A Study on First-Order Fuzzy Initial Value Problems with Positive and Negative Fuzzy Number Coefficients by Fuzzy Laplace Transform

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Abstract. In this paper, fuzzy initial value problems for first-order fuzzy differential equations with positive and negative fuzzy number coefficients are studied by fuzzy Laplace transform. Solutions are found under the approach of generalized differentiability. Examples are solved. Figures of the solutions are drawn using the Mathematica program. Finally, conclusions are given.

Keywords: Fuzzy Initial Value Problem, Generalized Differentiability, Fuzzy Laplace Transform, Positive and Negative Fuzzy Numbers.

Fuzzy Laplace Dönüşümüyle Pozitif ve Negatif Fuzzy Sayı Katsayılı Birinci-Mertebe Fuzzy Başlangıç Değer Problemleri Üzerine Bir Çalışma

Özet. Bu çalışmada, fuzzy Laplace dönüşümüyle pozitif ve negatif fuzzy sayı katsayılı birinci-mertebe fuzzy diferansiyel denklemler için fuzzy başlangıç değer problemleri çalışıldı. Çözümler genelleştirilmiş diferansiyellenebilirlik yaklaşımı altında bulundu. Örnekler çözüldü. Çözümlerin şekilleri Mathematica programı kullanarak çizildi. Son olarak, sonuçlar verildi.

Anahtar Kelimeler Fuzzy Başlangıç Değer Problem, Genelleştirilmiş Diferansiyellenebilirlik, Fuzzy Laplace Dönüşüm, Pozitif ve Negatif Fuzzy Sayılar.

1. INTRODUCTION

The topic of fuzzy differential equations has been rapidly grown in current years. The term of ''fuzzy differetial equation'' was introduced by Kandel and Byatt [1]. The fuzzy differential equations are extensively used in applied mathematics, physics and engineering. Many researchers study fuzzy differential equation with different approach. The first approach is on the Zadeh's extenciple [2]. The second approach is differential inclusion [3]. The third approach is Hukuhara derivative [4-5] or generalized derivative [6-8].

Fuzzy Laplace transform is very useful to solve fuzzy differential equation. In many papers, fuzzy Laplace transform was studied [9-11].

This paper is on solutions of fuzzy initial value problems for first-order fuzzy differential equations with positive and negative fuzzy number coefficients by fuzzy Laplace transform.

Definition 1.1 [12] A fuzzy number is a function $u: \mathbb{R} \to [0,1]$ satisfying the following properties:

u is normal, u is convex fuzzy set, u is upper s
emi-continuous on $\mathbb{R}$ and $cl\{x \in \mathbb{R}|u(x) > 0\}$ is compact, where cl denotes the closure of a subset. Let $\mathbb{R}_F$ be the space of fuzzy numbers.

Definition 1.2 [12] Let be $u \in \mathbb{R}_F$. The α-level set of u is $[u]^\alpha = \{x \in \mathbb{R}|u(x) \geq \alpha\}, 0 < \alpha \leq 1$.

If $\alpha = 0$, the support of u is $[u]^0 = cl\{x \in \mathbb{R}|u(x) > 0\}$.

Definition 1.3 [13] A fuzzy number u in parametric form is a pair $[\underline{u}_\alpha, \overline{u}_\alpha]$ of functions $\underline{u}_\alpha, \overline{u}_\alpha, 0 \leq \alpha \leq 1$, which satisfy the following requirements:

1. $\underline{u}_\alpha$ is bounded non-decreasing left-continuous in (0,1], right-continuous at $\alpha = 0$.

2. $\overline{u}_\alpha$ is bounded non-increasing left-continuous in (0,1] , right-continuous at $\alpha = 0$.

3. $\underline{u}_\alpha \leq \overline{u}_\alpha, 0 \leq \alpha \leq 1$.

