





RP-T-fuzzy soft subrings and ideals of soft rings

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Abstract

In this paper we introduce a concept which is called RP-T-fuzzy soft subring and examine some properties of the restricted intersection, the restricted union, the Λ -intersection and the product of their families. A condition to make the restricted union of RP-T-fuzzy soft subrings to be RP-T-fuzzy soft subring is determined. A correlation between the RP-T-fuzzy soft subring of a soft ring and α -level sets of this soft ring is demonstrated. The RP-T-fuzzy soft subrings under some binary operations are investigated. Moreover, the image and pre-image of RP-T-fuzzy soft subrings under fuzzy soft homomorphisms is examined. Finally, we present the concept of RP-T-fuzzy soft ideal and we investigate the analogue properties for them.

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1. Introduction

Most of the notions in all fields of the real world have uncertainties and vagueness. In 1965, Zadeh initiated fuzzy sets (or fuzzy subsets) as a class of objects with a continuum of grades of membership to deal with uncertain concepts [1]. Many researchers have established its connection with almost every topic since fuzzy sets introduced. The fuzzy sets were applied to algebra by presenting fuzzy groups by Rosenfeld [2] in 1971. The notion of fuzzy subrings was introduced by Liu in 1982 [3]. Dixit et. al. provide an internal description of the fuzzy subring and fuzzy ideal generated by a finite fuzzy subset of a ring [4].

In 1999, Molodtsov introduced the theory of the soft set has a large area of use for solving uncertain problems [5]. Maji et. al. put forth some basic algebraic features such as equality, union and intersection, null and absolute of two soft sets, and also its complement [6]. Ali et. al. discuss some situations at Maji et. al's study and give some new notions [7]. The theory of soft sets can be combined with some other theories. In 2007, Aktaş and Çağman combined soft sets with algebraic concepts and they defined the notions of soft group, soft subgroup, soft normal subgroup and soft homomorphism of soft sets [8]. Acar et. al. [9] introduce initial concepts of soft rings. Atagün and Sezgin [10] study the algebraic soft substructures of rings, fields and modules.

In 2001, Maji et. al. amalgamated soft and fuzzy sets and they defined fuzzy soft sets [11]. In 2009, Aygünoğlu and Aygün introduced a fuzzy soft group as a new concept. Defined fuzzy soft function, they also introduced a fuzzy soft homomorphism of fuzzy soft groups. Moreover, they gave the concept of normal fuzzy soft group and they investigated some of its basic properties [12]. Pazar Varol et. al. [13] introduce the concept of a fuzzy soft ring and study some of their algebraic features. İnan and Öztürk [14] present the notion of a fuzzy soft ring and $(\varepsilon, \varepsilon \wedge q)$ -fuzzy soft subring that is a generalization of the fuzzy soft ring. Çelik et. al. [15] provide a fuzzy extension of soft rings introduced by Acar et al [9]. Some recent papers show that investigations related to the theory of soft set continue rapidly [16- 26].

Akın and Karakaya [16] propose new algebraic notion which is called UP-fuzzy soft subset of a soft set, where U denotes a universal set and P denotes a set of parameters. They present the notions SP-fuzzy soft semigroup and SP-fuzzy soft left (right) ideal of a soft SP-fuzzy soft semigroup. Then, Akın [17] define GP-fuzzy soft groups. In this paper, the concept of an RP-T-fuzzy soft subring is introduced. The concept of RP-T-fuzzy soft subring, where T is a t-norm, is a combination of the notion of UP-fuzzy soft subset and ring theory. Some properties of the restricted intersection, Λ -intersection and product of their families are studied. It is demonstrated that the restricted union of RP-T-fuzzy soft subrings of a soft ring is an RP-T-fuzzy soft subring of this soft ring if T is

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an infinitely \vee -distributive t-norm. Relations of the RP-T-fuzzy soft subring of a soft ring and α -level sets of this soft ring are investigated. Some RP-T-fuzzy soft subrings which are obtained by some binary operations are examined. Moreover, the image and pre-image of RP-T-fuzzy soft subrings under a fuzzy soft homomorphism is investigated. Finally, this paper presents the concept of RP-T-fuzzy soft ideal and gives some analogue properties for them.

