Cumhuriyet Sci. J., 43(2) (2022) 294-298
DOI: https://doi.org/10.17776/csj.1064559

Cumhuriyet Science Journal

| csj.cumhuriyet.edu.tr | Founded: 2002 ISSN: 2587-2680 e-ISSN: 2587-246X Publisher: Sivas Cumhuriyet University

Properties of J_p-Statistical Convergence

Canan Sümbül [1,a], Cemal Belen[2,b], Mustafa Yıldırım [1,c,*]

[1] *Department of Mathematics, Faculty of Science, Sivas Cumhuriyet University, Sivas, Türkiye.*
[2] *Department of Mathematics Education, Ordu University, Ordu, Türkiye.*
**Corresponding author*

Research Article

History
Received: 08/02/2022
Accepted: 19/04/2022

ABSTRACT

In this study, different characterizations of J_p-statistically convergent sequences are given. The main features of J_p-statistically convergent sequences are investigated and the relationship between J_p-statistically convergent sequences and J_p-statistically Cauchy sequences is examined. The properties provided by the set of bounded and J_p statistical convergent sequences is shown. It is given that the statistical limit is unique. Furthermore, a sequence that J_p-statistical converges to the number L has a subsequence that converges to the same number of L, is shown. The analogs of J_p statistical convergent sequences is studied.

Keywords: Power series method, J_p-statistical convergence, J_p-statistical Cauchy

[a] *cnnhhs@gmail.com* *https://orcid.org/0000-0002-8905-1247*
[c] *yildirim@cumhuriyet.edu.tr* *https://orcid.org/0000-0002-8880-5457*
[b] *cbelen52@gmail.com* *https://orcid.org/0000-0002-8832-1524*

Introduction

Statistical convergence is a generalization of the concept of convergence in the Cauchy sense. The idea of statistical convergence was introduced under the name of "almost convergence" in the first edition [1] of Zygmund's monograph, published in 1935. The term "statistical convergence" was used by Fast [2] and Steinhaus [3] independently of each other. Also, statistical convergence was studied by Buck [4] in 1953 with the expression of "convergence in density".

Fridy [5] introduced the concept of the statistical Cauchy sequence and presented a characterization of statistical convergence without needing to know the statistical limit. Statistical convergence was considered as a regular summability method, and it was discussed in Schoenberg [6], Connor [7] and [8] .

Although statistical convergence is a new field of study, it has become an active area of research in recent years (see Belen et al [9], [10], Burgin and Duman [11], Connor and Kline [12], Çakallı and Khan [13], Et and Şengül [14], Freedman and Sember [15], Miller [16], Salat [17], Savaş and Mohiuddine [18]). Many researchers have done and still do studies on statistical convergence ([19], [20], [21] , [22]).

Ünver [23] defined the new density concept using the Abel method and presented a definition of a new version of statistical convergence via this density. Ünver and Orhan [24] gave a new density concept according to the power series method and the definitions of P_p-statistical convergence and strong P_p-convergence via this density. In the study, they gave a Krovkin-type approximation theorem. Belen et al. [25] defined the concepts of J_p-convergence respect to a power series method and strong J_p-convergence via a modulus function f. They examined the relationship between them. In addition, in the study, the concepts of J_p-statistical convergence and f-J_p-statistical convergence were given and the relationships between them were examined.

Now, let us remind the basic concepts used in this study.

Let $E \subset \mathbb{N}_0$, $E(n) := \{k \leq n : k \in E\}$ and $|E(n)|$ denote the cardinality of the set $E(n)$. If the limit $\delta(E) = \frac{\lim_{n\to\infty}|E(n)|}{(n+1)}$ exists, then the set $E \subset \mathbb{N}_0$ is said to have the usual density $\delta(E)$ [4]. The real number sequence $x = (x_k)$ is said to be statistically convergent to the number L, if the limit $\lim_{n\to\infty} \frac{1}{n+1}|\{k \leq n : |x_k - L| \geq \varepsilon\}| = 0$ for each $\varepsilon > 0$; i.e., $\delta(E_\varepsilon) = 0$ where $E_\varepsilon := \{k \leq n : |x_k - L| \geq \varepsilon\}$ and denoted by st-$\lim x = L$ [5].

