Cumhuriyet Sci. J., 43(3) (2022) 468-476
DOI: https://doi.org/10.17776/csj.1087221

Cumhuriyet Science Journal

| csj.cumhuriyet.edu.tr | Founded: 2002 ISSN: 2587-2680 e-ISSN: 2587-246X Publisher: Sivas Cumhuriyet University

Solutions of Time Fractional fKdV Equation Using the Residual Power Series Method

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Research Article

History
Received: 14/03/2022
Accepted: 02/08/2022

ABSTRACT

The fifth-order Korteweg-de Vries (fKdV) equation is a nonlinear model in various long wave physical phenomena. The residual power series method (RPSM) is used to gain the approximate solutions of the time fractional fKdV equation in this study. Basic definitions of fractional derivatives are described in the Caputo sense. The solutions of the time fractional fKdV equation with easily computable components are calculated as a quick convergent series. When compared to exact solutions, the RPSM provides good accuracy for approximate solutions. The reliability of the proposed method is also illustrated with the aid of table and graphs. Cleary observed from the results that the suggested method is suitable and simple for similar type of the time fractional nonlinear differential equations.

Keywords: Fractional partial differential equation, Fifth-order Korteweg-de Vries equation, Residual power series method, Caputo derivative, Approximate solutions.

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Introduction

Nonlinear phenomena modeled as nonlinear partial differential equations occur in many fields of science such as, mathematical biology, plasma physics, nonlinear optics, quantum mechanics, hydrodynamics, fluid dynamics, and chemical kinetics. Among these equations, the fKdV equation has utilized to investigate numerous significant issues in nonlinear physical phenomena. The fKdV equation has emerged in important physical systems such as in the theory of shallow water waves, gravity capillary waves, large interior waves in densely layered oceans, ion sound waves in plasma, and sound waves in a crystal lattice. Besides, the most well-known fKdV equations are the Sawada-Kotera equation, the Lax equation, the Caudrey-Dodd-Gibbon equation, the Ito equation, and the Kaup-Kuperschmidt equation. So far, several methods have used for solving the fKdV equations. These methods are Adomian decomposition [1], Laplace decomposition [2], variational iteration [3], Hirota direct [4], extended direct algebraic [5], homotopy perturbation transform [6], modified variational iteration algorithm-I [7], and modified variational iteration algorithm-II [8].

In recent years, mathematicians and scientists have been interested in studying the solutions of fractional differential equations because of their various applications in fields such as physics, biology, mathematics, chemistry, viscoelasticity, ecology, turbulence, nanotechnology, ecology, aerodynamics, control theory, and so on [9-11]. In the literature, the homotopy analysis method [12, 13], the operational collocation method [13], the finite difference method [13], the homotopy analysis transform method [14], the generalized Adams-Bashforth Moulton method [15], and the Euler method [16] have been used in solving many fractional differential equations. So far, the time fractional fKDV equation is investigated by utilized homotopy perturbation transform [17], simplest equation [18], trial equation [19], Lie group analysis [20], generalized exp(-Ø(ξ))-expansion [21], novel hyperbolic and exponential ansatz [22] methods. However, the RPSM has not yet been used in the literature to solve the fractional fKdV equation. Hence, the goal of this study is to get approximate solutions of the time fractional fKdV equation

$$D_t^{\alpha}v(x,t)+v(x,t)v_x(x,t)-v(x,t)v_{xxx}(x,t)+v_{xxxxx}(x,t)=0,\quad 0<\alpha\le 1 \tag{1}$$

by utilizing the RPSM. Here, D_t^{α} represents the Caputo derivative of $v(x,t)$. The RPSM is offered by Abu Arqub [23] is an efficient method to find the values of the power series solution for fuzzy differential equations. Without perturbation, discretization, or linearization, the proposed method suggests a powerful and simple power series solution for differential equations. RPSM has also fewer processing requirements, require less time, and is more reliable compared to the Taylor series method. Besides, this method does not require comparing the coefficients of the corresponding terms or a recursion relationship. Moreover, the proposed method does not perform any transformation in the transition from simple linearity to complex nonlinearity and from the low order to higher order. In the literature, many fractional differential equations have also been solved by suggested method, for example, the Zakharov-Kuznetsov equation [24], the Klein-Gordon equation [25], the Boussinesq-Burger's equation [26], the foam drainage equation [27], the Swift-Holenberg equation [28], the Sharma-Tasso-Olever equation [29], the Fisher equation [30], the Vibration equation [31], the Navier-Stokes equation [32], and the biological population diffusion equations [33].