2. Preliminaries

2.1. Fuzzy subsets

Let U be a universe of discourse. A function f from U to $[0,1]$ is called a fuzzy subset of U . The family of all fuzzy subsets of U is denoted by $\mathcal{F}(U)$. Let $f, g \in \mathcal{F}(U)$. Then, $f \subseteq g$ means that $f(a) \leq g(a)$ for all $a \in U$. $f * g$ is a binary relation on $\mathcal{F}(U)$ defined by $(f * g)(u) = f(u) * g(u)$ for all $u \in U$, where " $*$ " is a binary relation on $[0,1]$. For $t \in [0,1]$, the set $f_t = \{a \in U | f(a) \geq t\}$ is called t -level set of f . Let $\Lambda \neq \emptyset$ be an index set and $\{f_i | i \in \Lambda\} \subseteq \mathcal{F}(U)$. Then, $(\bigwedge_{i \in \Lambda} f_i)(x) = \bigwedge_{i \in \Lambda} f_i(x)$ and $(\bigvee_{i \in \Lambda} f_i)(x) = \bigvee_{i \in \Lambda} f_i(x)$ and $(T_{i \in \Lambda} f_i)(x) = T_{i \in \Lambda} f_i(x)$ (See [1, 27]).

2.2. t-norms, t-conorms, negators and implications

A mapping $T: [0,1] \times [0,1] \rightarrow [0,1]$ which is increasing, associative, commutative and providing the boundary condition $T(u, 1) = u$ for all $u \in [0,1]$ is called a t-norm. On $[0,1]$, the largest t-norm is the standard minimum operator $T_M(u, v) = \min\{u, v\} = u \wedge v$ and the weakest t-norm is the drastic t-norm $T_D(u, v)$ which is defined as u if $v = 1$, v if $u = 1$, 0 otherwise. A mapping $S: [0,1] \times [0,1] \rightarrow [0,1]$ which is increasing, associative, commutative and providing the boundary condition (i.e., $S(u, 0) = u$ for all $u \in [0,1]$) is called a t-conorm on $[0,1]$. On $[0,1]$, the maximum operator $S_M(u, v) = \max\{u, v\} = u \vee v$ is the smallest t-conorm and the drastic t-conorm $S_W(u, v)$ which is defined as u if $v = 0$, v if $u = 0$ and 1 otherwise is the largest t-conorm. On $[0,1]$, the nilpotent t-conorm $S_N(u, v)$ is defined as $\max\{u, v\}$ if $u + v < 1$ and 0 otherwise. A mapping $N: [0,1] \rightarrow [0,1]$ which is decreasing and providing the conditions $N(1) = 0, N(0) = 1$ is referred as a negator N . The negator $N_S(u) = 1 - u$ for all $u \in [0,1]$ is called standard negator. An implication on $[0,1]$ is a mapping $I: [0,1] \times [0,1] \rightarrow [0,1]$ providing the conditions $I(1,1) = I(0,1) = I(0,0) = 1, I(1,0) = 0$. An S-implication based on S and S is an implication defined by $I(u, v) = S(N(u), v)$ for all $u, v \in [0,1]$, and R-implication (residual implication) based on a t-norm T is an implication defined by $I(u, v) = \bigvee_{T(u, \alpha) \leq v} \alpha$ for all $u, v \in [0,1]$ (See [28-30]). A t-norm T is called \vee -distributive if $T(a, b_1 \vee b_2) = T(a, b_1) \vee T(a, b_2)$ for all $a, b_1, b_2 \in [0,1]$. A t-norm T is called infinitely \vee -distributive if $T(a, \bigvee_{i \in \Lambda} b_i) = \bigvee_{i \in \Lambda} T(a, b_i)$ for all $a, b_i \in [0,1], i \in \Lambda$ (See [31, 32]).

2.3. Subrings, ideals, T-fuzzy subrings and T-fuzzy ideals

Let R denote a commutative ring and $\emptyset \neq I \subseteq R$. Then, I is called a subring of R if $a - b, a, b \in I$ for all $a, b \in I$. A subring I is a left ideal if $ra \in I$ for all $a \in I, r \in R$. Let $f \in \mathcal{F}(R)$ and T be a t-norm. Then, f is called a T-fuzzy subring if $f(x - y) \geq f(x)Tf(y)$ and $f(x.y) \geq f(x)Tf(y)$ for all $x, y \in R$. A T-fuzzy subring f is called a T-fuzzy left (right) ideal if $f(x.y) \geq f(y)$ ($f(x.y) \geq f(x)$) for all $x, y \in R$. f is called a T-fuzzy ideal (or two-sided ideal) if its both T-fuzzy left and right ideal (See [27, 33, 34]).

