



Linear And Nonlinear Intersubband Optical Absorptions In Multiple Quantum Wells Under The External Fields

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Received: 19.04.2019; Accepted: 05.09.2019

<http://dx.doi.org/10.17776/csj.556155>

Abstract. In this study, the effects of the external fields (electric and tilted magnetic fields) and well parameters on the optical absorption coefficients in GaAs/GaAlAs multiple (five) quantum wells under the applied electric and tilted magnetic fields has been investigated theoretically. Firstly, the energy eigenvalues and eigen functions of an electron confined in multiple quantum wells are calculated by analytically from Schrödinger equation using the transfer matrix method within the effective mass approximation, Secondly, the linear, nonlinear and total intersubband optical absorptions in GaAs/GaAlAs multiple quantum wells system are studied within the compact density-matrix approach. It is shown that the parameters such as strenghts of the external fields and θ -tilted angle values not only shift the peak positions in absorption spectrum but also considerably modify their potential heights. In generally, electronic and optical properties of the quantum wells are very sensitive to the applied external fields and well parameters. Therefore, we can conclude that the effect of the external fields can be used to tune and control the optical properties of interest in the range of the far-infrared electromagnetic spectrum.

Keywords: Multiple Quantum Wells, Optical Properties, Electric Field, Tilted Magnetic Field.

DIŞ ALANLAR ALTINDAKİ ÇOKLU KUANTUM KUYULARINDA LİNEER VE LİNEER OLMAYAN BAND İÇİ OPTİK SOĞURMA

Özet. Bu çalışmada, elektrik ve eğik manyetik alan altında GaAs/GaAlAs çoklu (beş kuantum kuyusu) kuantum kuyusunda dış alanların (elektrik ve eğik manyetik alan) ve kuyu parametrelerinin optik soğurma katsayısı üzerindeki etkileri teorik olarak incelenmiştir. İlk olarak, çoklu kuantum kuyularında kuşatılmış bir elektronun özdeğer ve özfonksiyonları, etkin-kütle yaklaşımı ile transfer-matris metodu kullanılarak Schrödinger denkleminde analitik olarak hesaplanmıştır. İkinci aşamada, GaAs/GaAlAs çoklu kuantum kuyu sisteminde doğrudan, dolaylı ve toplam bandiçi optik soğurma çalışılmış ve optik geçişler için kompakt yoğunluk matris yaklaşımı kullanılmıştır. Dış alan şiddetleri ve θ -eğiklik açıları gibi parametrelerin sadece soğurma spektrumdaki pik pozisyonlarını kaydırmakla kalmayıp, aynı zamanda yüksekliklerini de önemli ölçüde değiştirdiği gösterilmiştir. Genellikle, kuantum kuyularının elektronik ve optik özellikleri, uygulanan dış alanlara ve kuyu parametrelerine oldukça duyarlıdır. Bu nedenle, dış alanların etkisini, uzak kızılötesi elektromanyetik spektrum aralığında ilgili optik özellikleri ayarlamak ve kontrol etmek için kullanılabileceği sonucuna varabiliriz.

Anahtar Kelimeler: Çoklu Kuantum Kuyusu, Optik Özellikler, Elektrik Alan, Eğik Manyetik Alan.

1. INTRODUCTION

In recent years, with great advances in semiconductor growth technology, semiconductor quantum wells (QW) with different geometries such as square, parabolic, semiparabolic, rectangular have been grown and due to their superior electronic and optical properties have considerably attracted great deal of attention. In addition to the well-known square and parabolic quantum wells, such as semi-parabolic, graded, V-shaped, inverse parabolic, PoschTeller, Tietz-Hua have been intensively studied also [1-13]. The electronic and optical properties of the QWs are very sensitive to the applied external fields. These and quantum confinement effects leads to the formation of discrete energy levels within the well, which results in significant optical properties in the semiconductor QW system compared with that in the bulk material [14,15]. Optical properties of these semiconductor QWs have the potential for device applications such as far-infrared laser amplifiers, high-speed electro-optical modulators, photodetectors, etc.[16-18].