Preliminaries

In this section, we examine some definitions and theorems for the fractional power series and the Caputo derivative. More detailed information about these can be found in [34,35].

Definition 2.1. [34] The Riemann-Liouville fractional integral operator with order α is expressed as

$$J^{\alpha}f(x) = \begin{cases} \dfrac{1}{\Gamma(\alpha)}\displaystyle\int_0^x (x-t)^{\alpha-1}f(t)dt, & \alpha > 0,\ x > 0 \\ f(x), & \alpha = 0. \end{cases}$$

Definition 2.2. [34] The Caputo fractional derivative with order α is defined as

$$\begin{aligned} D^{\alpha}f(x) &= J^{n-\alpha}\mathrm{D}^{n}f(x) \\ &= \frac{1}{\Gamma(n-\alpha)}\int_0^x (x-t)^{n-\alpha-1}\frac{d^n}{dt^n}f(t)dt, \quad x > 0, \quad n-1 < \alpha < n \in \mathbb{Z}^{+} \end{aligned}$$

where D^n is the classic differential operator. Utilizing the Caputo derivative, the following is also gained

$$D^{\alpha}x^{\beta} = 0, \qquad \beta < \alpha,$$
$$D^{\alpha}x^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta+1-\alpha)}x^{\beta-\alpha}, \qquad \beta \geq \alpha.$$

Definition 2.3. [34] For n is the smallest integer which exceeds α, the Caputo time fractional differential operator of order α of $v(x,t)$ is defined as

$$D_t^{\alpha}v(x,t) = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_0^t (t-\tau)^{n-\alpha-1}\frac{\partial^n v(x,\tau)}{\partial\tau^n}d\tau, & n-1 < \alpha < n \\ \dfrac{\partial^n v(x,t)}{\partial t^n}, & \alpha = n \in \mathbb{N}. \end{cases}$$

Definition 2.4. [35] A power series expanding which is called a fractional power series at $t = t_0$ of the form

$$\sum_{n=0}^{\infty} c_n(t-t_0)^{n\alpha} = c_0 + c_1(t-t_0)^{\alpha} + c_2(t-t_0)^{2\alpha} + \cdots, \quad 0 \leq n-1 < \alpha \leq n, \quad t \geq t_0,$$

where the constants c_n's are called the coefficients of the series and t is a variable.

Theorem 2.1. [35] Assume that f has a fractional power series at $t = t_0$ of the manner

$$f(t) = \sum_{n=0}^{\infty} c_n(t-t_0)^{n\alpha}, \qquad 0 \leq n-1 < \alpha \leq n, \quad t_0 \leq t < t_0 + R.$$

If $D^{n\alpha}f(t)$ are continuous on (t_0, t_0+R), then

$$c_n = \frac{D^{n\alpha}f(t_0)}{\Gamma(n\alpha+1)}, \qquad n = 0,1,2,\ldots,$$

where $D^{n\alpha} = D^{\alpha}.D^{\alpha}\ldots D^{\alpha}$, and R is the radius of convergence.