2.4. Soft sets and soft rings.

In this part, we present some known definitions of topics of soft sets and soft rings.

Definition 1 [5] Let U be an initial universe set and P be a set of parameters. The power set of U is denoted by $P(U)$ and A is a subset of P . A pair (F, A) is called a soft set over U where F is a mapping given by $F: A \rightarrow P(U)$. The pair (U, P) denotes the collection of all soft sets on U with the attributes from P and is called a soft class [35].

Definition 2 [5] Let (F, A) and (G, B) be two soft sets over U , (F, A) is called a soft subset of (G, B) , denoted by $(F, A) \subseteq (G, B)$, if

(i) $A \subseteq B$.

(ii) $F(x) \subseteq G(x)$ for each $x \in A$.

Definition 3 [6] Let (F, A) be soft set over U . Then,

(i) (F, A) is said to be a null soft set, denoted by Φ , if $F(x) = \emptyset$ for all $x \in A$.

(ii) (F, A) is said to be an absolute soft set, denoted by \tilde{A} , if $F(x) = U$ for all $x \in A$.

Definition 4 [36] For a soft set (F, A) , the set $\text{Supp}(F, A) = \{x \in A | F(x) \neq \emptyset\}$ is called the support of the subset (F, A) . If $\text{Supp}(F, A) \neq \emptyset$, then the soft set (F, A) is called non-null.

Definition 5 [6, 7, 36- 39] Let $\{(F_i, A_i) | i \in \Lambda\}$ be a family of soft sets in a soft class (U, P) . Then,

(i) The restricted intersection of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $(\cap_r)_{i \in \Lambda}(F_i, A_i)$, is the soft set (F, A) defined as $A = \cap_{i \in \Lambda} A_i, F(x) = \cap_{i \in \Lambda} F_i(x) (\forall x \in A)$.

(ii) The extended intersection of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $(\cap_e)_{i \in \Lambda}(F_i, A_i)$, is the soft set (F, A) defined as $A = \cup_{i \in \Lambda} A_i, F(x) = \cap_{i \in \Lambda(x)} F_i(x) (\forall x \in A)$, where $\Lambda(x) = \{i | x \in A_i\}$.

(iii) The restricted union of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $(\cup_r)_{i \in \Lambda}(F_i, A_i)$, is the soft set (F, A) defined as $A = \cap_{i \in \Lambda} A_i, F(x) = \cup_{i \in \Lambda} F_i(x) (\forall x \in A)$.

(iv) The extended union of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $(\cup_e)_{i \in \Lambda}(F_i, A_i)$, is the soft set (F, A) defined as $A = \cup_{i \in \Lambda} A_i, F(x) = \cup_{i \in \Lambda(x)} F_i(x) (\forall x \in A)$.

(v) The \wedge –intersection of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $\wedge_{i \in \Lambda} (F_i, A_i)$, is the soft set (F, A) defined as $A = \prod_{i \in \Lambda} A_i, F((x_i)_{i \in \Lambda}) = \cap_{i \in \Lambda} F_i(x_i) (\forall (x_i)_{i \in \Lambda} \in A)$.

(vi) The \vee –union of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $\vee_{i \in \Lambda} (F_i, A_i)$, is the soft set (F, A) defined as $A = \prod_{i \in \Lambda} A_i, F((x_i)_{i \in \Lambda}) = \cup_{i \in \Lambda} F_i(x_i) (\forall (x_i)_{i \in \Lambda} \in A)$.

(vii) The product of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $\prod_{i \in \Lambda} (F_i, A_i)$, is the soft set (F, A) defined as $A = \prod_{i \in \Lambda} A_i, F((x_i)_{i \in \Lambda}) = \prod_{i \in \Lambda} F_i(x_i) (\forall (x_i)_{i \in \Lambda} \in A)$.

Definition 6 [37] Let (F, A) be a non-null soft set over a ring R . Then,

(i) (F, A) is called a soft ring over R if $F(x)$ is a subring of R for all $x \in \text{Supp}(F, A)$.

(ii) (F, A) is called a soft ideal over R if $F(x)$ is an ideal of R for all $x \in \text{Supp}(F, A)$.

2.5. Fuzzy soft sets and Fuzzy soft rings

In this part, we give some known and useful definitions of topics of fuzzy soft sets and fuzzy soft rings.

