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Neutrosophic triplets in some neutrosophic rings

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Abstract

In this paper, some mistakes about the neutrosophic triplets of some neutrosophic rings in the literature are pointed out and corrected. For this purpose, the neutrosophic triplets in neutrosophic rings $\langle Z \cup I \rangle$, $\langle Q \cup I \rangle$ and $\langle R \cup I \rangle$ where Z, Q and R denote the ring of integers, field of rationals and field of reals respectively are reinvestigated. It was claimed that $\langle Z \cup I \rangle$ has only trivial neutrosophic triplet in a paper which was recently published in Mathematics. But, as a result of the calculations, it was seen that $\langle Z \cup I \rangle$ has non-trivial neutrosophic triplets. Also neutrosophic triplets of the rings $\langle Q \cup I \rangle$ and $\langle R \cup I \rangle$ in the same literature was calculated incomplete.

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1. Introduction

The theories of neutrosophic set and neutrosophic triplets are introduced by F. Smarandache. The concept of neutrosophic set is a generalization of intuitionistic fuzzy sets [1]. F. Smarandache and M. Ali in [2, 3], for the first time, introduced the new notion of neutrosophic triplet group (NTG), which is another generalization of classical group. However, it is different from a group. In a NTG, the neutral element is different from the unit element of the classical group theory. By removing this restriction ([2,4]), it is defined neutrosophic extended triplet group (NETG) and the classical group is regarded as a special case of NETG. In [5], M. Ali, F. Smarandache and M. Khan introduced the concept of neutrosophic triplet ring and study some of its basic properties. Until now, for neutrosophic triplet group and neutrosophic triplet ring, some research articles (for example [3-7]) are published. But, at the same time, there are still some misunderstandings, mistakes about this new algebraic structure. In some papers, some authors clarified and corrected these misunderstandings and mistakes ([4,8,9]). This paper will clarify some misunderstandings, especially pointing out some erroneous conclusions in [7] and will try to give improved results.

2. Basic Concepts

In this section, we recall some of the basic concepts and properties associated with both neutrosophic triplets, neutrosophic groups and neutrosophic rings.

Definition 2.1 ([4]) Let N be a non-empty set together with a binary operation $*$. Then, N is called a neutrosophic extended triplet set if there exists a neutral of "a" (denoted by neut(a)) for any $a \in N$, and an opposite of "a" (denoted by anti(a)) for any $a \in N$, such that $neut(a) \in N$, $anti(a) \in N$ and

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$$a * neut(a) = neut(a) * a = a\ ,\ a * anti(a) = anti(a) * a = neut(a)\ .$$

The triplet (a, neut(a), anti(a)) is called a neutrosophic extended triplet.

Definition 2.2 ([4]) Let $(N,*)$ be a neutrosophic extended triplet set. Then, N is called a neutrosophic extended triplet group (NETG), if the following conditions are satisfied:

(1) $(N,*)$ is well-defined, i.e., $a*b \in N$, for all $a,b \in N$,
(2) $(N,*)$ is associative, i.e., $(a*b)*c = a*(b*c)$ for all $a,b,c \in N$.

A NETG N is called a commutative NETG if $a*b = b*a$ for all $a,b \in N$.

Definition 2.3 ([5]) Let N be a neutrosophic extended triplet set together with two binary operations $*$ and #. Then N is called a neutrosophic extended triplet ring (NETR) if the following conditions hold:

1. (N, $*$) is a commutative neutrosophic triplet group,
2. (N, #) is well-defined and associative,
3. $a\#(b*c) = (a\#b)*(a\#c)$ and $(b*c)\#a = (b\#a)*(c\#a)$ for all $a,b,c \in N$.

Remark 1. A NETR in general is not a classical ring.