Definition 1.4 [14] A fuzzy number u is called positive (negative), denoted by $u > 0$ ($u < 0$), if its membership function $u(x)$ satisfies $u(x) = 0, \forall x < 0$ ($x > 0$).

Definition 1.5 [12] If A is a symmetric triangular number with support $[\underline{a}, \overline{a}]$, the α-level set of A is

$$[A]^\alpha = \left[\underline{a} + \left(\frac{\overline{a}-\underline{a}}{2}\right)\alpha, \overline{a} - \left(\frac{\overline{a}-\underline{a}}{2}\right)\alpha\right].$$

Definition 1.6 [15] Let be $u, v \in \mathbb{R}_F$, $[u]^\alpha = [\underline{u}_\alpha, \overline{u}_\alpha]$, $[v]^\alpha = [\underline{v}_\alpha, \overline{v}_\alpha]$, the product $u.v$ is defined by

$$[u.v]^\alpha = [u]^\alpha.[v]^\alpha, \forall \alpha \in [0,1],$$

where

$$[u]^\alpha.[v]^\alpha = [\underline{u}_\alpha, \overline{u}_\alpha][\underline{v}_\alpha, \overline{v}_\alpha] = [\underline{w}_\alpha, \overline{w}_\alpha],$$

$$\underline{w}_\alpha = min\{\underline{u}_\alpha\underline{v}_\alpha, \underline{u}_\alpha\overline{v}_\alpha, \overline{u}_\alpha\underline{v}_\alpha, \overline{u}_\alpha\overline{v}_\alpha\}, \overline{w}_\alpha = max\{\underline{u}_\alpha\underline{v}_\alpha, \underline{u}_\alpha\overline{v}_\alpha, \overline{u}_\alpha\underline{v}_\alpha, \overline{u}_\alpha\overline{v}_\alpha\}.$$

Definition 1.7 [16] Let be $u, v \in \mathbb{R}_F$. If there exists $w \in \mathbb{R}_F$ such that $u = v + w$ then w is called the H-difference of u and v and it is denoted $u \ominus v$.

Definition 1.8 [16] Let be $f: [a, b] \to \mathbb{R}_F$ and $t_0 \in [a, b]$. If there exists $f'(t_0) \in \mathbb{R}_F$ such that for all $h > 0$ sufficiently small, $\exists f(t_0 + h) \ominus f(t_0), f(t_0) \ominus f(t_0 - h)$ and the limits hold

$$\lim_{h \to 0} \frac{f(t_0 + h) \ominus f(t_0)}{h} = \lim_{h \to 0} \frac{f(t_0) \ominus f(t_0 - h)}{h} = f'(t_0),$$

f is (1)-differentiable at t_0. If there exists $f'(t_0) \in \mathbb{R}_F$ such that for all $h > 0$ sufficiently small, $\exists f(t_0) \ominus f(t_0 + h), f(t_0 - h) \ominus f(t_0)$ and the limits hold

$$\lim_{h \to 0} \frac{f(t_0) \ominus f(t_0 + h)}{-h} = \lim_{h \to 0} \frac{f(t_0 - h) \ominus f(t_0)}{-h} = f'(t_0),$$

f is (2)-differentiable.

Definition 1.9 [13] The fuzzy Laplace transform of fuzzy function f is

$$F(s) = L(f(t)) = \int_0^\infty e^{-st} f(t)dt = \left[\lim_{\rho\to\infty} \int_0^\rho e^{-st} \underline{f}(t)dt, \lim_{\rho\to\infty} \int_0^\rho e^{-st} \overline{f}(t)dt\right].$$

$$F(s,\alpha) = L([f(t)]^\alpha) = \left[L\left(\underline{f_\alpha}(t)\right), L\left(\overline{f}_\alpha(t)\right)\right],$$

$$L\left(\underline{f_\alpha}(t)\right) = \int_0^\infty e^{-st} \underline{f_\alpha}(t)dt = \lim_{\rho\to\infty} \int_0^\rho e^{-st} \underline{f_\alpha}(t)dt,$$

$$L\left(\overline{f}_\alpha(t)\right) = \int_0^\infty e^{-st} \overline{f}_\alpha(t)dt = \lim_{\rho\to\infty} \int_0^\rho e^{-st} \overline{f}_\alpha(t)dt.$$

Theorem 1.1 [9] Let $f'(t)$ be an integrable fuzzy function and $f(t)$ is primitive of $f'(t)$ on $(0, \infty]$.