Theorem 2.2. [35] Assume that $v(x,t)$ is a multiple fractional power series at $t = t_0$ of the form

$$v(x,t) = \sum_{n=0}^{\infty} f_n(x)(t-t_0)^{n\alpha}, \qquad x \in I, \quad 0 \leq n-1 < \alpha \leq n, \quad t_0 \leq t < t_0 + R.$$

When $D_t^{n\alpha}v(x,t)$ are continuous on $I\times(t_0,t_0+R)$, $f_n(x)$ are described by

$$f_n(x)=\frac{D_t^{n\alpha}v(x,t_0)}{\Gamma(n\alpha+1)}, \qquad n=0,1,2,\dots.$$

Here, $D_t^{n\alpha}=\frac{\partial^{n\alpha}}{\partial t^{n\alpha}}=\frac{\partial^{\alpha}}{\partial t^{\alpha}}\cdot\frac{\partial^{\alpha}}{\partial t^{\alpha}}\cdots\frac{\partial^{\alpha}}{\partial t^{\alpha}}$, and $R=\min_{c\in I}R_c$, that R_c is effect domain of convergency of the fractional power series $\sum_{n=0}^{\infty}f_n(c)(t-t_0)^{n\alpha}$.

Basic Idea of Suggested Method

In this part of the paper, we examine a solution procedure for the suggested method. To present the basic idea of proposed method, we study the nonlinear fractional differential equation in the form

$$D_t^{\alpha}v(x,t)=N(v)+R(v), \qquad 0<\alpha\le 1, \quad t>0, \tag{2}$$

by the initial condition

$$v(x,0)=f(x).$$

Here, $D_t^{\alpha}v(x,t)$ represents the Caputo derivative of $v(x,t)$, $N(v)$ and $R(v)$ denote nonlinear and linear terms, respectively. The RPSM proposes the solution for Eq. (2) with a fractional power series at $t=0$,

$$v(x,t)=\sum_{n=0}^{\infty}f_n(x)\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \qquad x\in I, \quad 0<\alpha\le 1, \quad 0\le t<R.$$

Then, the kth truncated series of $v(x,t)$, that is $v_k(x,t)$ can be given as

$$v_k(x,t)=\sum_{n=0}^{k}f_n(x)\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \qquad x\in I,\ 0<\alpha\le 1, \quad 0\le t<R, \tag{3}$$

where $v_0=f_0(x)=v(x,0)=f(x)$. Eq. (3) can be also expressed as

$$v_k(x,t)=f(x)+\sum_{n=1}^{k}f_n(x)\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \qquad x\in I, \quad 0<\alpha\le 1, \quad 0\le t<R, \quad k=1,2,\dots. \tag{4}$$

In order to obtain the $f_n(x)$ in series expansion (4), the residual function for Eq. (1) is given below:

$$Res_v(x,t)=D_t^{\alpha}v(x,t)-N(v)-R(v).$$

Therefore, the k-th residual function $Res_{v,k}$ is

$$Res_{v,k}(x,t)=D_t^{\alpha}v_k(x,t)-N(v_k)-R(v_k). \tag{5}$$

As in [23, 36-39], some effective relations of RPSM are described as follows:

$$Res_v(x,t)=0,$$

$\lim_{k\to\infty}Res_{v,k}(x,t)=Res_v(x,t)$ for $x\in I$ and $t\ge 0$,

$$D_t^{n\alpha}Res_v(x,0)=D_t^{n\alpha}Res_{v,k}(x,0)=0, \qquad n=0,1,\dots,k. \tag{6}$$

The RPSM and its applications are based on these relations.

The RPSM is clarified by substituting kth truncated series of $v(x,t)$ in Eq. (5) and computing the fractional derivative $D_t^{(k-1)\alpha}$ of $Res_{v,k}(x,t)$ for $k=1,2,\dots$. Then, utilizing the relation (6), the algebraic equation in the form

$$D_t^{(k-1)\alpha}Res_{v,k}(x,0)=0, \qquad 0<\alpha\le 1, \quad 0\le t<R, \quad t=0, \quad k=1,2,\dots. \tag{7}$$