Let I denote the indeterminate which satisfies $I^2 = I$. Z, Q and R denote the ring of integers, the field of rationals and the field of reals, respectively. The neutrosophic ring of integers, the field of neutrosophic rationals and the field of neutrosophic reals with usual addition and multiplication in all the three rings as the following, respectively:

$$\langle Z \cup I \rangle = \{a + bI : a,b \in Z\}\ ,$$
$$\langle Q \cup I \rangle = \{a + bI : a,b \in Q\}\ ,$$
$$\langle R \cup I \rangle = \{a + bI : a,b \in R\}\ .$$

3. Results and Discussion

In [7], the authors claimed that $\langle Z \cup I \rangle$ has no non-trivial neutrosophic triplets. In this section, we find the neutrosophic triplets in rings $\langle Z \cup I \rangle$, $\langle Q \cup I \rangle$ and $\langle R \cup I \rangle$. We see that $\langle Z \cup I \rangle$ has non-trivial neutrosophic triplets.

Theorem 3.1 The neutrosophic triplets in ring $\langle Z \cup I \rangle$ are as the following:
i) $(0,\ 0, u+vI)$ where $u,v \in Z$,
(ii) $(I, I, u+(1-u)I)$, $(-I, I, u-(1+u)I)$ where $u \in Z$,
(iii) $(1,1,1)$ and $(-1,1,-1)$,
(iv) $(1-2I, 1, 1-2I)$ and $(-1+2I, 1, -1+2I)$,
(v) $(1-I,\ 1-I,\ 1+vI)$ and $(-1+I,\ 1-I,\ -1+vI)$ for $v \in Z$.
Proof. Let $a+bI \in Z \cup I$, $neut(a+bI) = x+yI$ and $anti(a+bI) = u+vI$ where $a,b,x,\ y,u,v \in Q$. Then we have

$$(a+bI)(x+yI) = a+bI \tag{3.1}$$

and

$$(a+bI)(u+vI)=x+yI\,. \tag{3.2}$$

Using the Eq.(3.1), we get

$$ax=a \ \text{ and } \ ay+bx+by=b \tag{3.3}$$

and using the Eq. (3.2) we get

$$au=x \text{ and } av+bu+bv=y\,. \tag{3.4}$$

Hence we have the following cases:

a) If $a=0$ and $b=0$, using the Eqs. (3.3) and (3.4), we have $x=0,\ y=0$. Hence $\text{neut}(0+0I)=0+0I$ and $\text{anti}(0+0I)=u+vI$, where $u,v\in Z$. Then the elements $(0,\ 0,u+vI)$ where $u,v\in Z$ are neutrosophic triplets in $\langle Z\cup I\rangle$.

b) If $a=0$ and $b\neq 0$, using the Eqs. (3.3), (3.4), we have $x=0,\ y=1$ and $b(u+v)=1$. Hence $(b=1 \text{ and } v=1-u)$ or $(b=-1 \text{ and } v=-1-u)$. So we obtain $\text{neut}(I)=I$, $\text{anti}(I)=u+(1-u)I$, $\text{neut}(-I)=I$ and $\text{anti}(-I)=u-(1+u)I$ where $u\in Z$. Then the elements $(I,I,u+(1-u)I)$ and $(-I,I,u-(1+u)I)$ where $u\in Z$ are neutrosophic triplets in $\langle Z\cup I\rangle$.

c) If $a\neq 0$ and $b=0$, we have $x=1,\ y=0,\ v=0$ and $au=1$ using the Eqs. (3.3),(3.4). Hence it must be $a=u=1$ or $a=u=-1$. So we obtain $\text{neut}(1)=1$, $\text{anti}(1)=1$, $\text{neut}(-1)=1$ and $\text{anti}(-1)=-1$. Then the elements $(1,1,1)$ and $(-1,1,-1)$ are neutrosophic triplets in $\langle Z\cup I\rangle$.

d) Let $a\neq 0$ and $b\neq 0$. We have $x=1$ and $(a+b)y=0$ using the Eqs. (3.3). In this case, $y=0$ or $b=-a$.