1. If f is (1)-differentiable, $L(f'(t)) = sL(f(t)) \ominus f(0)$.

2. If f is (2)-differentiable, $L(f'(t)) = (-f(0)) \ominus (-sL(f(t)))$.

2. RESULTS AND DISCUSSION

Consider the fuzzy problem

$$u'(t) + [\mu]^\alpha u(t) = [\rho]^\alpha, \; t > 0 \tag{1}$$

$$u(0) = [\omega]^\alpha, \tag{2}$$

by the fuzzy Laplace transform, where $[\mu]^\alpha = \left[\underline{\mu_\alpha}, \overline{\mu}_\alpha\right]$, $[\rho]^\alpha = \left[\underline{\rho_\alpha}, \overline{\rho}_\alpha\right]$, $[\omega]^\alpha = \left[\underline{\omega_\alpha}, \overline{\omega}_\alpha\right]$ are symmetric triangular fuzzy numbers and $u(t)$ is positive fuzzy function. In this paper, (i) solution means that u is (i)-differentiable, i=1,2.

I) The case of positive fuzzy number coefficient

Let be $[\mu]^\alpha = \left[\underline{\mu_\alpha}, \overline{\mu}_\alpha\right]$ positive fuzzy number.

1) Let be u (1)-differentiable. Then, using the fuzzy Laplace transform, from the fuzzy differential equation (1)

$$sL(u(t)) \ominus u(0) + [\mu]^\alpha L(u(t)) = L([\rho]^\alpha)$$

is obtained. Using the Hukuhara difference and fuzzy arithmetic, we have

$$sL\left(\underline{u_\alpha}(t)\right) - \underline{u_\alpha}(0) + \underline{\mu_\alpha} L\left(\underline{u_\alpha}(t)\right) = L\left(\underline{\rho_\alpha}\right),$$

$$sL(\overline{u}_\alpha(t)) - s\overline{u}_\alpha(0) + \overline{\mu}_\alpha L(\overline{u}_\alpha(t)) = L(\overline{\rho}_\alpha).$$

Also, using the initial condition (2), we obtain

$$L\left(\underline{u_\alpha}(t)\right) = \frac{\underline{\rho_\alpha}}{s\left(s + \underline{\mu_\alpha}\right)} + \frac{\underline{\omega_\alpha}}{s + \underline{\mu_\alpha}},$$

$$L(\overline{u}_\alpha(t)) = \frac{\overline{\rho}_\alpha}{s(s + \overline{\mu}_\alpha)} + \frac{\overline{\omega}_\alpha}{s + \overline{\mu}_\alpha}.$$

Taking the inverse Laplace transform of this equations, (1) solution is obtained as

$$\underline{u}_\alpha(t) = e^{-\underline{\mu}_\alpha t}\left(\underline{\omega}_\alpha - \frac{\underline{\rho}_\alpha}{\underline{\mu}_\alpha}\right) + \frac{\underline{\rho}_\alpha}{\underline{\mu}_\alpha},$$

$$\overline{u}_\alpha(t) = e^{-\overline{\mu}_\alpha t}\left(\overline{\omega}_\alpha - \frac{\overline{\rho}_\alpha}{\overline{\mu}_\alpha}\right) + \frac{\overline{\rho}_\alpha}{\overline{\mu}_\alpha},$$

$$[u(t)]^\alpha = \left[\underline{u}_\alpha(t), \overline{u}_\alpha(t)\right].$$