Solutions of the Time Fractional fKdV Equation

In this section, we consider Eq. (1) by the initial condition

$$v(x,0)=e^{x}. \tag{8}$$

The exact solution for Eq. (1) when $\alpha=1$ is [1]

$$v(x,t)=e^{x-t}.$$

For Eq. (1), we express the following residual function as

$$Res_{v}(x,t)=D_{t}^{\alpha}v(x,t)+v(x,t)\frac{\partial}{\partial x}v(x,t)-v(x,t)\frac{\partial^{3}}{\partial x^{3}}v(x,t)+\frac{\partial^{5}}{\partial x^{5}}v(x,t),$$

and k-th residual function $Res_{v,k}$,

$$Res_{v,k}(x,t)=D_{t}^{\alpha}v_{k}(x,t)+v_{k}(x,t)\frac{\partial}{\partial x}v_{k}(x,t)-v_{k}(x,t)\frac{\partial^{3}}{\partial x^{3}}v_{k}(x,t)+\frac{\partial^{5}}{\partial x^{5}}v_{k}(x,t). \tag{9}$$

In order to gain coefficient $f_1(x)$, we consider $k=1$ in Eq. (9) and we get

$$Res_{v,1}(x,t)=D_{t}^{\alpha}v_{1}(x,t)+v_{1}(x,t)\frac{\partial}{\partial x}v_{1}(x,t)-v_{1}(x,t)\frac{\partial^{3}}{\partial x^{3}}v_{1}(x,t)+\frac{\partial^{5}}{\partial x^{5}}v_{1}(x,t),$$

where

$$v_{1}(x,t)=f(x)+f_{1}(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)},$$

for

$$v_{0}=f_{0}(x)=f(x)=v(x,0)=e^{x}.$$

Hence, we gain

$$\begin{aligned}Res_{v,1}(x,t)=f_{1}(x)&+\left(f(x)+f_{1}(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right)\left(f'(x)+f_{1}{}'(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right)\\&-\left(f(x)+f_{1}(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right)\left(f'''(x)+f_{1}'''(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right)+f^{(5)}(x)+f_{1}^{(5)}(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)}.\end{aligned}$$

From Eq. (7), we get $Res_{v,1}(x,0)=0$, and thus

$$f_{1}(x)=-e^{x}.$$

Therefore, the first RPS solution of Eq. (1) is

$$v_{1}(x,t)=e^{x}-e^{x}\frac{t^{\alpha}}{\Gamma(\alpha+1)}.$$

Similarly, substituting $k=2$ in Eq. (9) to yield the coefficient $f_2(x)$, we get

$$Res_{v,2}(x,t)=D_{t}^{\alpha}v_{2}(x,t)+v_{2}(x,t)\frac{\partial}{\partial x}v_{2}(x,t)-v_{2}(x,t)\frac{\partial^{3}}{\partial x^{3}}v_{2}(x,t)+\frac{\partial^{5}}{\partial x^{5}}v_{2}(x,t),$$

where

$$v_{2}(x,t)=f(x)+f_{1}(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)}+f_{2}(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}.$$

Therefore, we have

$$Res_{v,2}(x,t) = f_1(x) + f_2(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + \left(f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\left(f'(x) + f_1'(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2'(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right) - \left(f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\left(f'''(x) + f_1'''(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2'''(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right) + f^{(5)}(x) + f_1^{(5)}(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2^{(5)}(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}.$$

From Eq. (7), we gain $D_t^{\alpha} Res_{v,2}(x,0) = 0$, and hence

$f_2(x) = e^x$.

Therefore, the second RPS solution of Eq. (1) is

$$v_2(x,t) = e^x - e^x\frac{t^{\alpha}}{\Gamma(\alpha+1)} + e^x\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}.$$

Likewise, substituting $k = 3$ in Eq. (9) to obtain the coefficient $f_3(x)$, we have

$$Res_{v,3}(x,t) = D_t^{\alpha} v_3(x,t) + v_3(x,t)\frac{\partial}{\partial x}v_3(x,t) - v_3(x,t)\frac{\partial^3}{\partial x^3}v_3(x,t) + \frac{\partial^5}{\partial x^5}v_3(x,t),$$

where

$$v_3(x,t) = f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}.$$

Therefore, we get

$$Res_{v,3}(x,t) = f_1(x) + f_2(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_3(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \left(f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\right)\left(f'(x) + f_1'(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2'(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3'(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\right) - \left(f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\right)\left(f'''(x) + f_1'''(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2'''(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3'''(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\right) + f^{(5)}(x) + f_1^{(5)}(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2^{(5)}(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3^{(5)}(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}.$$

From Eq. (7), we gain $D_t^{2\alpha} Res_{v,3}(x,0) = 0$, and hence

$f_3(x) = -e^x$.