d_i) If y=0, puting x=1 and y=0 in the Eqs. in (3.4), we get $au=1$ and $av+bu+bv=0$. Hence in case $a=u=1$ we have $b=v=-2$ and in case $a=u=-1$ we get $b=v=2$. So $(1-2I,1,1-2I)$ and $(-1+2I,1,-1+2I)$ are neutrosophic triplets in $\langle Z\cup I\rangle$.

d_{ii}) If $b=-a$, puting $x=1$ and $b=-a$ in Eqs. in (3.4) we have $au=1$ and $y=-1$. Then neutrosophic triplets are $(1-I,\ 1-I,\ 1+vI)$ for $v\in Z$ when $a=u=1,\ y=-1$ and $(-1+I,\ 1-I,\ -1+vI)$ for $v\in Z$ when $a=u=-1,\ y=-1$.

Theorem 3.2 The neutrosophic triplets in the ring $\langle Q\cup I\rangle$ are as the following:

i) $(0,\ 0,u+vI)$ where $u,v\in Q$,

(ii) $(bI,I,u+vI)$ where $b\neq 0, u+v=\frac{1}{b}$ and $b,u,v\in Q$,

(iii) $\left(a,1,\frac{1}{a}\right)$ where $a\neq 0$ and $a\in Q$,

(iv) $\left(a+bI,1,\frac{1}{a}-\frac{b}{a(a+b)}I\right)$ where $a\neq 0, b\neq 0$ and $a+b\neq 0$,

(v) $\left(a-aI,\ 1-I,\ \frac{1}{a}+vI\right)$ where $a\neq 0,\ v\in Q$.

Proof. Let $a+bI\in Q\cup I$, $\text{neut}(a+bI)=x+yI$ and $\text{anti}(a+bI)=u+vI$ where $a,b,x,\ y,u,v\in Q$. Then the Eqs. (3.1)-(3.4) are true.

Hence we have the following cases:

a) If $a=0$ and $b=0$, using the Eqs. (3.3) and (3.4), we have $x=0,\ y=0$. Hence $\text{neut}(0+0I)=0+0I$ and $\text{anti}(0+0I)=u+vI$, where $u,v\in Q$. Then the elements $(0,\ 0,u+vI)$ where $u,v\in Q$ are neutrosophic triplets in $\langle Q\cup I\rangle$.

b) If $a=0$ and $b\neq 0$, using the Eqs. (3.3), (3.4), we have $x=0,\ y=1$ and $b(u+v)=1$. Hence we get $u+v=\frac{1}{b}$. So we obtain $\text{neut}(bI)=I$, $\text{anti}(bI)=u+vI$ where $u+v=\frac{1}{b}$. Then the elements $(bI,I,u+vI)$ where $u+v=\frac{1}{b}$ are neutrosophic triplets in $\langle Q\cup I\rangle$.

c) If $a\neq 0$ and $b=0$, we have $x=1,\ y=0,\ v=0$ and $au=1$ using the Eqs. (3.3), (3.4). Hence it must be $u=\frac{1}{a}$. So we obtain $\text{neut}(a)=1$, $\text{anti}(a)=\frac{1}{a}$. Then the elements $\left(a,1,\frac{1}{a}\right)$ where $a\neq 0$ and $a\in Q$ are neutrosophic triplets in $\langle Q\cup I\rangle$.

d) Let $a\neq 0$ and $b\neq 0$. We have $x=1$ and $(a+b)y=0$ using the Eqs. (3.3). In this case, $y=0$ or $a+b=0$.