Now let's introduce the J_p convergence given in Boss [26].

Let $\mathbb{N}_0$ be the set of non-negative integers. Let $(p_k)_{k\in\mathbb{N}_0}$ be a sequence of non-negative integers where $p_0 > 0$, satisfying

$$P_n = \sum_{k=1}^{n} p_k \to \infty, (n \to \infty) \quad (1)$$

and

$$p(t) = \sum_{k=1}^{\infty} p_k t^k < \infty, (\text{for } 0 < t < 1) \quad (2)$$

(In other words, $p(t)$ has radius of convergence $R = 1$).

Let $x = (x_k)_{k\in\mathbb{N}_0}$ be a sequence of real numbers. In this case, the power series method J_p is defined as follows:

If for every $0 < t < 1$, $p_x(t) = \sum_{k=1}^{\infty} p_k t^k x_k$ converges and $\lim_{t\to 1^-} \frac{p_x(t)}{p(t)} = L$, then (x_k) is called J_p-convergent to

L the sequence via the power series method and it is denoted as $x_k \to L\ (J_p)$. If $x_k \to L\ (J_p)$ as $x_k \to L$, the J_p-method is called regular. It is known that condition (1) or, equivalently, condition $p(t) \to \infty$ when $t \to 1^-$ guarantees the regularity of method J_p (see, [4]). Therefore, assuming (1), we will consider only regular J_p-methods.

Let $E \subset \mathbb{N}_0$ be any set. If $\delta_{J_p}(E) = \lim_{t\to 1^-} \frac{1}{p(t)} \sum_{k\in E} p_k t^k = 0$ exists, then $\delta_{J_p}(E)$ is called the J_p-density of the set E. If $\lim_{t\to 1^-} \frac{1}{p(t)} \sum_{k\in E_\varepsilon} p_k t^k = 0$ for every $\varepsilon > 0$, i.e., $\delta_{J_p}(E_\varepsilon) = 0$, then the number L of the sequence $x = (x_k)$ is said to be J_p-statistically convergent. The set of all J_p-statistically convergent sequences will be denoted by st_{J_p} [24].

In this study, some expected properties of the J_p-statistical convergent sequence space are examined.

Main Results

In this section, we prove that if a sequence $x = (x_k)$ is J_p-statistical convergent then there is a subsequence of $x = (x_k)$ which is convergence to the same number in ordinary sense. Also, we show that the J_p-statistical limit is unique, and we give the relationship between J_p-statistical Cauchy sequences and J_p-statistical convergent sequences.

Theorem 2.1 A real sequence $x = (x_k)$ is J_p-statistical convergent to a number ℓ if and only if there exists a subset $K := \{k \in \mathbb{N} : k = 1,2,\dots\}$ such that $\delta_{J_p}(K) = 1$ and

$$\lim_{\substack{k\to\infty \\ k\in K}} x_k = \ell$$

Proof. Necessity. Let $x = (x_k)$ be J_p-statistical convergent to ℓ.

$$K_r := \left\{k \in \mathbb{N} : |x_k - \ell| \geq \frac{1}{r}\right\}$$

and

$$M_r := \left\{k \in \mathbb{N} : |x_k - \ell| < \frac{1}{r}\right\}, r = 1,2,\dots .$$

In this case, we get $\delta_{J_p}(K_r) = 0$ and

$$M_1 \supset M_2 \supset \cdots \supset M_i \supset M_{i+1} \supset \cdots \quad (3)$$

$$\delta_{J_p}(M_r) = 1. \quad (4)$$

Now, we have to show that (x_k) converges to ℓ for $k \in M_r$. Assume that (x_k) is not convergent to ℓ. In this case, there is an $\varepsilon > 0$ for the infinitely many terms, such that

$$|x_k - \ell| \geq \varepsilon.$$

Define

$$M_\varepsilon = \{k : |x_k - \ell| < \varepsilon\} \text{ and } \varepsilon > \frac{1}{r} \quad (r = 1,2,\dots).$$

Hence

$$\delta_{J_p}(M_\varepsilon) = 0 \quad (5)$$

and $M_r \subset M_\varepsilon$ from (3). So we have $\delta_{J_p}(M_r) = 0$, which is a contradiction with (4). Then (x_k) is convergent to ℓ.

