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On Fixed Point Results for Generalized Contractions in Non-Newtonian Metric Spaces

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ABSTRACT

The work of non-Newtonian calculus was begun in 1972. This calculus provides a different area to the classical one. Non-Newtonian metric concept was defined in 2002 by Basar and Cakmak. Then Binbaşıoğlu et al. had given the metric spaces of non-Newtonian in 2016. Also, they started to the fixed-point theory by defining some topological properties in non-Newtonian metric spaces.
In this work, we give some fixed-point theorems and results for self-mappings satisfying certain conditions in the non-Newtonian metric spaces.

Keywords: Fixed point, Non-Newtonian metric space, Contraction mapping, Generalized contraction mapping.

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Introduction

There exist too many studies on fixed-point theory in different spaces [1-12]. Also, there are many applications of the theory and mappings that meet certain conditions of contraction and have been a crucial area of different research works.

The non-Newtonian calculus is alternative to what is customary. The non-Newtonian calculus in various fields including information technology, fractal geometry, economic growth, finance, wave theory, quantum physics, in medicine for examples tumor therapy, cancer-chemotherapy, in mathematics for examples functional analysis, differential equations, approximation theory, problems of decision making, and chaos theory has many applications. The non-Newtonian metric concept was defined in 2002 by Basar et al. and then Binbaşıoğlu et al. gave the metric spaces of non-Newtonian in 2016. Also, they started to study on the fixed-point theory in non-Newtonian metric spaces.

In this work, we present fixed-point theorems and results for self-mappings satisfying certain conditions in the non-Newtonian metric spaces.

Preliminaries

We mention that some basic knowledge related to structure of non-Newtonian calculus.

Definition

A generator is called as an injective function from $\mathbb{R}$ to a subset of $\mathbb{R}$ [6].

Remark

Every generator generates an arithmetic. An arithmetic is generated by a generator [6].

Remark

Let us take the function $\beta\colon \mathbb{R} \to \mathbb{R}^{+}, a \to \beta(a) = e^{a} = b$. If $\beta =$ exp, then the function generates the geometrical arithmetics [6].

Remark

Assume that the function β is a generator, i.e., if $\beta = I$, then β generates the usual arithmetic, where I is an identity mapping [6].

Definition

The β-integers are produced as follows;
β-zero, β-one and similarly all β-integers are denoted as, ..., $\beta(-1)$, $\beta(0)$, $\beta(1)$, ...

Let us take any generator β with range A. Then for $a, b \in \mathbb{R}$, the operations β-addition, β-substraction, β-multiplication, β-division and β-order are defined as follows,

$$a \dot{+} b = \beta\{\beta^{-1}(a) + \beta^{-1}(b)\},$$
$$a \dot{-} b = \beta\{\beta^{-1}(a) - \beta^{-1}(b)\},$$
$$a \dot{\times} b = \beta\{\beta^{-1}(a) \times \beta^{-1}(b)\},$$
$$a \dot{/} b = \beta\{\beta^{-1}(a) \div \beta^{-1}(b)\},$$
$$a \dot{<} b = \beta(a) < \beta(b).$$

The set $\mathbb{R}(N) = \{\beta(a) : a \in \mathbb{R}\}$, is non-Newtonian real numbers set.
For $a \in A \subset \mathbb{R}(N)$, the β-square is described as $a \dot{\times} a$ and denoted with a^{2N}. The notation $\sqrt{a}^{N}$ denotes $k = \beta\{\sqrt{\beta^{-1}(a)}\}$. The β –square is equal to a and which means $k^{2N} = a$.
During this work, a^{pN} denotes the concept of pth non-Newtonian exponent.