Therefore, the third RPS solution of Eq. (1) is

$$v_3(x,t) = e^x - e^x\frac{t^{\alpha}}{\Gamma(\alpha+1)} + e^x\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - e^x\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}.$$

Using the same process for $k = 4$, the following is obtained as

$f_4(x) = e^x$,

$$v_4(x,t) = e^x - e^x\frac{t^{\alpha}}{\Gamma(\alpha+1)} + e^x\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - e^x\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + e^x\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}.$$

To validate the accuracy and efficiency of the suggested method, the numerical comparisons of the fourth RPS solution with the exact solution for $\alpha = 1$ and different values of x and t are illustrated in Table 1. Clearly observed from Table 1 that the absolute error is being smaller when the value of the t is decreasing

Table 1. Comparison between the solutions v_4 (x,t) and the exact solution for α=1.

x	*t*	*v_4 (x,t)*	*Exact solution*	*Absolute error*
-10	0	$4.53999x10^{-5}$	$4.53999x10^{-5}$	0
	0.2	$3.71704x10^{-5}$	$3.71703x10^{-5}$	$1.x10^{-10}$
	0.4	$3.04361x10^{-5}$	$3.04325x10^{-5}$	$3.6299x10^{-9}$
	0.6	$2.49427x10^{-5}$	$2.4916x10^{-5}$	$2.67117x10^{-8}$
	0.8	$2.05087x10^{-5}$	$2.03995x10^{-5}$	$1.09158x10^{-7}$
	1	$1.7025x10^{-5}$	$1.67017x10^{-5}$	$3.23273x10^{-7}$
-5	0	$6.73795x10^{-3}$	$6.73795x10^{-3}$	0
	0.2	$5.51658x10^{-3}$	$5.51656x10^{-3}$	$1.73856 x10^{-8}$
	0.4	$4.51712x10^{-3}$	$4.51658x10^{-3}$	$5.38726x10^{-7}$
	0.6	$3.70183x10^{-3}$	$3.69786x10^{-3}$	$3.96436x10^{-6}$
	0.8	$3.04376x10^{-3}$	$3.02755x10^{-3}$	$1.62005x10^{-5}$
	1	$2.52673x10^{-3}$	$2.47875x10^{-3}$	$4.79779x10^{-5}$
0	0	1	1	0
	0.2	$8.18733x10^{-1}$	$8.18731x10^{-1}$	$2.58026x10^{-6}$
	0.4	$6.704x10^{-1}$	$6.7032x10^{-1}$	$7.9954x10^{-5}$
	0.6	$5.494x10^{-1}$	$5.48812x10^{-1}$	$5.88364x10^{-4}$
	0.8	$4.51733x10^{-1}$	$4.49329x10^{-1}$	$2.40437x10^{-3}$
	1	$3.75x10^{-1}$	$3.67879x10^{-1}$	$7.12056x10^{-3}$
5	0	$1.48413x10^{2}$	$1.48413x10^{2}$	0
	0.2	$1.21511x10^{2}$	$1.2151x10^{2}$	$3.82944x10^{-4}$
	0.4	$9.94962x10^{1}$	$9.94843x10^{1}$	$1.18662x10^{-2}$
	0.6	$8.15382x10^{1}$	$8.14509x10^{1}$	$8.73209x10^{-2}$
	0.8	$6.70432x10^{1}$	$6.66863x10^{1}$	$3.5684x10^{-1}$
	1	$5.56549x10^{1}$	$5.45982x10^{1}$	$1.05678x10^{0}$
10	0	$2.20265x10^{4}$	$2.20265x10^{4}$	0
	0.2	$1.80338x10^{4}$	$1.80337x10^{4}$	$5.68339x10^{-2}$
	0.4	$1.47665x10^{4}$	$1.47648x10^{4}$	$1.7611x10^{0}$
	0.6	$1.21013x10^{4}$	$1.20884x10^{4}$	$1.29596x10^{1}$
	0.8	$9.95009x10^{3}$	$9.89713x10^{3}$	$5.29598x10^{1}$
	1	$8.25992x10^{3}$	$8.10308x10^{3}$	$1.56841x10^{2}$