Sufficiency. Suppose that there is a subset $K := \{k \in \mathbb{N} : k = 1,2,\dots\}$ such that $\delta_{J_p}(K) = 1$ and

$$\lim_{\substack{k\to\infty \\ k\in K}} x_k = \ell$$

Therefore, for every $\varepsilon > 0$ there is a $N \in \mathbb{N}$ such that

$$|x_k - \ell| < \varepsilon, \forall k \geq N \text{ and } k \in K.$$

Since

$$K_\varepsilon = \{k : |x_k - \ell| \geq \varepsilon\} \subseteq \mathbb{N} - \{k_{N+j} : j \in \mathbb{N} \text{ and } k_{N+j} \in K\}$$

we have

$$\delta_{J_p}(K_\varepsilon) \leq 1 - 1 = 0.$$

Thus, $x = (x_k)$ is statistically convergent to ℓ.

Theorem 2.2 Let the sequence $x = (x_k)$ be J_p-statistical convergent to a number L. In this case, there is a sequence y that converges to the number L and a sequence z that J_p-statistical convergences to zero such that $x = y + z$.

Proof. Let the sequence $x = (x_k)$ be J_p-statistical convergent to a number L. For the set

$$E_j = \left\{k \leq n : |x_k - L| \geq \frac{1}{j}\right\}$$

with $N_0 = 0$ and $n \geq N_j (j = 1,2,\dots)$, we can find an increasing sequence of positive numbers (N_j) such that $\delta_{J_p}(E_j) < \frac{1}{j}$. Now let's define the y and z sequences as follows. Take $z_k = 0$ and $y_k = x_k$ when $N_0 < k \leq N_1$. For $\frac{1}{j} \geq 1$, let $N_j < k \leq N_{j+1}$. $z_k = 0$ and $y_k = x_k$ when $|x_k - L| < \frac{1}{j}$ and finally, when $|x_k - L| \geq \frac{1}{j}$, let $z_k = x_k - L$ and $y_k = L$. It is clear that we can write $x = y + z$. Now, we claim that the sequence y is convergent to L. Let $\varepsilon > 0$ be given, let us choose j such that $\varepsilon > \frac{1}{j}$. For $k \leq N_j$, if

$$|x_k - L| \geq \frac{1}{j} \text{ then } |y_k - L| = |L - L| = 0$$

and if

$$|x_k - L| < \frac{1}{j} \text{ then } |y_k - L| = |x_k - L| < \frac{1}{j} < \varepsilon$$

so $\lim_k y_k = L$ is obtained. Now, let us see $st_{J_p} - \lim z = 0$. We should show that

$$\lim_{t\to 1^-}\frac{1}{p(t)}\sum_{k\in E_z} p_k t^k = 0$$

for $E_z = \{k \le n: z_k \ne 0\}$. Since

$$\{k \le n: |z_k| \ge \varepsilon\} \subset \{k \le n: z_k \ne 0\}$$

for every $\varepsilon > 0$, we have

$$\delta_{J_p}(\{k \le n: |z_k| \ge \varepsilon\}) \le \delta_{J_p}(\{k \le n: z_k \ne 0\}).$$

Now if $\delta > 0$, $j \in \mathbb{N}$ and $\frac{1}{j} < \delta$ we have to show that $\delta_{J_p}(\{k \le n: z_k \ne 0\}) < \delta$ for every $n > N_j$. Let $N_j < k \le N_{j+1}$, then $z_k \ne 0$ is possible only with $|x_k - L| \ge \frac{1}{j}$. So if $N_j < k \le N_{j+1}$ then

$$\{k \le n: z_k \ne 0\} = \left\{k \le n: |x_k - L| \ge \frac{1}{j}\right\}.$$

Therefore, if $N_v < k \le N_{v+1}$ and $v > j$ implies that

$$\delta_{J_p}(\{k \le n: z_k \ne 0\}) \le \delta_{J_p}\left(\left\{k \le n: |x_k - L| \ge \frac{1}{v}\right\}\right) < \frac{1}{v} < \frac{1}{j} < \delta.$$

Thus, the proof is complete.