$|a|_N$ denotes the β −absolute value for a number $a \in A \subset \mathbb{R}(N)$ defined by $\beta(|\beta^{-1}(a)|)$ and so

$$\sqrt{a^{2N}}^N = |a|_N = \beta(|\beta^{-1}(a)|).$$

Thus,

$$|a|_N = \beta(|\beta^{-1}(a)|) = \begin{cases} a, & \beta(0) \dot{<} a, \\ \beta(0), & \beta(0) = a, \\ \beta(0) \dot{-} a, & \beta(0) \dot{>} a. \end{cases}$$

Let us take any $c \in \mathbb{R}(N)$. If $c \dot{>} \beta(0)$, then c is called a positive non-Newtonian real number. If $c \dot{<} \beta(0)$, then c is called a non-Newtonian negative real number. If $c = \beta(0)$, then c is called an unsigned non-Newtonian real number. Non-Newtonian positive and negative real numbers are denoted by $\mathbb{R}^+(N)$ and $\mathbb{R}^-(N)$ respectively [6].

Definition

Let us take $X \neq \emptyset$ and suppose that $d_N : X \times X \to \mathbb{R}^+(N)$ satisfies the following conditions for $a, b, c \in$ X;

(NM1) $d_N(a,b) = \beta(0)$ iff $a = b$,
(NM2) $d_N(a,b) = d_N(b,a)$,
(NM3) $d_N(a,b) \dot{<} d_N(a,c) \dot{+} d_N(c,b)$.

Then d_N is a non-Newtonian metric on X. Also (X, d_N) is a non-Newtonian metric space [6].

Example

Assume that d_N is defined as $d_N(a,b) = |a \dot{-} b|_N$ for all $a, b \in \mathbb{R}(N)$, then $(\mathbb{R}(N), d_N)$ is a non-Newtonian metric space [6].

Definition

A sequence (a_n) in a non-Newtonian metric space $X = (X, d_N)$ is non-Newtonian convergent if taken any $n_0 = n_0(\varepsilon) \in$ N, $a \in X$ there exists $\varepsilon \dot{>} \beta(0)$ such that for all $n > n_0$, $d_N(a_n, a) \dot{<} \varepsilon$ and it is shown with $\lim_{n\to\infty} a_n = a$ or $a_n \overset{N}{\to} a, n \to \infty$ [5].

Definition

A sequence (a_n) in a non-Newtonian metric space $X = (X, d_N)$ is non-Newtonian Cauchy if taken any $n_0 = n_0(\varepsilon) \in$ N, $a \in X$ there exists $\varepsilon \dot{>} \beta(0)$ such that for all $m, n > n_0$, $d_N(a_n, a_m) \dot{<} \varepsilon$. The non-Newtonian metric space (X, d_N) is non-Newtonian complete if every non-Newtonian Cauchy sequence is non-Newtonian convergent [5].

Remark

Let k, l, m, n, p be non-Newtonian positive real numbers with $k \dot{+} l \dot{+} m \dot{+} n \dot{+} p \dot{<} \beta(1),\ l = m, n = p.$

If $r = (k \dot{+} l \dot{+} n) \dot{\times} (\beta(1) \dot{-} m \dot{-} n)^{-1}$ and $s = (k \dot{+} m \dot{+} p) \dot{\times} (\beta(1) \dot{-} l \dot{-} p)^{-1}$, then $r \dot{\times} s \dot{<} \beta(1)$.
If $l = m$ then

$$r \dot{\times} s = \frac{k \dot{+} l \dot{+} n}{\beta(1) \dot{-} m \dot{-} n} \dot{\times} \frac{k \dot{+} m \dot{+} p}{\beta(1) \dot{-} l \dot{-} p} = \frac{k \dot{+} m \dot{+} n}{\beta(1) \dot{-} l \dot{-} p} \dot{\times} \frac{k \dot{+} l \dot{+} p}{\beta(1) \dot{-} m \dot{-} n} \dot{<} \beta(1),$$

and if $n = p$ then

$$r \dot{\times} s = \frac{k \dot{+} m \dot{+} n}{\beta(1) \dot{-} m \dot{-} n} \dot{\times} \frac{k \dot{+} m \dot{+} p}{\beta(1) \dot{-} l \dot{-} p} = \frac{k \dot{+} l \dot{+} p}{\beta(1) \dot{-} m \dot{-} n} \dot{\times} \frac{k \dot{+} m \dot{+} n}{\beta(1) \dot{-} l \dot{-} p} \dot{<} \beta(1).$$