d_i) If y=0 and $a+b\neq 0$, puting x=1 and y=0 in the Eqs. in (3.4), we get $au=1$ and $av+bu+bv=0$. Hence we have $u=\frac{1}{a}$ and $v=-\frac{b}{a(a+b)}$. So $\text{neut}(a+bI)=1$ and $\text{anti}(a+bI)=\frac{1}{a}-\frac{b}{a(a+b)}I$. That is, the elements $\left(a+bI,1,\frac{1}{a}-\frac{b}{a(a+b)}I\right)$ where $a\neq 0,\ b\neq 0$ and $a+b\neq 0$ are neutrosophic triplets in $\langle Q\cup I\rangle$.

d_{ii}) If $b=-a$ and $y\neq 0$, puting $x=1$ and $b=-a$ in Eqs. in (3.4), we have $au=1$ and $y=-1$. Then neutrosophic triplets are $\left(a-aI,\ 1-I,\ \frac{1}{a}+vI\right)$ where $a\neq 0$.

Remark 3.3 According to Theorem 3.2 (v), Example 1 in [7] is missing. The elements $\left(a-aI,1-I,\frac{1}{a}-\frac{1}{a}I\right)$ are neutrosophic triplets. But the elements $\left(a-aI,\ 1-I,\ \frac{1}{a}+vI\right)$ for $a,v\in Q$ and $a\neq 0$ which contain the elements $\left(a-aI,1-I,\frac{1}{a}-\frac{1}{a}I\right)$ are neutrosophic triplets. According to Example 1 in [7], the element $\left(2-2I,1-I,\frac{1}{2}+4I\right)$ is not a neutrosophic element. But this is a neutrosophic element. Also, since $a\in Q^*$, the elements $\left(a-aI,1-I,\frac{1}{a}-\frac{1}{a}I\right)$ contain the elements $\left(\frac{1}{a}-\frac{1}{a}I,1-I,a-aI\right)$. That is, no need to write the elements $\left(\frac{1}{a}-\frac{1}{a}I,1-I,a-aI\right)$ as new neutrosophic triplets. Same is valid for $\left(aI,\ I,\ \frac{I}{a}\right)$ and $\left(\frac{I}{a},\ I,\ aI\right)$, $\left(a+bI,\ 1,\ \frac{1}{a}-\frac{b}{a(a+b)}I\right)$ and $\left(\frac{1}{a}-\frac{b}{a(a+b)}I,\ 1,\ a+bI\right)$.

Remark 3.4 In $Q\cup I$, for any $0\neq a\in Q$, neut($a-aI$)=$1-I$ and anti($a-aI$)$=\left\{\frac{1}{a}+vI:v\in Q\right\}$. Hence the collection of the neutrosophic triplets of $Q\cup I$ with neutral $1-I$ is the set

$$N=\left\{\left(a-aI,1-I,\frac{1}{a}+vI\right):a,v\in Q\ \text{and}\ a\neq 0\right\}.$$

So the set M in Theorem 3 in [7] is not true. Actually $M\subset N$. The set N is not a group under component-wise product.

Theorem 3.5 The set of the neutrosophic triplets of $Z \cup I$ with neutral 1 is a commutative group of order 4 under component-wise product.

Proof. The set is

$$N = \{(1,1,1),(-1,1,-1),(1-2I,1,1-2I),(-1+2I,1,-1+2I)\}.$$

It is easily seen that the set N is closed under component-wise product. $(1,1,1)$ is the identity element, the inverse of every element is itself. Also, the set is associative.

In this paper, some mistakes about the neutrosophic triplets of neutrosophic rings in [7] are pointed out and corrected. For this purpose, the neutrosophic triplets in neutrosophic rings $\langle Z \cup I \rangle$, $\langle Q \cup I \rangle$ and $\langle R \cup I \rangle$ are reinvestigated. As a result of the calculations, it was seen that $\langle Z \cup I \rangle$ has non-trivial neutrosophic triplets. Also neutrosophic triplets of the rings $\langle Q \cup I \rangle$ and $\langle R \cup I \rangle$ in [7] was calculated incomplete. In this paper, we gave all neutrosophic triplets in those rings.

Conflicts of interest

The authors state that they did not have a conflict of interest.

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