Corollary 2.1 If the sequence $x = (x_k)$ is J_p-statistical convergent to the number L, then $\exists (x_{n_k}) \subset (x_n) \ni x_{n_k} \to L$.

Theorem 2.3 If $x = (x_k)$ be a sequence such that $st_{J_p} - \lim x = L$, then L is determined uniquely.
Proof. Assume that $x = (x_k)$ is J_p -statistically convergent to two different numbers L and K. i.e., $st_{J_p} - \lim x = L$ and $st_{J_p} - \lim x = K$. Let us choose $L < K$. If we choose $\varepsilon = \frac{K-L}{3}$, then

$$(L-\varepsilon, L+\varepsilon) \cap (K-\varepsilon, K+\varepsilon) = \emptyset.$$

Also, since $st_{J_p} - \lim x = L$ and $st_{J_p} - \lim x = K$

$$\delta_{J_p}(\{k \le n: |x_k - L| \ge \varepsilon\}) = 0$$
$$\delta_{J_p}(\{k \le n: |x_k - K| \ge \varepsilon\}) = 0$$

then

$$\delta_{J_p}(\{k \le n: |x_k - L| < \varepsilon\}) = 1$$
$$\delta_{J_p}(\{k \le n: |x_k - K| < \varepsilon\}) = 1.$$

Hence, we get $\{k \le n: |x_k - L| < \varepsilon\} \cap \{k \le n: |x_k - K| < \varepsilon\} \ne \emptyset$. This is a contradiction, as the sets are disjoint. Hence the theorem is proved.
The following theorem shows that the statistical convergence method is linear.

Theorem 2.4 Let $x = (x_k)$ and $y = (y_k)$ be two real sequences.

(i) $st_{J_p} - \lim x = L_1$ and $st_{J_p} - \lim y = L_2$ implies $st_{J_p} - \lim(x+y) = L_1 + L_2$.

(ii) $st_{J_p} - \lim x = L_1$ and $\alpha \in R$ implies $st_{J_p} - \lim(\alpha x) = \alpha L_1$.

Proof. (i) Let $st_{J_p} - \lim x = L_1$ and $st_{J_p} - \lim y = L_2$. For the set $A_1 = \left\{k \le n: |x_k - L_1| \ge \frac{\varepsilon}{2}\right\}$ since $\delta_{J_p}(A_1) = 0$, there is $k_1 \in \mathbb{N}$ such that $|x_k - L_1| < \frac{\varepsilon}{2}$ for every $k > k_1$ and $k \in (\mathbb{N} - A_1)$ when $\varepsilon > 0$. For the set $A_2 = \left\{k \le n: |y_k - L_2| \ge \frac{\varepsilon}{2}\right\}$ since $\delta_{J_p}(A_2) = 0$, there is $k_2 \in \mathbb{N}$ such that $|y_k - L_2| < \frac{\varepsilon}{2}$ for every $k > k_2$ and $k \in (\mathbb{N} - A_2)$ when $\varepsilon > 0$. Let define $k_0 := \max\{k_1, k_2\}$. Let show $|x_k + y_k - L_1 - L_2| < \varepsilon$ for every and every $k \in (\mathbb{N} - (A_1 \cap A_2))$ and every $k > k_0$. Since $\delta_{J_p}(A_1) = 0$ and $\delta_{J_p}(A_2) = 0$, then $\delta_{J_p}(A_1 \cap A_2) = 0$. In that case for $k > k_0$

$$\begin{aligned}|x_k + y_k - L_1 - L_2| &< |x_k - L_1| + |y_k - L_2| \\ &\frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon\end{aligned}$$

and for every $\varepsilon > 0$

$$\delta_{J_p}(\{k \le n: |x_k + y_k - L_1 - L_2| \ge \varepsilon\}) = 0.$$

This gives $st_{J_p} - \lim(x+y) = L_1 + L_2$
(ii) If $\alpha = 0$, we have nothing to prove. Let us assume that $\alpha \ne 0$.