Definition 7 [11] Let U be an initial universe set and P be a set of parameters. A pair (f, E) is called a fuzzy soft set over U where $f: E \rightarrow \widetilde{F(U)}$ is a mapping. The pair (\widetilde{U}, P) denotes the collection of all fuzzy soft sets on U with the attributes from P and is called a fuzzy soft class (See [40]).

Definition 8 [11] Let (f, E) be a fuzzy soft set over U . For each $\alpha \in [0,1]$, the set $(f, E)_\alpha = (f_\alpha, E)$ is called an α -level set of (f, E) where $f_\alpha(a) = \{x \in U | f(a)(x) \geq \alpha\}$ for each $a \in E$. Obviously, $(f, E)_\alpha$ is a soft set over U .

Definition 9 [11] Let (f, E) and (g, H) be two fuzzy soft sets over U , (f, E) is called a fuzzy soft subset of (g, H) , denoted by $(f, E) \subseteq (g, H)$, if (i) $E \subseteq H$, (ii) for each $a \in E$, $f(a) \leq g(a)$.

Definition 10 [11, 15, 40] Let $\{(f_i, E_i) | i \in \Lambda\}$ be a family of fuzzy soft sets in a fuzzy soft class (U, P) . Then,

(i) The restricted intersection of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $(\wedge_r)_{i \in \Lambda}(f_i, E_i)$, is a fuzzy soft set (f, E) , $E = \bigcap_{i \in \Lambda} E_i$ and for all $x \in E$, $f(x) = \wedge_{i \in \Lambda} f_i(x)$.

(ii) The extended intersection of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $(\wedge_e)_{i \in \Lambda}(f_i, E_i)$, is a fuzzy soft set (f, E) , $E = \bigcup_{i \in \Lambda} E_i$ and for all $x \in E$, $f(x) = \wedge_{i \in \Lambda(x)} f_i(x)$ where $\Lambda(x) = \{i | x \in E_i\}$.

(iii) The restricted union of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $(\vee_r)_{i \in \Lambda}(f_i, E_i)$, is a fuzzy soft set (f, E) , $E = \bigcap_{i \in \Lambda} E_i$ and for all $x \in E$, $f(x) = \vee_{i \in \Lambda} f_i(x)$.

(iv) The extended union of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $(\vee_e)_{i \in \Lambda}(f_i, E_i)$, is a fuzzy soft set (f, E) , $E = \bigcup_{i \in \Lambda} E_i$ and for all $x \in E$, $f(x) = \vee_{i \in \Lambda} f_i(x)$.

Definition 11 [11, 15] Let $\{(f_i, E_i) | i \in \Lambda\}$ be a family of fuzzy soft sets in a fuzzy soft class $(\widetilde{U}, \widetilde{P})$. Then,

(i) The fuzzy \wedge -intersection of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $\bigwedge_{i \in \Lambda} (f_i, E_i)$, is a soft set (f, E) defined as $E = \prod_{i \in \Lambda} E_i$, $f((x_i)_{i \in \Lambda}) = \bigwedge_{i \in \Lambda} f_i(x_i) (\forall (x_i)_{i \in \Lambda} \in E)$.

(ii) The fuzzy \vee -union of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $\bigvee_{i \in \Lambda} (f_i, E_i)$, is a soft set (f, E) defined as $E = \prod_{i \in \Lambda} E_i$, $f((x_i)_{i \in \Lambda}) = \bigvee_{i \in \Lambda} f_i(x_i) (\forall (x_i)_{i \in \Lambda} \in E)$.

(iii) The product of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $\prod_{i \in \Lambda} (f_i, E_i)$, is a fuzzy soft set (f, E) defined as $E = \prod_{i \in \Lambda} E_i$, $f((x_i)_{i \in \Lambda}) = \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} f_j(x_j))$.

Definition 12 [15] Suppose that \oplus is a binary operation on the power set of P and \otimes is a binary operation on $F(U)$. Then, for any two fuzzy soft sets $(f, E), (g, A) \in (\widetilde{R}, \widetilde{P})$, $(f, E) \oplus_{\otimes} (g, A)$ is defined as the Φ -fuzzy soft set (h, C) , where $C = E \oplus A$ and

$$h(x) = \begin{cases} f(x) & \text{if } x \in E \setminus A, \\ g(x) & \text{if } x \in A \setminus E, \\ f(x) \otimes g(x) & \text{if } x \in E \cap A, \\ \Phi & \text{otherwise} \end{cases}$$

for all $x \in C$, where Φ is arbitrary fuzzy set of U . Clearly, " \oplus_{\otimes} " is a binary operation on $(\widetilde{R}, \widetilde{P})$.