In a parabolic potential well eigenenergies of two-dimensional electrons subjected to a tilted magnetic field have been solved analytically by Maan [19]. İ.Sökmen at all. have completely solve the Schrödinger equation using a square multiple quantum wells as the confinement potential and it has obtained analytical solutions with applying an orthogonal transformation to two-dimensional semiconductor heterostructures under externally tilted magnetic field [20]. They have made the Hamiltonian of the system separable in terms of the new coordinates after implementation of successive transformations [20-23]. In this manuscript, It have been completely solved the Schrödinger equation using multiple square wells potentials as the confining potential and obtained analytical solutions without making any approximations as in Refs [20-23] for two dimensional semiconductor heterostructures under externally applied electric and tilted magnetic fields. In this system, the electric field is applied along the growth direction (z-direction) of the multiple square quantum wells whereas the magnetic field is applied to the (x-z) in-plane. θ is the tilt angle between the direction of magnetic field and x-axis. After solving the Hamiltonian of the system, the multiple quantum wells become narrower (wider) when the θ -tilted angle increases (decreases) because the multiple quantum well widths are proportional with $L\cos\theta$. Furthermore, potential heights of the multiple quantum wells are proportional with $V_0\cos^2\theta$ so their potential heights decrease when the θ -tilted angle increases.

In recent years, several theoretical studies have been made on linear and nonlinear intersubband optical absorption (AC) and refractive index changes(RIC) in single QWs and multi-quantum wells under applied electric and magnetic fields [24-32]. In this study, the effects of the electric and magnetic field strength, different tilt angles (i.e. direction of the magnetic field) on total absorption coefficient including the linear and third order nonlinear terms for the transitions between the ground and other excited states of an electron in multiple (five) quantum wells are investigated.

2. THEORY

Within the framework of an effective mass approximation, the electron Hamiltonian for the multiple (five) quantum wells in the presence of magnetic field \vec{B} applied in x-z plane and electric field \vec{F} applied in the z-direction (perpendicular to the growth direction), is given by:

$$H = \frac{1}{2m^*} (\vec{p} + e\vec{A})^2 + V(z). \quad (1)$$

where m^* , \vec{p} and e are the effective mass, momentum vector and charge of the electron, respectively. The magnetic field can be described by the vector potential $\vec{A} = (0, x B \sin \theta - z B \cos \theta, 0)$ which is applied in the (x-z) plane and where θ is the angle between the magnetic field and the x-axis. The magnetic field have been defined as $\vec{B} = (B \cos \theta, 0, B \sin \theta)$ by using vector potential in Coulomb gauge. $V(z)$ is the confinement potential of the electron and its functional form is defined by

$$V(z) = V_0 \sum_{i=1}^N [S(z_{L_i} - z) + S(z - z_{R_i}) - (N - 1)] - eFz, \quad (2)$$

where $S(z)$ is the step function, for i.th quantum well $z=z_{L_i}$ and $z=z_{R_i}$ are the left and right well boundaries, respectively. N is the total number of quantum well. The wave function of the system by using the translation symmetry in the y direction can be defined as

$$\psi(r) = \text{Exp}(ik_y y) \varphi(x, z) \quad (3)$$

After using the successive transformations which are proposed by Refs.[20-23], Hamiltonian can be separable in terms of the new coordinates and the Schrödinger equation of the system taken into consideration without making any approximation for the heterostructures subject to the electric and tilted magnetic field can be solved. After these transformations, Hamiltonian in the z' -direction is obtained as follows

$$H = \frac{p_{z'}^2}{2m^*} + \frac{1}{2} m^* \omega^2 (z'_0 - z')^2 + V(z') \quad (4)$$

where $\omega (= eB/m^*)$ is the cyclotron frequency, $z'_0 (= \hbar k_y / eB = a_H^2 k_y)$ is the position of the orbit center, $a_H (= (\hbar/m^* \omega)^{1/2})$ is the magnetic length,

$$V(z') = V_0 \cos^2 \theta \sum_{i=1}^N [S(z'_{L_i} - z') + S(z' - z'_{R_i}) - (N - 1)] + eF \cos \theta z' \quad (5)$$

is the confinement potential in z' -direction. And eigenfunction of the system is obtained

$$\psi(r) = \text{Exp}(ik_y y') \chi(x') \phi(z') \quad (6)$$

If we define $\xi = \tilde{u} - 2\tilde{\beta}$ together with the dimensionless variables- $\tilde{u} = \left(\frac{\sqrt{2}}{a_H}\right) (z'_0 - z')$, $\tilde{E}_{z'} = E_{z'} / \hbar \omega$ and $\tilde{V}_0 = V_0 / \hbar \omega$ for z' -direction, Schrödinger equation in the new coordinate system is written as follows

$$\frac{d^2\phi(\xi)}{d\xi^2} + \left\{ \left(m + \frac{1}{2}\right) - \frac{1}{4}\xi^2 \right\} \phi(\xi) = 0, \quad (7)$$

where m is the quantum number and the solution of this equation which corresponds to the motion in the z' -direction is the Weber functions.