$$\begin{aligned}\delta_{J_p}(\{k \le n: |\alpha x_k - \alpha L_1| \ge \varepsilon\}) &= \delta_{J_p}(\{k \le n: |\alpha||x_k - L_1| \ge \varepsilon\}) \\ &\le \delta_{J_p}\left(\left\{k \le n: |x_k - L_1| \ge \frac{\varepsilon}{|\alpha|}\right\}\right). \\ &= 0\end{aligned}$$

So $st_{J_p} - \lim(\alpha x) = \alpha L_1$ is obtained.

Theorem 2.5 The space $st_{J_p} \cap \ell_\infty$ is a closed subspace of the normed space ℓ_∞.
Proof. Let $x^{(n)} \in st_{J_p} \cap \ell_\infty$ and $x^{(n)} \to x \in \ell_\infty$. Since $x_k \in st_{J_p} \cap \ell_\infty$ there are real numbers a_n such that

$$st_{J_p} - \lim_k x_k^{(n)} = a_n (n = 1,2,\ldots).$$

Since $x^{(n)} \to x$, for every $\varepsilon > 0$, there is a number $N = N(\varepsilon) \in \mathbb{N}$ such that

$$\left|x^{(p)} - x^{(n)}\right| < \varepsilon/3 \quad (6)$$

where $p \ge n \ge N$. Here, $|.|$ denotes the norm in a vector space. From Theorem 2.1, $\mathbb{N}$ has a subset of K_1 with $\delta_{J_p}(K_1) = 1$ and

$$\lim_{\substack{k \\ k\in K_1}} x_k^{(n)} = a_n. \tag{7}$$

Since $\delta_{J_p}(K_1) = 1$, let us take $k_1 \in K_1$. From (7),

$$\left|x_{k_1}^{(p)} - a_p\right| < \varepsilon/3. \tag{8}$$

TThus, for every $p \geq n \geq N$ from (6), we have

$$|a_p - a_n| \leq \left|a_p - x_{k_1}^{(p)}\right| + \left|x_{k_1}^{(p)} - x_{k_1}^{(n)}\right| + \left|x_{k_1}^{(n)} - a_n\right| < \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon.$$

Therefore (a_n) is a Cauchy sequence and hence (a_n) is convergent. Let

$$\lim_n a_n = a. \tag{9}$$

We should show that x is J_p-statistical convergence to a. Since $x^{(n)} \to x$, for every $\varepsilon > 0$, there is a $N_1(\varepsilon)$ such that

$$\left|x_j^{(n)} - x_j\right| < \varepsilon/3$$

where every $j \geq N_1(\varepsilon)$. Also, from (9), for every $\varepsilon > 0$ there is a $N_2(\varepsilon) \in \mathbb{N}$ such that

$$\left|a_j - a\right| < \varepsilon/3$$

where every $j \geq N_2(\varepsilon)$. Again, since $st_{J_p} \lim x^{(n)} = a_n$, there is a set $K \subseteq \mathbb{N}$ with $\delta_{J_p}(K) = 1$ and $N_3(\varepsilon) \in \mathbb{N}$ for every $\varepsilon > 0$ such that

$$\left|x_j^{(n)} - a_n\right| < \varepsilon/3$$

when $j \in K$ and all $j \geq N_3(\varepsilon)$. Let us say $max\{N_1(\varepsilon), N_2(\varepsilon), N_3(\varepsilon)\} = N_4(\varepsilon)$. In this case

$$\left|x_j - a\right| \leq \left|x_j^{(n)} - x_j\right| + \left|x_j^{(n)} - a_n\right| + \left|a_j - a\right| < \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon$$

is obtained for a given $\varepsilon > 0$ and all $j \geq N_4(\varepsilon)$, $j \in K$. Therefore $st_{J_p} \lim x = a$, i.e., $x \in st_{J_p} \cap \ell_\infty$. So $st_{J_p} \cap \ell_\infty$ is a closed subspace of ℓ_∞.