Definition 13 [15] Let $(f, E_1), (g, E_2)$ be fuzzy soft sets in a fuzzy soft class $(\widetilde{U}, \widetilde{P})$. Then, the fuzzy product of them, denoted by $(f, E_1) \widetilde{\times} (g, E_2)$, is the soft set (h, C) defined as $C = E_2 \times E_1$, $h(a, b) = f(a).g(b)$ for all $a \in E_1, b \in E_2$.

Definition 14 [14, 15] Let (f, E) be a fuzzy soft set over R . Then,

(i) (f, E) is said to be T-fuzzy soft ring over R if $f(x)$ is a T-fuzzy subring of R for all $x \in E$,

(ii) (f, E) is said to be T-fuzzy soft ideal over R if $f(x)$ is T-fuzzy ideal of R for $x \in E$.

Definition 15 [14] Let (f, A) be a fuzzy soft ring over R . A fuzzy soft set (g, B) over R is called a fuzzy soft ideal of (f, A) if and only if

- (i) $B \subseteq A$,
- (ii) $g(x)$ is a fuzzy ideal of $f(x)$ for all $x \in \text{Supp}(g, B)$,
- (iii) $g(x) \leq f(x)$ for all $x \in \text{Supp}(g, B)$.

Definition 16 [12] Let (f, A) and (g, B) fuzzy soft sets on the classes $(\widetilde{U}_1, \widetilde{P}_1)$ and $(\widetilde{U}_2, \widetilde{P}_2)$, respectively and let $\varphi: U_1 \rightarrow U_2, \psi: A \rightarrow B$ be functions. Then, the pair (φ, ψ) is called a fuzzy soft function from U_1 to U_2 .

Definition 17 [12] Let (f, A) and (g, B) fuzzy soft sets on the classes $(\widetilde{U}_1, \widetilde{P}_1)$ and $(\widetilde{U}_2, \widetilde{P}_2)$, respectively and let (φ, ψ) be a fuzzy soft function from U_1 to U_2 . Then,

a) The image of (f, A) under (φ, ψ) denoted by $(\varphi, \psi)(f, A)$, is the fuzzy soft set on the class (U_2, P_2) defined by $(\varphi, \psi)(f, A) = (\varphi(f), \psi(A))$, where;

$$\varphi(f)(b)(y) = \begin{cases} \bigvee_{\varphi(x)=y} \bigvee_{\psi(a)=b} f(a)(x) & \text{if } \exists \in \varphi^{-1}(y), \\ 0 & \text{otherwise.} \end{cases}$$

b) The pre-image of (g, B) under the fuzzy soft function (φ, ψ) denoted by $(\varphi, \psi)^{-1}(g, B)$, is defined by $(\varphi, \psi)^{-1}(g, B) = (\varphi^{-1}(g), \psi^{-1}(B))$, where $\varphi^{-1}(g)(a)(x) = g(\psi(a))(\varphi(x)), (\forall a \in \psi^{-1}(B), \forall x \in U_1)$

Definition 18 [13] Let (φ, ψ) be a fuzzy soft function from R to S . If φ is a ring homomorphism from R to S , then (φ, ψ) is said to be a fuzzy soft homomorphism. If φ is an isomorphism and ψ is a one-to-one mapping from A onto B , then (φ, ψ) is said to be a fuzzy soft isomorphism.

2.6. SP-fuzzy soft subsemigroup

Akın and Karakaya present a fuzzy soft set of a crisp soft set as a new concept. They introduced some new algebraic notions which are called SP-fuzzy soft semigroup and SP-fuzzy soft left (right) ideal of a soft semigroup.

Definition 19 [16] Let (F, A) be a soft set in a soft class (U, P) and (f, E) be a fuzzy soft set in the fuzzy soft class $(\widetilde{U}, \widetilde{P})$. Then, (f, E) is said to be a UP-fuzzy soft subset of (F, A) , denoted by $(f, E) \subseteq_{UP} (F, A)$, if $E \subseteq A$ and $f(x)$ is a fuzzy subset of $F(x)$ for all $x \in E$.