After the eigenvalues and their corresponding wave functions for the Hamiltonian in the Eq. (4) are obtained, the expressions including the linear, third order nonlinear and total absorption coefficient (AC) can be define clearly as follows [23-31 and references therein], respectively:

$$\alpha^{(1)}(\omega) = \omega \sqrt{\frac{\mu}{\varepsilon_R}} \frac{|M_{ji}|^2 \sigma_V \hbar \Gamma_{ij}}{(\Delta E - \hbar\omega)^2 + (\hbar\Gamma_{ij})^2} \quad (8)$$

$$\alpha^{(3)}(\omega, I) = -2\omega \sqrt{\frac{\mu}{\varepsilon_R}} \left(\frac{I}{\varepsilon_0 n_r c} \right) \frac{|M_{ji}|^4 \sigma_V \hbar \Gamma_{ij}}{[(\Delta E - \hbar\omega)^2 + (\hbar\Gamma_{ij})^2]^2} \quad (9)$$

$$\times \left(1 - \frac{|M_{jj} - M_{ii}|^2}{|2M_{ji}|^2} \frac{(\Delta E - \hbar\omega)^2 - (\hbar\Gamma_{ij})^2 + 2(\Delta E)(\Delta E - \hbar\omega)}{(\Delta E)^2 + (\hbar\Gamma_{ij})^2} \right),$$

$$\alpha(\omega, I) = \alpha^{(1)}(\omega) + \alpha^{(3)}(\omega, I) \quad (10)$$

In these equations, n_r is the refractive index, μ is the permeability of the system, σ_V is electron density, ε_0 is the permittivity of free space, Γ_{12} is the relaxation rate which is equals to the inverse relaxation time T_{12} , c is the speed of the light in free space, $I (= \frac{2n_r}{\mu c} |E(\omega)|^2)$ is the optical intensity of the incident electromagnetic wave with an angular frequency ω which leads to the intersubband optical transitions, ε_R is the real part of the permittivity, $\Delta E = E_j - E_i$ is the energy difference between the ground and first excited levels, $M_{ij} = \left| \left\langle \phi(\xi)_i \left| \xi \right| \phi(\xi)_j \right\rangle \right|$ ($i=1$ and $j=2$) is the dimensionless electric dipole moment matrix element.

3.RESULTS AND DISCUSSIONS

The values of the using parameters in calculations are: $\varepsilon_0 = 12.58$, $m^* = 0.067 m_0$ (where m_0 is the free electron mass), $V_0 = 228$ [meV] (which corresponding to $x=0.3$ for Aluminum concentration),

$$n_r = 3.2, \quad T_{12} = 0.148$$
 [ps], $I = 2.0 \times 10^9$ [W/m²], $\Gamma_{12} = 1/T_{12}$, $\mu_0 = 4\pi \times 10^{-7}$ [Hm⁻¹],

$\sigma_V = 3.0 \times 10^{22}$ [m⁻³]. Effective well width, effective barrier height and the orbit center are chosen as $L_T = \frac{\sqrt{2}}{a_H} (L_w + L_b) \cos \theta$ (where L_w is well width, L_b is barrier width), $V_{eff} = \frac{V_0}{\hbar\omega} \cos^2 \theta$ and $z'_0 = [2L_T + \frac{\sqrt{2}L_w}{2a_H} \cos \theta]$. All results are for $L_w = 80$ [Å] and $L_b = 15$ [Å].

The schematic diagram of the multiple (five) quantum wells and squared wavefunctions of the electron in the ground and first excited level for $F=0$ (blackline) and $F=30\text{kV/cm}$ (redline) is seen in Fig.1. In addition, effect of the θ -tilted angle on the system is also clearly seen. The energy levels are quantized by the combined effects of the magnetic field and barriers of the multiple quantum wells. The second energy level is in an extended state in which is dominant effect of the magnetic confinement as seen in $\theta > 60$ (in Fig.1(c)). When the electric field is applied, electron becomes localize in the right side of the well (triangle region, (please see Figs.1(a), (b) and(c)).