Theorem 2.6 The space $st_{J_p} \cap \ell_\infty$ is nowhere dense in ℓ_∞.
Proof. Since every closed subspace of an arbitrary normed space S different from S is nowhere dense in S (Neubrum et al. 1968), it is sufficient to show that it is only $st_{J_p} \cap \ell_\infty \neq \ell_\infty$. Let

$$p_k = \begin{cases} 1, & k = n^2, n \in \mathbb{N}_0 \\ 0, & otherwise. \end{cases}$$

and

$$x_k = \begin{cases} 1, & k = n^2, n \in \mathbb{N}_0 \\ 0, & otherwise. \end{cases}.$$

Then x is not J_p-statistical convergent but bounded. Hence, $st_{J_p} \cap \ell_\infty \neq \ell_\infty$.

Definition 2.1 $x = (x_k)$ is said to be J_p-statistical Cauchy sequence if for every $\varepsilon > 0$ there exists a $N(\varepsilon) \in N$ such that $\delta_{J_p}(\{k \leq n: |x_k - x_N| < \varepsilon\}) = 1$.

Theorem 27 A sequence $x = (x_k)$ is J_p −statistical convergent if and only if $x = (x_k)$ is J_p −statistical Cauchy.
Proof. Let (x_k) be J_p −statistical convergent to L. In this case, $\delta_{J_p}(\{k \leq n: |x_k - \ell| \geq \varepsilon\}) = 0$ for every $\varepsilon > 0$. Let us choose N as $|x_N - \ell| \geq \varepsilon$ and define the sets as

$$A_\varepsilon = \{k \leq n: |x_k - x_N| \geq \varepsilon\},$$
$$B_\varepsilon = \{k \leq n: |x_k - \ell| \geq \varepsilon\},$$
$$C_\varepsilon = \{k = N \leq n: |x_N - \ell| \geq \varepsilon\}$$

In this case, it is clear that $A_\varepsilon \subseteq B_\varepsilon \cup C_\varepsilon$. From here, $\delta_{J_p}(A_\varepsilon) \leq \delta_{J_p}(B_\varepsilon) + \delta_{J_p}(C_\varepsilon) = 0$ is obtained. So x is J_p-statistical Cauchy sequence. Conversely, let x be J_p-statistical Cauchy, but not J_p −statistical convergent. In this case, there exists N such that $\delta_{J_p}(A_\varepsilon) = 0$. Therefore,

$$\delta_{J_p}(\{k \leq n: |x_k - x_N| < \varepsilon\}) = 1.$$

Specifically, if $|x_k - \ell| < \varepsilon/2$ we can write

$$|x_k - x_N| \leq 2|x_k - \ell| < \varepsilon. \tag{10}$$

Since x is not J_p-statistical convergent, $\delta_{J_p}(B_\varepsilon) = 1$. That is

$$\delta_{J_p}(\{k \leq n: |x_k - \ell| < \varepsilon\}) = 0.$$

Thus from (10),

$$\delta_{J_p}(\{k \leq n: |x_k - x_N| < \varepsilon\}) = 0$$

i.e., $\delta_{J_p}(A_\varepsilon) = 1$. This is a contradiction. So, x is J_p −statistical convergent.

Conclusion

In this study, different characterizations of Jp-statistically convergent sequences are given. The main features of Jp-statistical convergent sequences are investigated and the relationship between Jp-statistical convergent sequences and Jp-statistical Cauchy sequences is examined.

Acknowledgment

The authors would like to thank the anonymous reviewers for their suggestions about paper.