2) Let be u (2)-differentiable. Using the fuzzy Laplace transform, from the fuzzy differential equation (1)

$$-u(0) \ominus \left(-sL\big(u(t)\big)\right) + [\mu]^\alpha L\big(u(t)\big) = L([\rho]^\alpha),$$

that is, the equations

$$-\overline{u}_\alpha(0) + sL\big(\overline{u}_\alpha(t)\big) + \underline{\mu}_\alpha L\left(\underline{u}_\alpha(t)\right) = L\left(\underline{\rho}_\alpha\right),$$

$$-\underline{u}_\alpha(0) + sL\left(\underline{u}_\alpha(t)\right) + \overline{\mu}_\alpha L\big(\overline{u}_\alpha(t)\big) = L\big(\overline{\rho}_\alpha\big)$$

are obtained. From this, we have

$$sL\big(\overline{u}_\alpha(t)\big) + \underline{\mu}_\alpha L\left(\underline{u}_\alpha(t)\right) = \frac{\underline{\rho}_\alpha}{s} + \overline{\omega}_\alpha \tag{3}$$

$$sL\left(\underline{u}_\alpha(t)\right) + \overline{\mu}_\alpha L\big(\overline{u}_\alpha(t)\big) = \frac{\overline{\rho}_\alpha}{s} + \underline{\omega}_\alpha. \tag{4}$$

If $L\big(\overline{u}_\alpha(t)\big)$ in the equation (4) is replaced by the equation (3),

$$L\left(\underline{u}_\alpha(t)\right) = \left(\overline{\rho}_\alpha - \overline{\mu}_\alpha \overline{\omega}_\alpha\right)\left(\frac{1}{s^2 - \underline{\mu}_\alpha \overline{\mu}_\alpha}\right) + \underline{\omega}_\alpha\left(\frac{s}{s^2 - \underline{\mu}_\alpha \overline{\mu}_\alpha}\right) - \frac{\underline{\rho}_\alpha \overline{\mu}_\alpha}{s\left(s^2 - \underline{\mu}_\alpha \overline{\mu}_\alpha\right)}$$

is obtained. Taking the inverse Laplace transform of this equation, the lower solution is obtained as

$$\underline{u}_\alpha(t) = \frac{\left(\overline{\rho}_\alpha - \overline{\mu}_\alpha \overline{\omega}_\alpha\right)}{\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}}\sin\left(\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}\, t\right) + \underline{\omega}_\alpha \cos\left(\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}\, t\right) - \frac{\underline{\rho}_\alpha}{\underline{\mu}_\alpha}\left(\frac{e^{\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}\, t} + e^{-\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}\, t}}{2} - 1\right).$$

Similarly, the upper solution is obtained as

$$\overline{u}_\alpha(t) = \frac{\left(\underline{\rho}_\alpha - \underline{\mu}_\alpha \underline{\omega}_\alpha\right)}{\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}}\sin\left(\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}\, t\right) + \overline{\omega}_\alpha \cos\left(\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}\, t\right) - \frac{\overline{\rho}_\alpha}{\overline{\mu}_\alpha}\left(\frac{e^{\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}\, t} + e^{-\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}\, t}}{2} - 1\right).$$

Example 2.1 Consider the fuzzy initial value problem

$$u'(t) + [1]^\alpha u(t) = [2]^\alpha, \ t > 0 \tag{5}$$

$$u(0) = [1]^\alpha, \tag{6}$$

using the fuzzy Laplace transform, where $[1]^\alpha = [\alpha, 2 - \alpha]$, $[2]^\alpha = [1 + \alpha, 3 - \alpha]$.