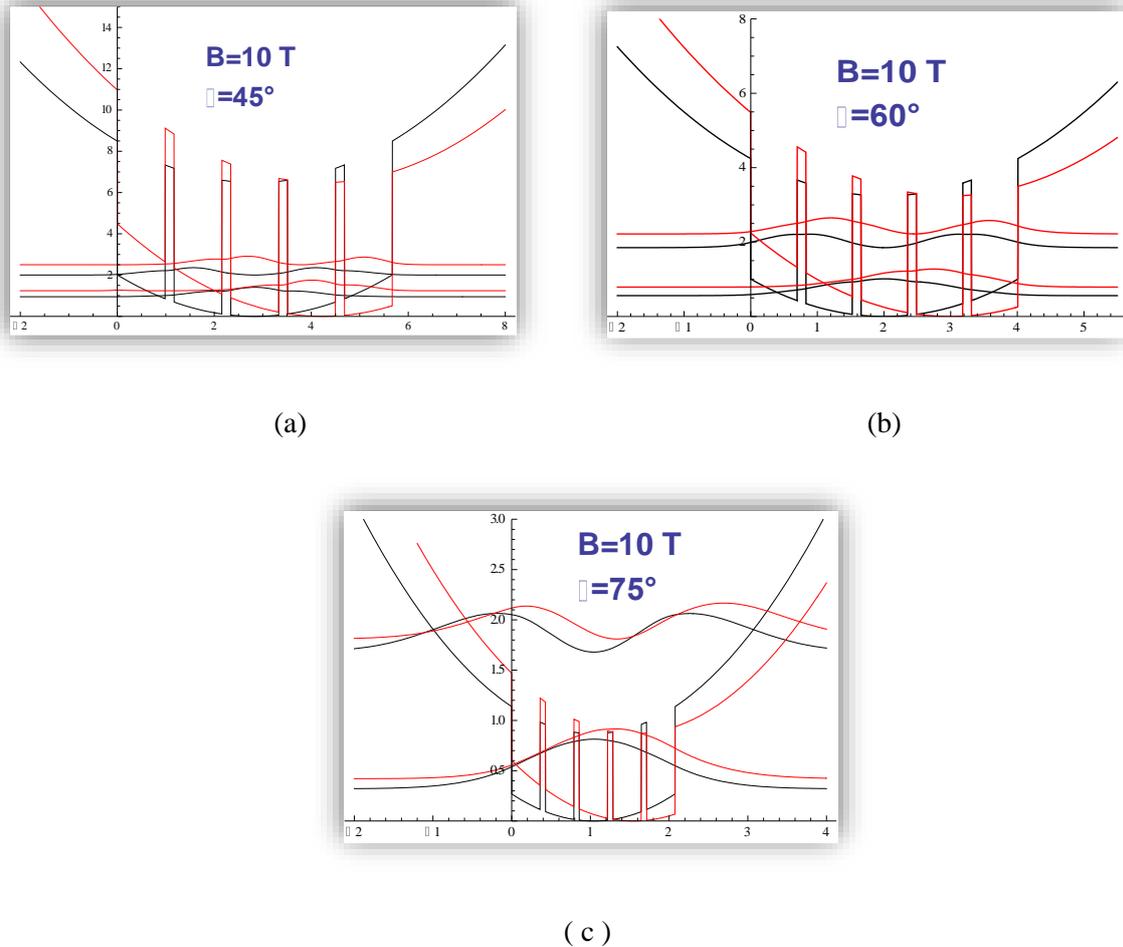


Figure1.The schematic diagram of the multiple (five) quantum wells and squared wavefunctions of the electron in the ground and first excited level for $F=0$ (blackline) and $F=30\text{kV/cm}$ (redline) a) $\theta = 45^\circ$ b) $\theta = 60^\circ$ c) $\theta = 75^\circ$

In Fig.2, it's plotted the first six electronic energy levels of the multiple quantum wells system as a function of the z' for three different tilt angles for $F=0$ (in a, b, c) and $F=30\text{kV/cm}$ (in d,e,f). These results clearly show that there are two different types of the energy states; the states confined in multiple quantum wells (the lowest states in Fig 2 (a, b and c)) and extended states (the higher states in Fig.2 (a, b and c)). The energies of the lower states are less than effective potential height in small θ -tilt angle degrees (see Fig.2(a, b, c)). The effective potential heights decrease as θ -tilt angle degree increases. Because the effective potential height is proportional with $\cos^2 \theta$. Electronic energy levels are located at Landau levels as they move away from the center of multiple quantum wells (i.e. in larger z' values).