In Figure 1, for $-10 \leq x \leq 10$ and $0 \leq t \leq 1$ when $\alpha = 1$, the comparison between the $v_4(x,t)$ solution and the exact solution is illustrated. When equal parameters are selected, the fourth RPS solutions have similar shapes to the exact solutions, as seen in Figure 1.

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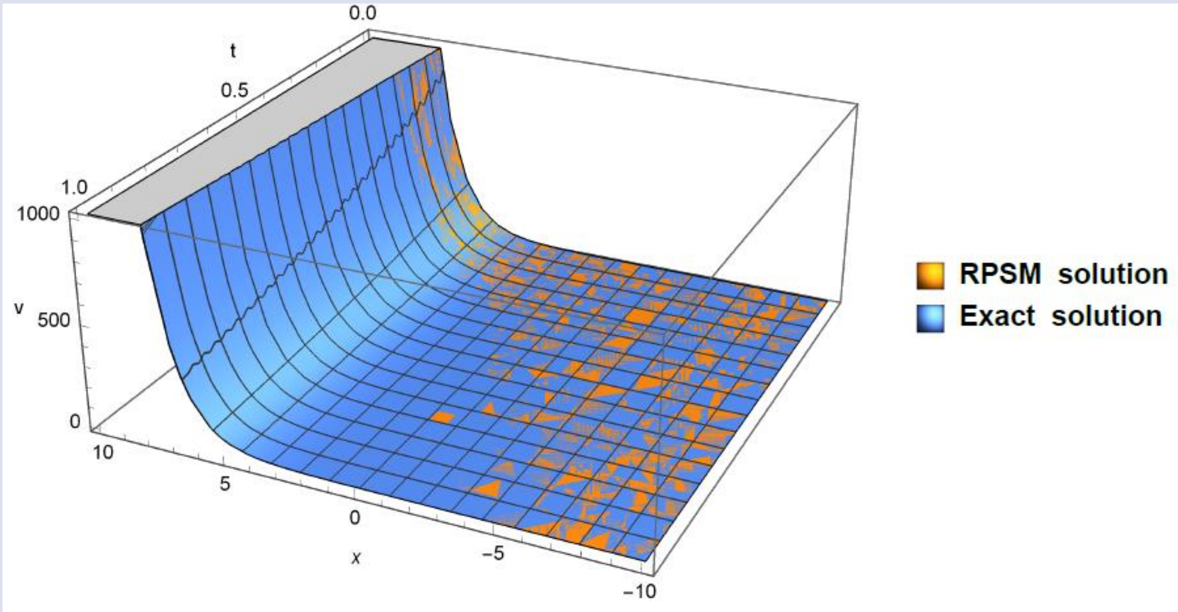

Figure 1. The graph of the exact solution and the $v_4(x,t)$ of Eq. (1) when $\alpha = 1$.

The RPS solution $v_4(x,t)$ is illustrated in Figure 2, for $-20 \leq x \leq 20$ and $0 \leq t \leq 25$ when $\alpha = 0.2$, $\alpha = 0.5$, $\alpha = 0.8$, $\alpha = 1$. When $\alpha = 1$ is chosen among the different values of α, the $v_4(x,t)$ is closest to the exact solution.