If u is (1)-differentiable, (1) solution is

$$\underline{u}_\alpha(t) = e^{-\alpha t}\left(\alpha - \left(\frac{1+\alpha}{\alpha}\right)\right) + \left(\frac{1+\alpha}{\alpha}\right),$$

$$\overline{u}_\alpha(t) = e^{-(2-\alpha)t}\left((2-\alpha) - \left(\frac{3-\alpha}{2-\alpha}\right)\right) + \frac{3-\alpha}{2-\alpha},$$

$$[u(t)]^\alpha = [\underline{u}_\alpha(t), \overline{u}_\alpha(t)].$$

If u is (2)-differentiable, (2) solution is

$$\underline{u}_\alpha(t) = \left(\frac{(3-\alpha)-(2-\alpha)^2}{\sqrt{\alpha(2-\alpha)}}\right)\sin\left(\sqrt{\alpha(2-\alpha)}t\right) + \alpha\cos\left(\sqrt{\alpha(2-\alpha)}t\right) - \left(\frac{1+\alpha}{\alpha}\right)\left(\frac{e^{\sqrt{\alpha(2-\alpha)}t}+e^{-\sqrt{\alpha(2-\alpha)}t}}{2} - 1\right),$$

$$\overline{u}_\alpha(t) = \left(\frac{(1+\alpha)-\alpha^2}{\sqrt{\alpha(2-\alpha)}}\right)\sin\left(\sqrt{\alpha(2-\alpha)}t\right) + (2-\alpha)\cos\left(\sqrt{\alpha(2-\alpha)}t\right)$$

$$- \left(\frac{3-\alpha}{2-\alpha}\right)\left(\frac{e^{\sqrt{\alpha(2-\alpha)}t}+e^{-\sqrt{\alpha(2-\alpha)}t}}{2} - 1\right),$$

$$[u(t)]^\alpha = [\underline{u}_\alpha(t), \overline{u}_\alpha(t)].$$
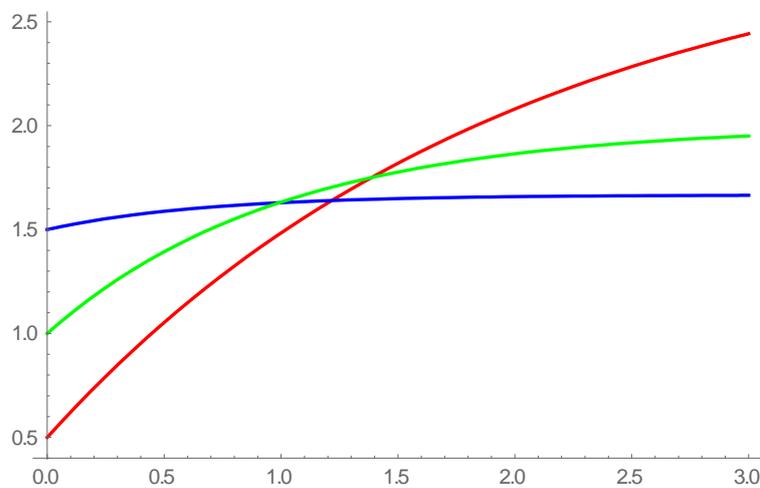
Figure 1 (1) solution for $\alpha = 0.5$
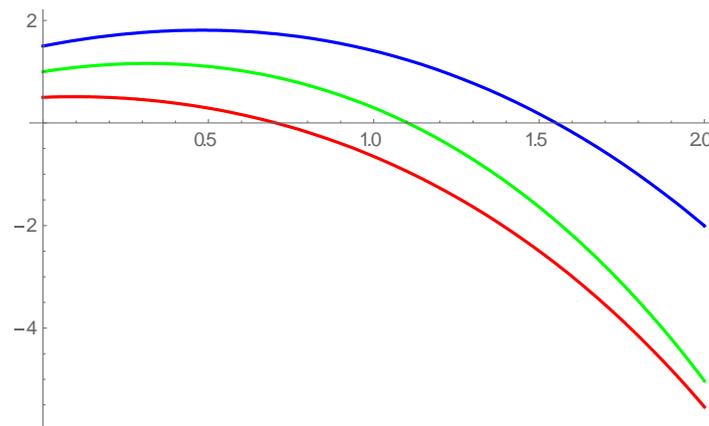
Figure 2 (2) solution for $\alpha = 0.5$