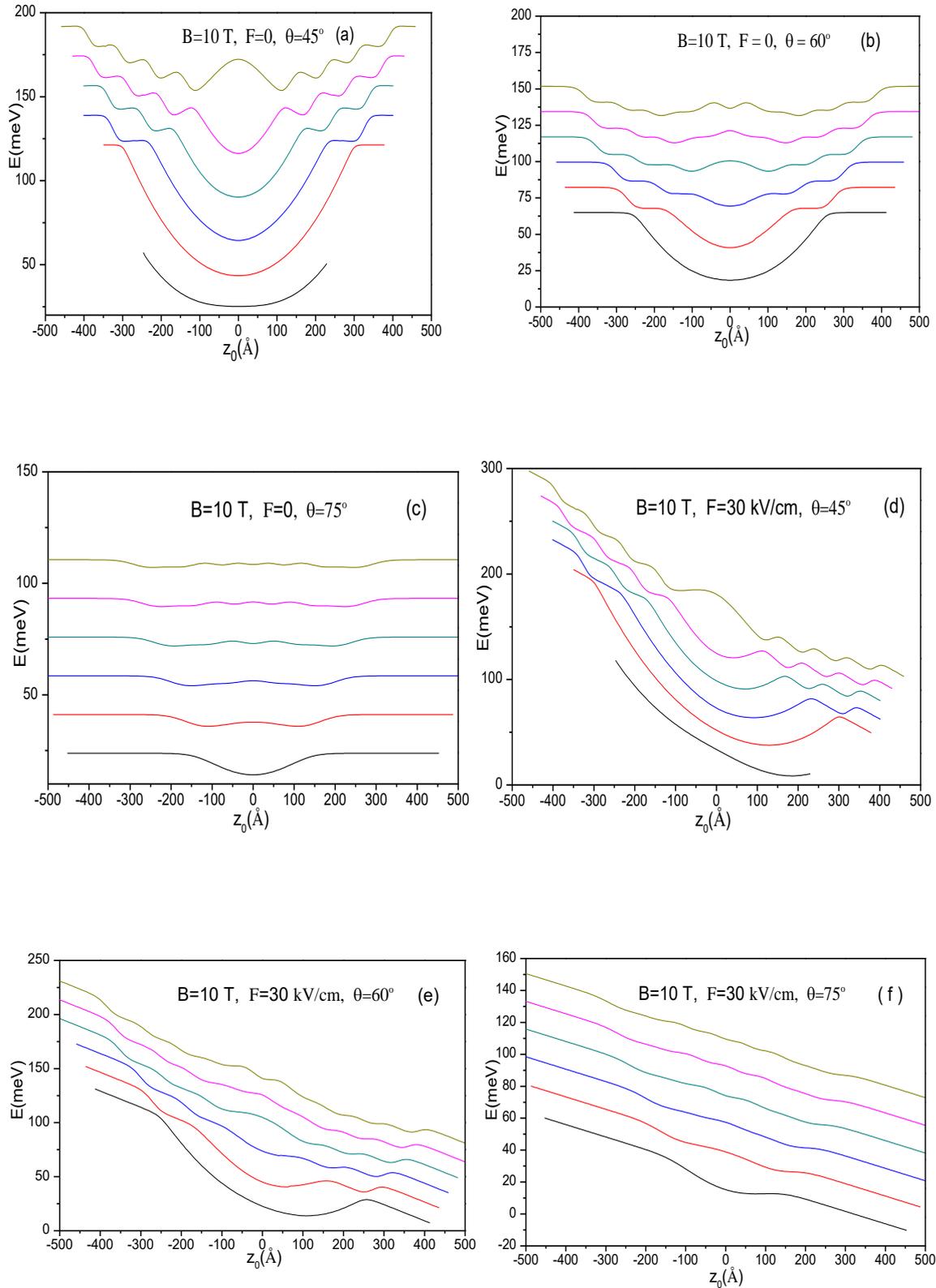
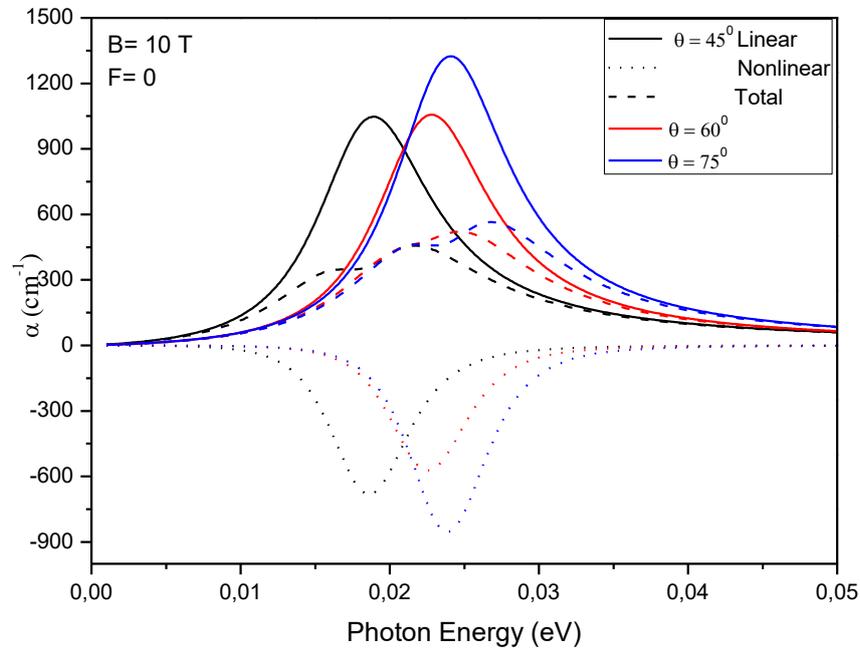


