\mathrm{Im}\, f(0) = -k(\sigma_{abs} + \sigma_{inel}) / 4\pi \tag{10}$$

where

$$\sigma_{abs.} = NV\left\langle (\delta\eta_0)^2 \right\rangle, \tag{11}$$

$$\sigma_{inel.} = N\sigma_{el.} NV <(\delta\eta)^2>, \tag{12}$$

σ_{abs} cross section of absorption and σ_{inel} cross section of inelastic scattering. Elastic scattering related unitarily condition with all inelastic processes. Since the cross section of inelastic scattering in the approximation of a heavy target is proportional to the cross section of elastic scattering [4].

Elastic scattering cross section is equal to

$$\sigma_{el.}(k) = \int \left| f(k,\theta) \right|^2 d\Omega\,, \tag{13}$$

and expression (13) can be represented as a series expansion in the multiplicity of elastic scattering

$$\begin{aligned} \sigma_{el.}(k) &= N_1 \int \left| f(k,\theta) \right|^2 d\Omega + \\ &+ \frac{N_2}{k^2} \int dk' \left| f(k') f(k-k') \right|^2 d\Omega + \ldots \end{aligned}\,, \tag{14}$$

where

$$Ni = \frac{1}{\sigma_1 i!} \int \exp(-\sigma_1 \delta\eta r)^i \, dr\,, \tag{15}$$

here σ_1 a total scattering cross section, related to the scattering center:

$$\sigma_1 = 4\pi \frac{V}{V_0}\left\langle \left(\frac{\delta\eta}{\eta_0}\right)^2 \right\rangle \ , \tag{16}$$

where V_0 - the volume per one scattering nucleus, and *i*- the number of scattering nuclei per unit volume of the crystal. Parameterized optical potential in the form of the Woods-Saxon was chosen

$$V(r) = V_R f_R(r) + iV_I f_I(r),\ f_{R,I}(r) = \left(1 + \exp\left[\left(r - r_{R,I}A^{1/3}\right)/a_{R,I}\right]\right)^{-1}. \tag{17}$$

Here, $V_R = \mathrm{Re}V_0, V_I = \mathrm{Im}V_0$, V_0 parameter is the depth of the optical potential. Here, six parameters are reliable and can be installed only three, two of which belong to the imaginary part of the OP, and one real.

These cross sections (11), (12) and (13) describes the processes in which the number of particles in the scattering system remains the same, namely, elastic scattering, the scattering of particles with excitation and scattering the scattering system, accompanied by partial or total decay of the scattering system.

Optical theorem relates the refractive index of substance with the scattering cross section of individual atoms and nuclei of which consist the material. It has been known for many years that, if the optical potential is parameterized by local analytical expression, then the parameters must be allowed to vary with the energy. It is usual to represent the real part of the potential by Saxon-Woods form and to allow only the potential depth to vary energy. Optical potentials obtained by analysis of elastic scattering are also widely used to generate the distorted waves used to analyse the cross-sections of many reactions, and these analyses have proved to be a powerful tool in determining nuclear structures

Now suppose that the plate substance contains of *N* localized impurity - scatters and these centres scatters spherically symmetric wave with the scattering length a_l [5]

$$a_l = \frac{1}{kctg\delta_l} = \begin{cases} (\delta_l - \pi)/k & \text{for} \quad |\delta_l - \pi| << 1, \\ \delta_l / k & \text{for} \quad |\delta_l| << 1 \end{cases} \ , \tag{18}$$

where δ_l scattering phase. Hence if the scattering potential field is small compared with the centrifugal force term, for r such that $kr \sim i + 1/2$, for large n and the phase δ_i is small ($\delta_i \ll 1$) hence one is in the validity regime of Born's approximation.