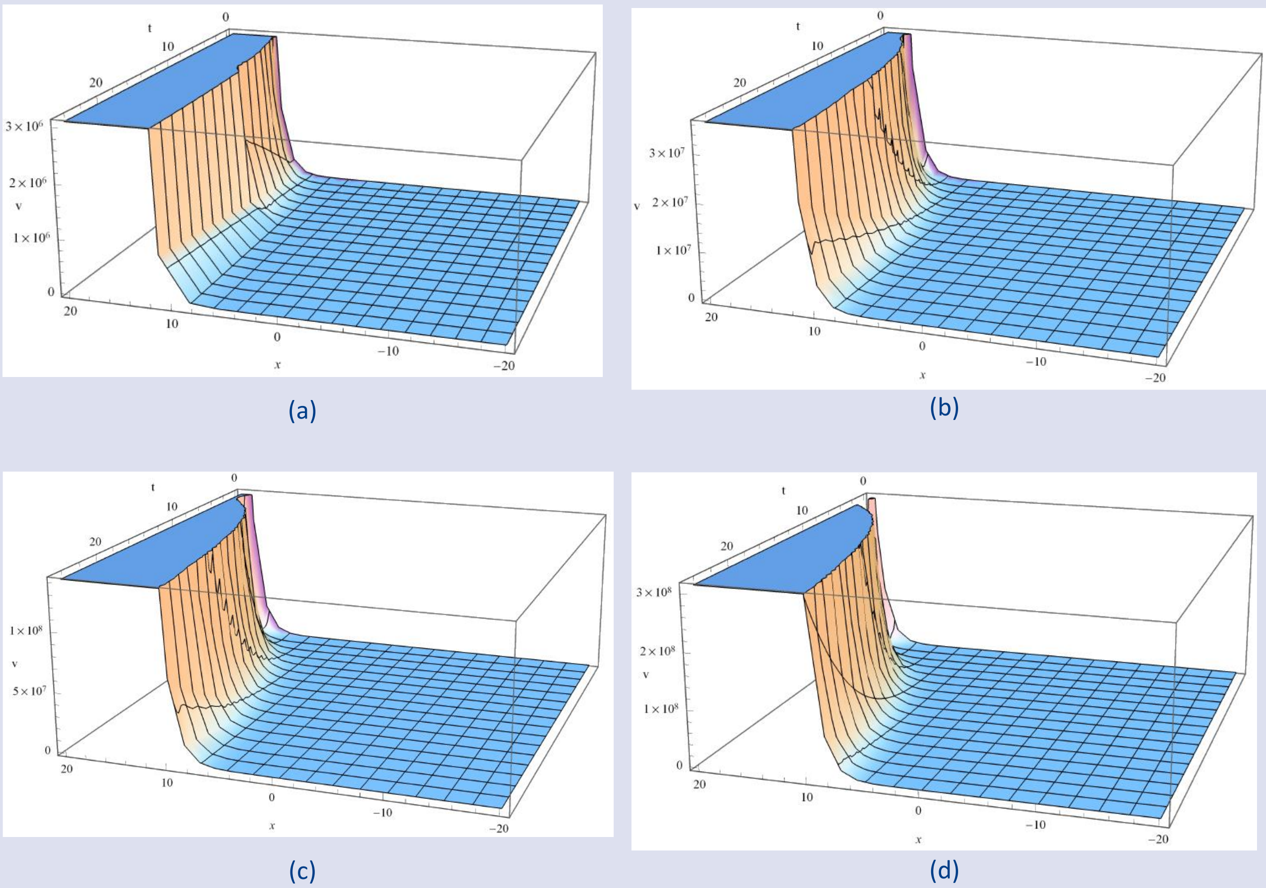

Figure 2. 3D graph of the fourth RPS solution of Eq. (1): (a) $v_4(x,t)$ when $\alpha = 0.2$, (b) $v_4(x,t)$ when $\alpha = 0.5$, (c) $v_4(x,t)$ when $\alpha = 0.8$, (d) $v_4(x,t)$ when $\alpha = 1$.