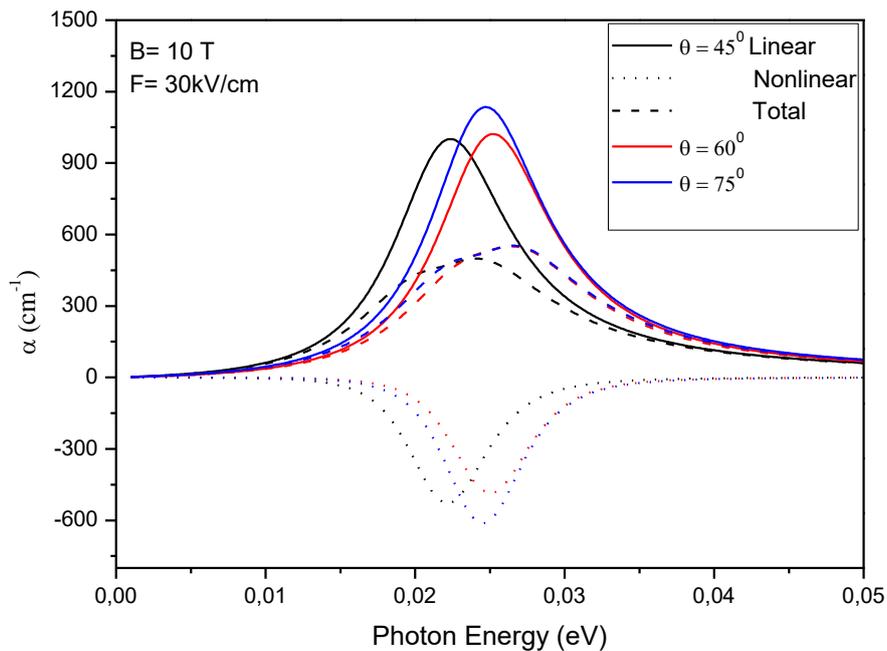
Figure 2. The first six electronic energy levels of the multiple quantum wells system as a function of the z' for three different tilt angles for $F=0$ (in a,b,c) and $F=30$ kV/cm (in d,e,f).

It shows the variations of linear, nonlinear and total ACs as a function of the incident photon energy for different tilt angles that are given in Fig. 3: a) $F=0$ b) $F=30\text{kV/cm}$ electric field values.