Main Results

Theorem

Let d_N be a non-Newtonian complete metric on X and c, d be positive integers. If a mapping $K: X \to X$ satisfies

$$d_N(K^c a, K^d b) \dot{\leq} k \dot{\times} d_N(a,b) \dot{+} l \dot{\times} d_N(a, K^c a)$$
$$\dot{+} m \dot{\times} d_N(b, K^d b) \dot{+} n \dot{\times} d_N(a, K^d b) \dot{+} p \dot{\times} d_N(b, K^c a)$$

for all $a, b \in X$, where k, l, m, n, p are non-Newtonian positive real numbers with $k \dot{+} l \dot{+} m \dot{+} n \dot{+} p \dot{<} \beta(1),\ l = m, n = p,$ then K has a unique fixed-point.

Proof

Take $a_0 \in X$, $t \geq \beta(0)$, we construct

$$a_{2t+1} = K^c a_{2t},$$
$$a_{2t+2} = K^d a_{2t+1}.$$

Then

$$d_N(a_{2t+1}, a_{2t+2}) = d_N(K^c a_{2t}, K^d a_{2t+1})$$
$$\dot{\leq} k \dot{\times} d_N(a_{2t}, a_{2t+1}) \dot{+} l \dot{\times} d_N(a_{2t}, K^c a_{2t}) \dot{+} m \dot{\times} d_N(a_{2t+1}, K^d a_{2t+1})$$
$$\dot{+} n \dot{\times} d_N(a_{2t}, K^d a_{2t+1}) \dot{+} p(a_{2t+1}, K^c a_{2t})$$
$$\dot{\leq} (k \dot{+} l) \dot{\times} d_N(a_{2t}, a_{2t+1}) \dot{+} m \dot{\times} d_N(a_{2t+1}, a_{2t+2}) \dot{+} n \dot{\times} d_N(a_{2t}, a_{2t+2})$$
$$\dot{\leq} (k \dot{+} l \dot{+} n) \dot{\times} d_N(a_{2t}, a_{2t+1}) \dot{+} (m \dot{+} n) \dot{\times} d_N(a_{2t+1}, a_{2t+2}).$$

It implies that

$$\dot{\le}(\beta(1) \dot{-} m \dot{-} n) \dot{\times} d_N(a_{2t+1}, a_{2t+2}) \dot{\le} (k \dot{+} l \dot{+} n) \dot{\times} d_N(a_{2t}, a_{2t+1}).$$

So

$d_N(a_{2t+1}, a_{2t+2}) \dot{\le} r \dot{\times} d_N(a_{2t}, a_{2t+1})$, where $r = \frac{(k \dot{+} l \dot{+} n)}{\beta(1) \dot{-} m \dot{-} n}$.

$$\begin{aligned} d_N(a_{2t+2}, a_{2t+3}) &= d_N(K^c a_{2t+2}, K^d a_{2t+1}) \\ &\dot{\le} k \dot{\times} d_N(a_{2t+2}, a_{2t+1}) \dot{+} l \dot{\times} d_N(a_{2t+2}, K^c a_{2t+2}) \dot{+} m \dot{\times} d_N(a_{2t+1}, K^d a_{2t+1}) \\ &\dot{+} n \dot{\times} d_N(a_{2t+2}, K^d a_{2t+1}) \dot{+} p(a_{2t+1}, K^c a_{2t+2}) \\ &\dot{\le} k \dot{\times} d_N(a_{2t+2}, a_{2t+1}) \dot{+} l \dot{\times} d_N(a_{2t+2}, a_{2t+3}) \dot{+} m \dot{\times} d_N(a_{2t+1}, a_{2t+2}) \\ &\dot{+} n \dot{\times} d_N(a_{2t+2}, a_{2t+2}) \dot{+} p \dot{\times} d_N(a_{2t+1}, a_{2t+3}) \\ &\dot{\le} (k \dot{+} m \dot{+} p) \dot{\times} d_N(a_{2t+1}, a_{2t+2}) \dot{+} (l \dot{+} p) \dot{\times} d_N(a_{2t+2}, a_{2t+3}), \end{aligned}$$

implies that

$d_N(a_{2t+2}, a_{2t+3}) \dot{\le} s \dot{\times} d_N(a_{2t+1}, a_{2t+2})$,

where $s = \frac{(k \dot{+} m \dot{+} p)}{\beta(1) \dot{-} l \dot{-} p}$.