$red \rightarrow \underline{u}_\alpha(t), \; blue \rightarrow \overline{u}_\alpha(t), \; green \rightarrow u(t)$

From Definition 1.3 and since $[u(x)]^\alpha$ is positive fuzzy function, according to figure 1, (1) solution is a valid fuzzy number for $x \epsilon [0,1.21735]$, according to figure 2, (2) solution is a valid fuzzy number for $x \epsilon [0,0.704091]$.

II) The case of negative fuzzy number coefficient

Let be $[\mu]^\alpha = \left[\underline{\mu}_\alpha, \overline{\mu}_\alpha\right]$ negative fuzzy number.

1) Let be u (1)-differentiable. Then, using the fuzzy Laplace transform, the fuzzy differential equation (1) yields

$$sL\big(u(t)\big) \ominus u(0) + [\mu]^\alpha L\big(u(t)\big) = L([\rho]^\alpha).$$

From this, using the fuzzy arithmetic and Hukuhara difference

$$sL\left(\underline{u}_\alpha(t)\right) - \underline{u}_\alpha(0) + \underline{\mu}_\alpha L(\overline{u}_\alpha(t)) = L\left(\underline{\rho}_\alpha\right),$$

$$sL(\overline{u}_\alpha(t)) - s\overline{u}_\alpha(0) + \overline{\mu}_\alpha L\left(\underline{u}_\alpha(t)\right) = L(\overline{\rho}_\alpha)$$

are obtained. That is, we have the equations

$$sL\left(\underline{u}_\alpha(t)\right) + \underline{\mu}_\alpha L(\overline{u}_\alpha(t)) = \frac{\rho_\alpha}{s} + \underline{\omega}_\alpha, \tag{7}$$

$$sL(\overline{u}_\alpha(t)) + \overline{\mu}_\alpha L\left(\underline{u}_\alpha(t)\right) = \frac{\overline{\rho}_\alpha}{s} + \overline{\omega}_\alpha. \tag{8}$$

If $L\big(\overline{u}_\alpha(t)\big)$ in the equation (7) is replaced by the equation (8),

$$L\left(\underline{u}_\alpha(t)\right) = \left(\underline{\rho}_\alpha - \underline{\mu}_\alpha \overline{\omega}_\alpha\right)\left(\frac{1}{s^2 - \underline{\mu}_\alpha \overline{\mu}_\alpha}\right) + \underline{\omega}_\alpha \left(\frac{s}{s^2 - \underline{\mu}_\alpha \overline{\mu}_\alpha}\right) - \frac{\overline{\rho}_\alpha \underline{\mu}_\alpha}{s\left(s^2 - \underline{\mu}_\alpha \overline{\mu}_\alpha\right)}$$

is obtained. Then, the lower solution is

$$\underline{u}_\alpha(t) = \frac{\left(\underline{\rho}_\alpha - \underline{\mu}_\alpha \overline{\omega}_\alpha\right)}{\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}} \sin\left(\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha} t\right) + \underline{\omega}_\alpha \cos\left(\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha} t\right) - \frac{\overline{\rho}_\alpha}{\overline{\mu}_\alpha}\left(\frac{e^{\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha} t} + e^{-\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha} t}}{2} - 1\right).$$

Similarly, upper solution is

$$\overline{u}_\alpha(t) = \frac{\left(\overline{\rho}_\alpha - \overline{\mu}_\alpha \underline{\omega}_\alpha\right)}{\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha}} \sin\left(\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha} t\right) + \overline{\omega}_\alpha \cos\left(\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha} t\right) - \frac{\rho_\alpha}{\underline{\mu}_\alpha}\left(\frac{e^{\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha} t} + e^{-\sqrt{\underline{\mu}_\alpha \overline{\mu}_\alpha} t}}{2} - 1\right).$$