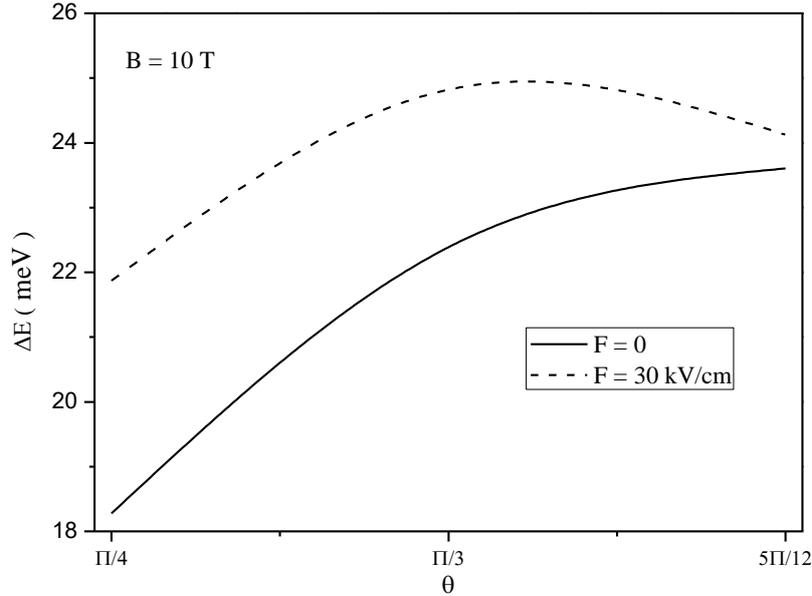
As the tilt angles increases, ΔE energy difference between the related energy levels increases and thus the total AC shifts to the blue and this behavior is seen clearly in Fig.3(a).



(a)



(b)



(c)

Figure 3. The variation of linear, nonlinear and total absorption coefficients as a function of the photon energy for different tilt angles and **a)** $F=0$ **b)** $F=30\text{ kV/cm}$ electric field values. **c)** The variation of the energy differences between the related energy levels- ΔE as a function of the tilt angles for two different electric field values.

When the electric field is applied, electron becomes localize in the right side of the well (Figs.1(a), (b) and(c) redline) since QWs bend in the opposite direction to the electric field and thus the peak positions of the ACs shifts towards to higher photon energies since ΔE increases with the electric field (Figs.3 (b)). It's seen that in Fig.3 (c) at $F=30\text{ kV/cm}$, as the tilt angle increases from $\theta=45^\circ$ to $\theta=60^\circ$ ΔE increases and thus total AC shifts to the blue. For further angle values ($\theta>60^\circ$), the peak of the total AC slows down shift to the blue since ΔE decreases. This behavior of total AC spectra is a direct consequence of the increment or decrement in the energy differences between the ground and first excited levels with increasing angle values, respectively. In addition, the presence of the electric field is eliminated saturation when θ tilt angle value increases (see Fig.3(b)). In order see better the effects of the electric field strength on the AC spectrum, it is plotted the variation of the energy differences between the related energy levels- ΔE as a function of the tilt angle for two different electric field values in Fig. 3 (c). This is expected, effective well width and effective potential height ($L_{eff} = \frac{\sqrt{2}L_w}{2a_H} \cos \theta$, $V_{eff} = \frac{V_0}{\hbar\omega} \cos^2 \theta$) decrease with increasing angle values and this behavior is pronounced for $\theta > 30^\circ$. In this case, electron in the ground and first excited level is localized in the bottom part of the well in $\theta \leq 60$ angle values, whereas for $\theta > 60$ electron in the first excited level is localized in the upper part (magnetic confinement) while its ground level is in the bottom part of the well as seen in Figs. 1(a) and (b).

As a result, it is investigated the effect of the electric field on total absorption coefficient including the linear and third order nonlinear terms in the multiple GaAs/GaAlAs quantum wells for a constant tilted

magnetic field. Furthermore, electric field strength as well as the effects of the magnetic field direction (i.e. different tilt angles) and the well parameters on the optical properties have been investigated.

4. CONCLUSION

In this study, it is investigated the effects of the electric field strength and direction of the magnetic field on total absorption coefficient including the linear and third order nonlinear terms in the GaAs/GaAlAs multiple quantum wells under the external electric and magnetic field. The obtained results show that absorption coefficient change is sensitive to the electric field and direction of the magnetic field. By changing the direction of the magnetic field and existence of the electric field, we can obtain a blue or red shift, without the need for the growth of different samples. This also gives a new degree of freedom in various device applications based on the intersubband transition of electrons.

Acknowledgements

This research was supported by The Scientific Research Project Fund of Cumhuriyet University (CUBAP) under the Project Number F-579.