Therefore, we get for each $t = 0,1,2,\ldots$

$$\begin{aligned} d_N(a_{2t+1}, a_{2t+2}) &\dot{\le} r \dot{\times} d_N(a_{2t}, a_{2t+1}) \\ &\dot{\le} r \dot{\times} s \dot{\times} d_N(a_{2t-1}, a_{2t}) \\ &\dot{\le} r \dot{\times} (r \dot{\times} s) \dot{\times} d_N(a_{2t-2}, a_{2t-1}) \\ &\dot{\le} \cdots \dot{\le} r \dot{\times} (r \dot{\times} s)^{tN} \dot{\times} d_N(a_0, a_1), \\ d_N(a_{2t+2}, a_{2t+3}) &\dot{\le} s \dot{\times} d_N(a_{2t+1}, a_{2t+2}) \\ &\dot{\le} \cdots \dot{\le} (r \dot{\times} s)^{(t \dot{+} 1)N} \dot{\times} d_N(a_0, a_1). \end{aligned}$$

So, for $y < z$ we have

$$\begin{aligned} d_N(a_{2y+1}, a_{2z+1}) &\dot{\le} d_N(a_{2y+1}, a_{2y+2}) \\ &\dot{+} d_N(a_{2y+2}, a_{2y+3}) \dot{+} \cdots \dot{+} d_N(a_{2z}, a_{2z+1}) \\ &\dot{\le} [r \dot{\times} \sum_{i=y}^{z-1} (r \dot{\times} s)^{iN} \dot{+} \sum_{i=y+1}^{z} (r \dot{\times} s)^{iN}] \dot{\times} d_N(a_0, a_1) \\ &\dot{\le} [\frac{r \dot{\times} (r \dot{\times} s)^{yN}}{\beta(1) \dot{-} r \dot{\times} s} \dot{+} \frac{(r \dot{\times} s)^{(y+1)N}}{\beta(1) \dot{-} r \dot{\times} s}] \dot{\times} d_N(a_0, a_1) \\ &\dot{\le} (\beta(1) \dot{+} r) \dot{\times} [\frac{(r \dot{\times} s)^{yN}}{\beta(1) \dot{-} r \dot{\times} s}] \dot{\times} d_N(a_0, a_1). \end{aligned}$$

Then we deduced

$$\begin{aligned} d_N(a_{2y}, a_{2z+1}) &\dot{\le} (\beta(1) \dot{+} r) \dot{\times} [\frac{(r \dot{\times} s)^{yN}}{\beta(1) \dot{-} r \dot{\times} s}] \dot{\times} d_N(a_0, a_1), \\ d_N(a_{2y}, a_{2z}) &\dot{\le} (\beta(1) \dot{+} r) \dot{\times} [\frac{(r \dot{\times} s)^{yN}}{\beta(1) \dot{-} r \dot{\times} s}] \dot{\times} d_N(a_0, a_1), \\ d_N(a_{2y+1}, a_{2z}) &\dot{\le} (\beta(1) \dot{+} r) \dot{\times} [\frac{(r \dot{\times} s)^{yN}}{\beta(1) \dot{-} r \dot{\times} s}] \dot{\times} d_N(a_0, a_1). \end{aligned}$$

For $0 < w < v$, $d_N(a_w, a_v) \dot{\le} q_w$, where

$q_w = (\beta(1) \dot{+} r) \dot{\times} [\frac{(r \dot{\times} s)^{yN}}{\beta(1) \dot{-} r \dot{\times} s}] \dot{\times} d_N(a_0, a_1)$ with an integer part of $\frac{w}{2}$.