2) Let be u (2)-differentiable. Then, using the fuzzy Laplace transform, from the fuzzy differential equation (1)

$$-u(0) \ominus \left(-sL(u(t))\right) + [\mu]^\alpha L(u(t)) = L([\rho]^\alpha),$$

that is,

$$-\overline{u}_\alpha(0) + sL(\overline{u}_\alpha(t)) + \underline{\mu}_\alpha L(\overline{u}_\alpha(t)) = L\left(\underline{\rho}_\alpha\right),$$

$$-\underline{u}_\alpha(0) + sL\left(\underline{u}_\alpha(t)\right) + \overline{\mu}_\alpha L\left(\underline{u}_\alpha(t)\right) = L(\overline{\rho}_\alpha)$$

are obtained. From this, we have

$$L\left(\underline{u}_\alpha(t)\right) = \frac{\overline{\rho}_\alpha}{s\left(s + \overline{\mu}_\alpha\right)} + \frac{\omega_\alpha}{s + \overline{\mu}_\alpha},$$

$$L(\overline{u}_\alpha(t)) = \frac{\rho_\alpha}{s\left(s + \underline{\mu}_\alpha\right)} + \frac{\overline{\omega}_\alpha}{s + \underline{\mu}_\alpha}.$$

Then, (2) solution is obtained as

$$\underline{u}_\alpha(t) = e^{-\overline{\mu}_\alpha t}\left(\omega_\alpha - \frac{\overline{\rho}_\alpha}{\overline{\mu}_\alpha}\right) + \frac{\overline{\rho}_\alpha}{\overline{\mu}_\alpha},$$

$$\overline{u}_\alpha(t) = e^{-\underline{\mu}_\alpha t}\left(\overline{\omega}_\alpha - \frac{\rho_\alpha}{\underline{\mu}_\alpha}\right) + \frac{\rho_\alpha}{\underline{\mu}_\alpha},$$

$$[u(t)]^\alpha = \left[\underline{u}_\alpha(t), \overline{u}_\alpha(t)\right].$$

Example 2.2 Consider the fuzzy initial value problem

$$u'(t) + [-1]^\alpha u(t) = [2]^\alpha, \ t > 0 \tag{9}$$

$$u(0) = [1]^\alpha, \tag{10}$$

using the fuzzy Laplace transform, where

$$[-1]^\alpha = [-2 + \alpha, -\alpha], \ [1]^\alpha = [\alpha, 2 - \alpha], \ [2]^\alpha = [1 + \alpha, 3 - \alpha].$$

If u is (1)-differentiable, (1) solution is

$$\underline{u}_\alpha(t) = \left(\frac{(1+\alpha) + (2-\alpha)^2}{\sqrt{\alpha(2-\alpha)}}\right) \sin\left(\sqrt{\alpha(2-\alpha)}t\right) + \alpha\cos\left(\sqrt{\alpha(2-\alpha)}t\right)$$

$$+ \left(\frac{3-\alpha}{\alpha}\right)\left(\frac{e^{\sqrt{\alpha(2-\alpha)}t} + e^{-\sqrt{\alpha(2-\alpha)}t}}{2} - 1\right),$$

$$\overline{u}_\alpha(t) = \left(\frac{(3-\alpha) + \alpha^2}{\sqrt{\alpha(2-\alpha)}}\right) \sin\left(\sqrt{\alpha(2-\alpha)}t\right) + (2-\alpha)\alpha\cos\left(\sqrt{\alpha(2-\alpha)}t\right)$$

$$+ \left(\frac{1+\alpha}{2-\alpha}\right)\left(\frac{e^{\sqrt{\alpha(2-\alpha)}t} + e^{-\sqrt{\alpha(2-\alpha)}t}}{2} - 1\right),$$

$$[u(t)]^\alpha = \left[\underline{u}_\alpha(t), \overline{u}_\alpha(t)\right].$$