References

- [1] Karabulut İ., Atav Ü., Şafak H., Tomak M., Eur. Phys. J. B 55 (2007) 283-288.
- [2] C. Mailhot, Y.C. Chang, T.C. McGill, Phys. Rev. B 26 (1982) 4449–4457.
- [3] Q. Guo, Y.P. Feng, H.C. Poon, C.K. Ong, Eur. Phys. J. B 9 (1999) 29–36.
- [4] F.Q. Zhao, X.X. Liang, S.L. Ban, Eur. Phys. J. B 33 (2003) 3–8.
- [5] Z.P. Wang, X.X. Liang, X. Wang, Eur. Phys. J. B 59 (2007) 41–46.
- [6] Y.B. Yu, S.N. Zhu, K.X. Guo, Phys. Lett. A 335 (2005) 175–181.
- [7] E. Kasapoglu, H. Sari, I. Sökmen, Surf. Rev. Lett. 13 (2006) 397–401.
- [8] E. Kasapoglu, I. Sökmen, Physica E 27 (2005) 198–203.
- [9] O. Aytekin, S. Turgut, M. Tomak, Physica E 44 (2012) 1612–1616.
- [10] E. Kasapoglu, S. Sakiroglu, F. Urgan, U. Yesilgul, C.A. Duque, I.Sökmen, Physica B 526 (2017) 127-131.
- [11] S. Baskoutas, A.F. Terzis, Physica E 40 (2008) 1367–1370.
- [12] S. Baskoutas, C. Garoufalis, A.F. Terzis, Eur. Phys. J. B 84 (2011) 241–247.
- [13] F. Urgan, E. Kasapoglu, I. Sökmen, Solid State Commun. 151 (2011) 1415–1419.
- [14] E. Rosencher, Ph. Bois, Phys. Rev. B 44, 11315 (1991).
- [15] M.K. Gurnick and T.A. Detemple, IEEE J. Quantum Electron. QE-19, 791 (1983).
- [16] İ. Karabulut, S. Baskoutas, J. Appl. Phys. 103, 073512 (2008).
- [17] C.H. Liu, B.R. Xu, Phys. Lett. A 372, 888 (2008). DOI: 10.1016/j.physleta.2007.08.046
- [18] B. Chen, K.X. Guo, Z.L. Liu, R.Z. Wang, Y.B. Zheng, B. Li, J. Phys.: Condens. Matter 20, 255214 (2008).

- [19] J.C. Maan, *Solid-State Sciences*, 53, edited by G. Bauer, F. Kuchar, H. Heinrich (1984).
- [20] I. Sökmen, H. Sari, S. Elagöz, Y. Ergün, S. Erzin, *Superlattices Microstruct.* 17, 3 (1995).
- [21] Y. Ergün, I. Sökmen, H. Sari, S. Elagoz, M.C. Arıkan, *Semicond. Sci. Technol.* 12, 802 (1997).
- [22] R. Amca, Y. Ergun, I. Sökmen, H. Sari, *Semicond. Sci. Technol.* 15, 1087 (2000).
- [23] R. Özbakır, *Can. J. Phys.* 96, (9), 999–1003 (2018)
- [24] E.M. Goldys, J.J. Shi, *Phys. Status Solidi (b)*, 210, 237 (1998).
- [25] S. Ünlü, İ. Karabulut, H. Safak, *Physica E* 33, 319 (2006).
- [26] F. Urgan, M. E. Mora-Ramos, C. A. Duque, E. Kasapoglu, H. Sari, I. Sökmen, *Superlattices Microstruct.* 66, 129 (2014).
- [27] E. Kasapoglu, C. A. Duque, H. Sari, I. Sökmen, *The European Physical Journal B*, 82, 13 (2011).
- [28] İ. Karabulut, Ü. Atav, H. Safak, M. Tomak, *Eur. Phys. J. B* 55, 283 (2007).
- [29] B. Chen, K.X. Guo, R.Z. Wang, Z.H. Zhang, Z.L. Liu, *Solid State Commun.* 149, 310 (2009).
- [30] D. Ahn, S.L. Chuang, *IEEE J. Quantum Electron* 23, 2196 (1987).
- [31] I. Karabulut, C.A. Duque, *Physica E* 43, 1405 (2011).
- [32] M.J. Karimi, A. Keshavarz, *Superlatt. Microstruct.* 50, 572 (2011).