So $\{a_w\}$ is non-Newtonian Cauchy. Since (X, d_N) is non-Newtonian complete, there exists $x \in X$ such that

$a_w \overset{N}{\rightarrow} x.$

For a non-Newtonian real number $0 \dot{<} \beta(e)$, choose $d_0 \in \mathbb{N}$ such that

$d_N(x, a_{2t}) \dot{<} \frac{\beta(e)}{\beta(3) \dot{\times} A}, d_N(a_{2t-1}, a_{2t}) \dot{<} \frac{\beta(e)}{\beta(3) \dot{\times} A}, d_N(x, a_{2t-1}) \dot{<} \frac{\beta(e)}{\beta(3) \dot{\times} A},$

for all $t \geq d_0$, where

$$A = max\{\frac{\beta(1) \dot{+} n}{\beta(1) \dot{-} l \dot{-} p}, \frac{k \dot{+} p}{\beta(1) \dot{-} l \dot{-} p}, \frac{m}{\beta(1) \dot{-} l \dot{-} p}\}.$$

Now,

$$\begin{aligned} d_N(x, K^v a) &\dot{\leq} d_N(x, a_{2t}) \dot{+} d_N(a_{2t}, K^v x) \\ &\dot{\leq} d_N(x, a_{2t}) \dot{+} d_N(K^w a_{2t-1}, K^v x) \\ &\dot{\leq} d_N(x, a_{2t}) \dot{+} k \dot{\times} d_N(x, a_{2t-1}) \dot{+} l \dot{\times} d_N(x, K^v x) \dot{+} m \dot{\times} d_N(a_{2t-1}, K^w a_{2t-1}) \\ &\dot{+} n \dot{\times} d_N(x, K^w a_{2t-1}) \dot{+} p \dot{\times} d_N(a_{2t-1}, K^v x) \\ &\dot{\leq} d_N(x, a_{2t}) \dot{+} k \dot{\times} d_N(x, a_{2t-1}) \dot{+} l \dot{\times} d_N(x, K^v x) \dot{+} m \dot{\times} d_N(a_{2t-1}, a_{2t}) \\ &\dot{+} n \dot{\times} d_N(x, a_{2t}) \dot{+} p \dot{\times} d_N(a_{2t-1}, x) \dot{+} p \dot{\times} d_N(x, K^v x) \\ &\dot{\leq} (\beta(1) \dot{+} n) \dot{\times} d_N(x, a_{2t}) \dot{+} (k \dot{+} p) \dot{\times} d_N(x, a_{2t-1}) \\ &\dot{+} m \dot{\times} d_N(a_{2t-1}, a_{2t}) \dot{+} (l \dot{+} p) \dot{\times} d_N(x, K^v x). \\ d_N(x, K^v x) &\dot{\leq} A \dot{\times} d_N(x, a_{2t}) \dot{+} A \dot{\times} d_N(x, a_{2t-1}) \dot{+} A \dot{\times} d_N(a_{2t-1}, a_{2t}) \\ &\dot{\leq} \frac{\beta(e)}{\beta(3)} \dot{+} \frac{\beta(e)}{\beta(3)} \dot{+} \frac{\beta(e)}{\beta(3)} = \beta(e). \end{aligned}$$

Therefore

$d_N(x, K^v x) \dot{\leq} \frac{\beta(e)}{\beta(y)}$ for every $y \in \mathbb{N}$. From $\frac{\beta(e)}{\beta(y)} \dot{-} d_N(x, K^v x) \dot{\geq} \beta(0)$ we have $d_N(x, K^v x) = \beta(0)$. This implies that $x = K^v x$.