If u is (2)-differentiable, (2) solution is

$$\underline{u}_\alpha(t) = e^{\alpha t}\left(\frac{\alpha^2 + 3 - \alpha}{\alpha}\right) - \left(\frac{3-\alpha}{\alpha}\right),$$

$$\overline{u}_\alpha(t) = e^{(2-\alpha)t}\left(\frac{(2-\alpha)^2 + 1 + \alpha}{2-\alpha}\right) - \left(\frac{1+\alpha}{2-\alpha}\right),$$

$$[u(t)]^\alpha = \left[\underline{u}_\alpha(t), \overline{u}_\alpha(t)\right].$$
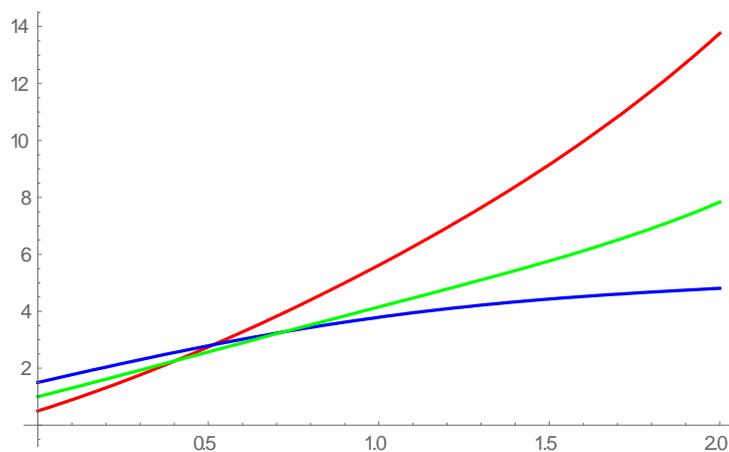
Figure 3 (1) solution for $\alpha = 0.5$
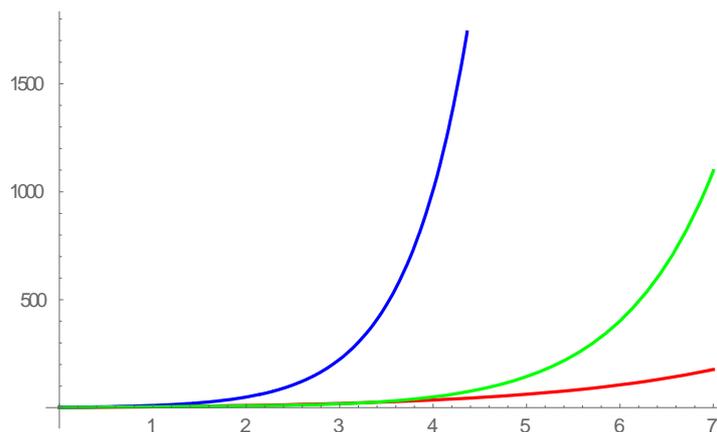
Figure 4 (2) solution for $\alpha = 0.5$

$red \rightarrow \underline{u}_\alpha(t), \ blue \rightarrow \overline{u}_\alpha(t), \ green \rightarrow u(t)$

From Definition 1.3 and since $[u(x)]^\alpha$ is positive fuzzy function, according to figure 3, (1) solution is a valid fuzzy number for $x \epsilon [0, 0.514863]$, according to figure 4, (2) solution i

s a valid fuzzy number for $t > 0$.

3. CONCLUSIONS

In this study, solutions of fuzzy initial value problems for first-order fuzzy differential equations with positive and negative fuzzy number coefficients are investigated by fuzzy Laplace transform. Generalized differentiability is used. Examples are solved and the figures of the solutions are drawn. It is shown that whether the solutions are valid fuzzy functions or not. Also, it is shown that solutions are valid fuzzy functions in different intervals for each α −cut.

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