Definition 20 [16] Let (f, E) be a UP-fuzzy soft subset of (F, A) . (F_α, E) called α -level soft subset of (f, E) , where $F_\alpha: E \rightarrow P(U)$ is defined by $F_\alpha(x) = \{a \in F(x) | f(x)(a) \geq \alpha\}$ for all $x \in E$.

Definition 21 [16] Let (f, E) be a UP-fuzzy soft subset of (F, A) . Then, the UP-fuzzy soft subset $(f, E)^c = (f^c, E)$ of (F, A) is called the complement of (f, E) , where for any $x \in E, f^c(x): F(x) \rightarrow [0,1]$ is defined by $f^c(x)(a) = 1 - f(x)(a)$ for all $a \in F(x)$.

Theorem 22 [16] Let $\{(f_i, E_i) | i \in \Lambda\}$ be the family of UP-fuzzy soft subset of (F, A) . Then, De Morgan rules are provided for restricted intersection and union, i.e.,

$$((\widetilde{\wedge}_r)_{i \in \Lambda} (f_i, E_i))^c = (\widetilde{\vee}_r)_{i \in \Lambda} (f_i, E_i)^c \text{ and } ((\widetilde{\vee}_r)_{i \in \Lambda} (f_i, E_i))^c = (\widetilde{\wedge}_r)_{i \in \Lambda} (f_i, E_i)^c.$$

3. Results

Throughout this paper, all fuzzy subsets are considered over commutative rings R and Z . T, S and I denote a t -norm, a t -conorm and an implication on $[0,1]$, respectively, unless otherwise stated. We begin by giving a definition of RP-T-fuzzy soft subring.

Definition 23 Let (F, A) be a soft subring in a soft class (R, P) and (f, E) be a fuzzy soft set in the fuzzy soft class $(\widetilde{R}, \widetilde{P})$. Then, (f, E) is called an RP-T-fuzzy soft subring of (F, A) if $E \subseteq A$ and $f(x)$ is a T-fuzzy subring of $F(x)$ for all $x \in E$.

Example 24 Let R be the ring $(\mathbb{Z}_{24}, +, \cdot)$, $P = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\}$, $E = \{e_2\}$ and let (F, A) be defined by $F(e_1) = \langle \bar{6} \rangle$, $F(e_2) = \langle \bar{4} \rangle$.

(i) Let $\alpha, \beta \in [0,1], \alpha \leq \beta$. Then, (f, E) defined by

$$f(e_2)(\bar{a}) = \begin{cases} \beta & \text{if } \bar{a} = 0, \\ \alpha & \text{if } \bar{a} \neq 0, \end{cases}$$

is an RP-T-fuzzy soft subring of (F, A) .

(ii) Let (g, E) be defined by

$$g(e_2)(\bar{a}) = \begin{cases} 0 & \text{if } 4 \mid a, \\ 1 & \text{if } 4 \nmid a. \end{cases}$$

Then, (g, E) is an RP-T-fuzzy soft subring of (F, A) . However, (g, E) is not a fuzzy soft subring of R .

Theorem 25 Let (F, A) be a soft ring over R and (f, E) be an RP-T-fuzzy soft subset of (F, A) .

(i) (f, E) is an RP-T-fuzzy soft subring of (F, A) if (F_α, E) is soft ring over R for all $\alpha \in [0,1]$.

(ii) Let (f, E) be an RP-T-fuzzy soft subring of (F, A) . Then, (F_α, E) is a soft ring over R for all $\alpha \in D_T$ if $f(x)_\alpha \neq \emptyset$ for all $x \in E$, where $D_T = \{\alpha \mid \alpha T \alpha = \alpha\}$.

Proof. Let $x \in E$.

(i) Let $\alpha = f(x)(a)Tf(x)(b)$ for any $a, b \in F(x)$. Thus, $f(x)(a) \geq \alpha$ and $f(x)(b) \geq \alpha$. So, $a, b \in F_\alpha(x)$. Thus, $a - b, a \cdot b \in F_\alpha(x)$ since $F_\alpha(x)$ is subring of R for all $x \in E$. So, $f(x)(a - b) \geq \alpha$ and $f(x)(a \cdot b) \geq \alpha$, i.e., $f(x)(a - b) \geq f(x)(a)Tf(x)(b)$ and $f(x)(a \cdot b) \geq f(x)(a)Tf(x)(b)$. Hence, $f(x): F(x) \rightarrow [0,1]$ is fuzzy subring for all $x \in E$. Therefore, (f, E) is an RP-T-fuzzy soft subring of (F, A) .