By using the inequality,

$$d_N(x, K^w x) \dot{\leq} d_N(x, a_{2t+1}) \dot{+} d_N(a_{2t+1}, K^w x),$$

now we show that $x = K^w x$.

$$\begin{aligned} d_N(Kx, x) = d_N(KK^v x, K^w x) &= d_N(K^v Kx, K^w x) \\ &\dot{\leq} k \dot{\times} d_N(Kx, x) \dot{+} l \dot{\times} d_N(Kx, K^v Kx) \\ &\dot{+} m \dot{\times} d_N(x, K^v x) \dot{+} n \dot{\times} d_N(Kx, K^w x) \dot{+} p \dot{\times} d_N(x, K^v Kx) \\ &\dot{\leq} k \dot{\times} d_N(Kx, x) \dot{+} l \dot{\times} d_N(Kx, Kx) \\ &\dot{+} m \dot{\times} d_N(x, x) \dot{+} n \dot{\times} d_N(Kx, x) \dot{+} p \dot{\times} d_N(x, Kx) \\ &= (k \dot{+} n \dot{+} p) \dot{\times} d_N(Kx, x). \end{aligned}$$

So x is a fixed-point of K.
We suppose that for some x^*, there exists another point $x^* \in X$ such that $x^* = Kx^*$. Thus, we have

$$\begin{aligned} d_N(x, x^*) &= d_N(K^v x, K^w x^*) \\ &\dot{\leq} k \dot{\times} d_N(x, x^*) \dot{+} l \dot{\times} d_N(x, K^v x) \\ &\dot{+} m \dot{\times} d_N(x^*, K^w x^*) \dot{+} n \dot{\times} d_N(x, K^w x^*) \dot{+} p \dot{\times} d_N(x^*, K^v x) \\ &\dot{\leq} k \dot{\times} d_N(x, x^*) \dot{+} l \dot{\times} d_N(x, x) \\ &\dot{+} m \dot{\times} d_N(x^*, x^*) \dot{+} n \dot{\times} d_N(x, x^*) \dot{+} p \dot{\times} d_N(x, x^*) \\ &\dot{\leq} (k \dot{+} n \dot{+} p) \dot{\times} d_N(x, x^*). \end{aligned}$$

Consequently, x^* is equal to x.

Theorem

Let d_N be non-Newtonian complete metric on X. If $K:X \rightarrow X$ satisfies

$$\begin{aligned} d_N(Ka, Kb) &\dot{\leq} k \dot{\times} d_N(a, b) \dot{+} l \dot{\times} d_N(a, Ka) \\ &\dot{+} m \dot{\times} d_N(b, Kb) \dot{+} n \dot{\times} d_N(a, Kb) \dot{+} p \dot{\times} d_N(b, Ka) \end{aligned}$$

for all $a, b \in X$, where k, l, m, n, p are non-Newtonian positive real numbers with $k \dot{+} l \dot{+} m \dot{+} n \dot{+} p \dot{<} \beta(1)$, then K has a unique fixed-point.

Proof

Since d_N is a non-Newtonian metric, the above inequality implies that

$$d_N(Ka, Kb) \dot{\leq} k \dot{\times} d_N(Ka, Kb) \dot{+} \frac{l \dot{+} m}{\beta(2)} \dot{\times} [d_N(a, Ka) \dot{+} d_N(b, Kb)] \dot{+} \frac{n \dot{+} p}{\beta(2)} \dot{\times} [d_N(a, Kb) \dot{+} d_N(b, Ka)].$$

If we substitute $K^v = K^w = K$ in the above theorem, we get the required result.

Corollary

Let (X, d_N) be a non-Newtonian complete metric space and v, w be positive integers. If a self-mapping K on X satisfies

$$d_N(K^v a, K^w b) \dot{\leq} k \dot{\times} d_N(a, b) \dot{+} l \dot{\times} d_N(a, K^v a) \dot{+} m \dot{\times} d_N(b, K^w b) \dot{+} n \dot{\times} d_N(a, K^w b) \dot{+} p \dot{\times} d_N(b, K^v a)$$

for all $a, b \in X$, where k, l, m, n, p be non-Newtonian positive real numbers with $k \dot{+} l \dot{+} m \dot{+} n \dot{+} p \dot{<} \beta(1), l = m, n = p,$ then K has a unique fixed-point.

Conclusion

In this paper, we use the concept of non-Newtonian metric space and present some new fixed-point theorems. We expect that our research results can offer a mathematical basis. In the future research, we will explore so concrete applications of the obtained results, here.

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Conflicts of Interest

The author states that did not has conflict of interests.

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