Conflicts of interest

The author states that did not have conflict of interests

References

[1] Zygmund A., Trigonometric Series. *3rd.ed. London: Cambridge Univ. Press,* (2003),

[2] Fast H., Sur la convergence statistique, *Colloq. Math*., 2 (1951) 241-244.

[3] Steinhaus H., Sur la convergence ordinaire et la convergence asymptotique, *Colloq. Math.*, 2 (1951) 73-74.

[4] Buck R. C., Generalized asymptotic density, *Amer. J. Math*., 75 (2) (1953) 335-346.

[5] Fridy, J. A., On statistical convergence, *Analysis* 5 (1985) 301-313.

[6] Schoenberg I. J. , The integrability of certain functions and related summability methods, Amer. *Math. Monthly,* 66(5) (1959) 361-375.

[7] Connor J., The statistical and strong $p-$Cesàro convergence of sequences, *Analysis*, 8 (1988) 47-63.

[8] Connor J., On strong matrix summability with respect to a modulus and statistical convergence*, Canad. Math. Bull*., 32 (1989) 194-198.

[9] Belen C., Mursaleen M.,Yildirim M., Statistical A-summability of double sequences and a Korovkin type approximation theorem, *Bull. Korean Math. Soc.*, 49 (4) (2012) 851–861.

[10] Belen C., Some Tauberian theorems for weighted means of bounded double sequences, *An. Ştiinţ. Univ. Al. I. Cuza Iaşi. Mat.* (N.S.), 63 (1) (2017) 115–122.

[11] Burgin M., Duman O., Statistical convergence and convergence in statistics, http://arxiv.org/abs/math/0612179. Retrieved, 2006.

[12] Connor J., Kline J., On statistical limit points and the consistency of statistical convergence*, J. Math. Anal. Appl*., 197 (2) (1996) 392-399.

[13] Çakallı H. and Khan M. K., Summability in topological spaces, *Appl. Math. Lett*. 24(3) (2011) 348-352.

[14] Et M. and Şengul H., Some Cesàro-type summability spaces of order and lacunary statistical convergence of order *, Filomat,* 28(8) (2014), 1593-1602.

[15] Freedman, A. R., Sember, J. J., Densities and summability, *Pacific J. Math*., 95(2) (1981) 293-305.

[16] Miller H. I., A measure theoretical subsequence characterization of statistical convergence, *Trans. Amer. Math. Soc*., 347(5) (1995) 1811-1819.

[17] Šalát, T., On statistically convergent sequences of real numbers, *Math. Slovaca,* 30 (1980) 139--150.

[18] Savas E., Mohiuddine S. A., λ-statistically convergent double sequences in probabilistic normed spaces, *Math. Slovaca,* 62(1) (2012), 99-108.

[19] Et M., Baliarsingh P., Şengül Kandemir H., Küçükaslan M., On μ -deferred statistical convergence and strongly deferred summable functions, *Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM,* 115(1-34) (2021).

[20] Sengül H., Et M., Lacunary statistical convergence of order (α,β) in topological groups, *Creat. Math. Inform*., 26(3), (2017), 339-344.

[21] Sengul H., Et M., f-lacunary statistical convergence and strong f-lacunary summability of order α, *Filomat,* 32(13) (2018) 4513-4521.

[22] Sengul H., Et M., On (λ,I)-statistical convergence of order α of sequences of function, *Proc. Nat. Acad. Sci. India Sect.* A, 88(2) (2018) 181--186.

[23] Ünver M., Abel summability in topological spaces, *Monatsh. Math.,* 178(4) (2015) 633-643.

[24] Ünver M., Orhan C., Statistical convergence with respect to power series methods and applications to approximation theory, *Numer. Func. Anal Opt*., 40(5) (2019) 535-547.

[25] Belen C., Yıldırım M., Sümbül C., On Statistical and Strong Convergence with Respect to a Modulus Function and a Power Series Method, *Filomat*, 34 (12) (2020) 3981-3993.A.

[26] Boos J., Classical and modern methods in summability, Oxford University Press, Oxford (2000).