(ii) Let $a, b \in F_\alpha(x)$ for $\alpha \in D_T$. Hereby, $f(x)(a) \geq \alpha$ and $f(x)(b) \geq \alpha$. So, $f(x)(a)Tf(x)(b) \geq \alpha T \alpha$. Thus, $f(x)(a - b) \geq \alpha$ and $f(x)(a \cdot b) \geq \alpha$ since $f(x)$ is subring of $F(x)$ for all $x \in E$. Therefore, $a - b, a \cdot b \in F_\alpha(x)$. So, (F_α, E) is a soft ring for all $\alpha \in D_T$.

Proposition 26 Let (f, E) be an RP-T-fuzzy soft subring of (F, A) . If (F_α, E) is a soft left or right ideal over R , then it is a soft ring over R .

Proof. It is straightforward.

Theorem 27 Let (f_i, E_i) be an RP-T-fuzzy soft subring of (F_i, A_i) for all $i \in \Lambda$. Then,

(i) $\bigcap_{i \in \Lambda}^r (f_i, E_i)$ is RP-T-fuzzy soft subring of $(\bigcap_{i \in \Lambda} (F_i, A_i))$ if $\bigcap_{i \in \Lambda} E_i \neq \emptyset$.

(ii) $\bigcap_{i \in \Lambda}^e (f_i, E_i)$ is RP-T-fuzzy soft subring of $(\bigcap_{i \in \Lambda} (F_i, A_i))$.

(iii) $\bigcap_{i \in \Lambda}^r (f_i, E_i)$ is RP-T-fuzzy soft subring of $(\bigcap_{i \in \Lambda} (F_i, A_i))$ if $\bigcap_{i \in \Lambda} E_i \neq \emptyset$.

(iv) $\bigwedge_{i \in \Lambda} (f_i, E_i)$ is RP-T-fuzzy soft subring of $\bigwedge_{i \in \Lambda} (F_i, A_i)$.

(v) $\prod_{i \in \Lambda} (f_i, E_i)$ is RP-T-fuzzy soft subring of $\prod_{i \in \Lambda} (F_i, A_i)$.

Proof. i) Let $\bigcap_{i \in \Lambda}^r (f_i, E_i) = (f, E)$ and $(\bigcap_{i \in \Lambda} (F_i, A_i)) = (F, A)$. Clearly $E = \bigcap_{i \in \Lambda} E_i \subseteq \bigcap_{i \in \Lambda} A_i = A$. Let $a, b \in F(x)$ for any $x \in E$

$$f(x)(a - b) = (\bigwedge_{i \in \Lambda} f_i(x))(a - b) = \bigwedge_{i \in \Lambda} f_i(x)(a - b) \geq \bigwedge_{i \in \Lambda} (f_i(x)(a)Tf_i(x)(b))$$

$$\geq \bigwedge_{i \in \Lambda} f_i(x)(a)T \bigwedge_{i \in \Lambda} f_i(x)(b) = (\bigwedge_{i \in \Lambda} f_i(x))(a)T(\bigwedge_{i \in \Lambda} f_i(x))(b)$$

$$= f(x)(a)Tf(x)(b).$$

$$f(x)(a \cdot b) = (\bigwedge_{i \in \Lambda} f_i(x))(a \cdot b) = \bigwedge_{i \in \Lambda} f_i(x)(a \cdot b) \geq \bigwedge_{i \in \Lambda} (f_i(x)(a)Tf_i(x)(b))$$

$$= \bigwedge_{i \in \Lambda} f_i(x)(a)T \bigwedge_{i \in \Lambda} f_i(x)(b) = (\bigwedge_{i \in \Lambda} f_i(x))(a)T(\bigwedge_{i \in \Lambda} f_i(x))(b)$$

$$= f(x)(a)Tf(x)(b).$$

Hence, $f(x)$ is RP-T-fuzzy soft subring of $F(x)$ for all $x \in E$. Thus, $\bigcap_{i \in \Lambda}^r (f_i, E_i)$ is RP-T-fuzzy ring of $(\bigcap_{i \in \Lambda} (F_i, A_i))$.

ii) The proof is similar to the proof of Theorem 3.4 (b) in [17] with the definitions T-fuzzy subring and RP-T-fuzzy soft subring.