For $-20 \leq x \leq 20$ and $t = 15$ at the different α values, the $v_4(x,t)$ is demonstrated in Figure 3. In this figure, the line with dots represents the solution at $\alpha = 0.2$, the unitary line represents the solution at $\alpha = 0.5$, the line with dashes represents the solution at $\alpha = 0.8$, and the line with dot-dash represents the solution at $\alpha = 1$. It is clear that from Figure 3, the frequency increases as x approaches to zero. Besides, clearly seen from Figure 3 that the $v_4(x,t)$ solution approaches the exact solution as the value of the α increases.

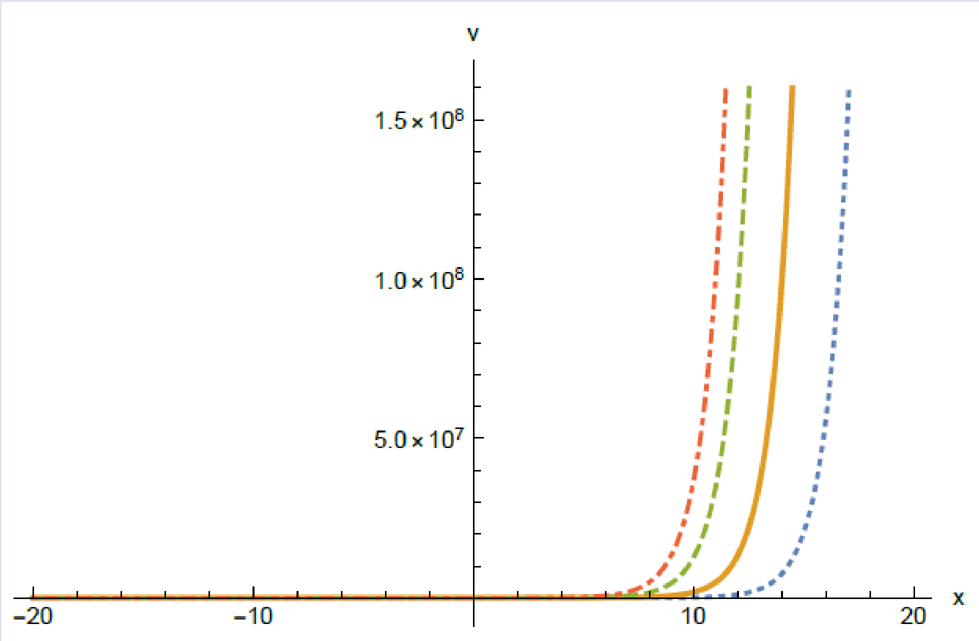

Figure 3. 2D graph of the $v_4(x, 15)$ for $\alpha = 0.2$, $\alpha = 0.5$, $\alpha = 0.8$, and $\alpha = 1$.

Conclusions

In this study, the RPSM was utilized to gain approximate solutions of the time fractional fKdV equation. These solutions were numerically compared to the exact solutions in Table 1. In this table, for $\alpha = 1$ and different values of x and t, the absolute errors of the RPS solutions were also introduced. In Table 1, when the numerical results were examined, the reliability of the proposed method for the time fractional fKdV equation had emerged. Besides, the fourth RPS solutions were demonstrated by 2D and 3D graphs. It could be seen in Figure 1 that the fourth RPS solution has similar shapes to the exact solution when equal parameters were chosen. The RPS solution $v_4(x,t)$ was illustrated for the different values of α in Figure 2 and Figure 3. All graphics were showed by the help of Mathematica. In addition, it was seen that RPSM achieved a high accuracy when the numerical results were analyzed in this paper.

When the RPSM is studied, it has more advantages than other methods in the literature. The RPSM is useful and effective method for solving nonlinear partial differential equations. The suggested method also does not require any linearization, transformation, discretization, or perturbation. Besides, this method does not need any small parameter for iterative solution. Moreover, by minimizing the residual error, the RPSM provides convergence of the series solution. Furthermore, by selecting a suitable initial estimate approximation, the proposed method can be used in nonlinear problems. As a result, the RPSM can be utilized to solve a wide range of fractional differential equations in mathematics and science.

Conflicts of interest

The author declares that there are no conflicts of interest.

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