Then, for the $z>d$ the wave can be expressed as

$$\psi_k(z > d) = e^{ikz} - a_l N d 2\pi \int r dr \frac{e^{ikr}}{r}. \tag{19}$$

This integral can be easily calculated using the coefficient convergence $\exp(-\omega r) \quad (\omega > 0)$

$$\int_z^\infty e^{ikr} dr = \lim_{\omega \to 0} \int_z^\infty e^{(ik-\omega)r} dr = -\frac{e^{ikz}}{ik} \ , \tag{20}$$

so that

$$\psi_k(z > d) = e^{ikz} - 2\pi a_l N d \frac{e^{ikz}}{k^2}. \qquad (21)$$

If (1) submit in the form

$$\psi_k(z > d) \approx e^{ikz}\left[1 + ik(n-1)d\right], \qquad (22)$$

and compare with (20), we obtain

$$n - 1 = -2\pi a_l N \frac{1}{k^2}. \qquad (23)$$

If the scattering is not spherically symmetric, which corresponds to the amplitude $f(\theta)$, we obtain:

$$n - 1 = -2\pi N \frac{1}{k^2} f(0), \qquad (24)$$

where $f(0)$ forward scattering amplitude because the refractive index describes the propagation of waves in the forward direction.

If the refractive index contains an imaginary part, it is necessary to divided (24) into two parts: real and imaginary:

$$\mathrm{Re}(n-1) = \frac{2\pi N}{k^2} \mathrm{Re}\, f_\kappa(0); \qquad (25)$$

$$\mathrm{Im}\, n = \frac{2\pi N}{k^2} \mathrm{Im}\, f_\kappa(0). \qquad (26)$$

Imaginary part of the refractive index determined by the condition of the decreasing the particle in the channel of elastic scattering.

Determining the effective cross section by the optical theorem (17) for the imaginary part of the refractive index obtain the following expression

$$\mathrm{Im}\, n = \frac{2\pi N^2}{k} \left\langle \left(\frac{\delta\eta}{\eta_0} \right)^2 \right\rangle. \qquad (27)$$

At $kf_k(0) << 1$ the imaginary part of the scattering amplitude is small and the refractive index can be considered real. Thus, in this case, scattering in density fluctuations does not contribute to the attenuation of the coherent neutron wave.

Knowing the effective wave number of the neutron wave in the medium and the refractive index can be calculated reflection and transmission coefficients for the neutron wave for the finite-volume substances.

3. CONCLUSION

These formulas have been obtained under the assumption that the imaginary part of the optical potential is a local operator. If do not resort to this hypothesis, have to deal with the time-consuming calculation of the sum of the vectors of the crystal lattice.

The above discussion applies to the case where there is only one isotope of one element present (especially an element with zero nuclear spin), however practically all real systems will have a distribution of both elements and isotopes of those elements. Moreover for thermal neutrons due to the shallow depth of penetration into the wall of the crystal must carefully consider the effect of the surface structure. The crystal surface is two-dimensional defect, distorting the frequency spectrum of vibrations of atoms located in the surface layer of the lattice.

The results are shown also, that if the wavelength of the incident particles becomes much greater than the thickness d of the target and the total cross section for scattering by the complete target becomes universal and does not depend on cross section for scattering by one nucleus.

Hence a study of the angular distribution of recoil tracks leads to important data for a theory of the field of the neutron. In fact the results of the experiments made to determine the field force consisting in the observations of the collisions of neutrons with material particles such as protons and electrons, have to be interpreted. All this requires the development of a theory of such collisions. The smallness of the field interaction between a neutron and a particle leads to the possibility of applying the approximate quantum theory of collisions of Born in elastic scattering of neutron with particles.

At first glance it is not enough accurate knowledge of such quantities as the wave function of the ground state of the nucleus, making the results of theoretical calculations of the cross sections is model dependent. In fact, it turns out that only the uncertainties in the particle density distribution of nucleons in a nucleus can significantly affect the results of these calculations. The sensitivity of the cross sections to the more complex characteristics of nuclear structure, such as the correlation functions of different ranks, is so low that in practical calculations during processing of experimental data correlations exist or can be ignored at all, or at best considered in some fairly simple model.

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