iii) The proof is similar to the proof of Theorem 3.4 (c) in [17] with the definitions T-fuzzy subring and RP-T-fuzzy soft subring.

iv) Let $\bigwedge_{i \in \Lambda} (f_i, E_i) = (f, E)$ and $\bigwedge_{i \in \Lambda} (F_i, A_i) = (F, A)$. Let $((x_i)_{i \in \Lambda}) \in E$ and $a, b \in F((x_i)_{i \in \Lambda})$. Then,

$$\begin{aligned} f((x_i)_{i \in \Lambda})(a - b) &= (\bigwedge_{i \in \Lambda} f_i(x_i))(a - b) = \bigwedge_{i \in \Lambda} (f_i(x_i)(a - b)) \\ &\geq \bigwedge_{i \in \Lambda} ((f_i(x_i)(a)T(f_i(x_i)(b))) = (\bigwedge_{i \in \Lambda} (f_i(x_i)(a))T(\bigwedge_{i \in \Lambda} (f_i(x_i)(b))) \\ &= (\bigwedge_{i \in \Lambda} (f_i(x_i))(a)T(\bigwedge_{i \in \Lambda} (f_i(x_i))(b)) = f((x_i)_{i \in \Lambda})(a)Tf((x_i)_{i \in \Lambda})(b). \end{aligned}$$

$$\begin{aligned} f((x_i)_{i \in \Lambda})(a \cdot b) &= (\bigwedge_{i \in \Lambda} f_i(x_i))(a \cdot b) = \bigwedge_{i \in \Lambda} (f_i(x_i)(a \cdot b)) \\ &\geq \bigwedge_{i \in \Lambda} ((f_i(x_i)(a)T(f_i(x_i)(b))) = (\bigwedge_{i \in \Lambda} (f_i(x_i)(a))T(\bigwedge_{i \in \Lambda} (f_i(x_i)(b))) \\ &= (\bigwedge_{i \in \Lambda} (f_i(x_i))(a)T(\bigwedge_{i \in \Lambda} (f_i(x_i))(b)) = f((x_i)_{i \in \Lambda})(a)Tf((x_i)_{i \in \Lambda})(b). \end{aligned}$$

Therefore, $\bigwedge_{i \in \Lambda} (f_i, E_i)$ is RP-T-fuzzy soft subring of $\bigwedge_{i \in \Lambda} (F_i, A_i)$.

v) Let $\prod_{i \in \Lambda} \tilde{(f_i, E_i)} = (f, E)$ and $\prod_{i \in \Lambda} (F_i, A_i) = (F, A)$. Clearly, $E = \prod_{i \in \Lambda} E_i \subseteq \prod_{i \in \Lambda} A_i = A$. Let $a, b \in F((x_i)_{i \in \Lambda})$ for any $(x_i)_{i \in \Lambda} \in E$.

$$\begin{aligned} f((x_i)_{i \in \Lambda})(a - b) &= \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} f_j(x_j))(a - b) = \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} f_j(x_j)(a - b)) \\ &\geq \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} (f_j(x_j)(a)Tf_j(x_j)(b))) \\ &\geq \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} f_j(x_j)(a)T \bigwedge_{j \in J} f_j(x_j)(b)) \\ &\geq \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} f_j(x_j))(a)T \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} f_j(x_j))(b) \\ &= f((x_i)_{i \in \Lambda})(a)Tf((x_i)_{i \in \Lambda})(b). \end{aligned}$$

$$\begin{aligned} f((x_i)_{i \in \Lambda})(a \cdot b) &= \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} f_j(x_j))(a \cdot b) = \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} f_j(x_j)(a \cdot b)) \\ &\geq \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} (f_j(x_j)(a)Tf_j(x_j)(b))) \\ &\geq \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} f_j(x_j)(a)T \bigwedge_{j \in J} f_j(x_j)(b)) \\ &\geq \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} f_j(x_j))(a)T \bigvee_{J \subseteq \Lambda} (\bigwedge_{j \in J} f_j(x_j))(b) \\ &= f((x_i)_{i \in \Lambda})(a)Tf((x_i)_{i \in \Lambda})(b). \end{aligned}$$

Therefore, $\prod_{i \in \Lambda} \tilde{(f_i, E_i)}$ is an RP-T-fuzzy soft subring of $\prod_{i \in \Lambda} (F_i, A_i)$ since $f((x_i)_{i \in \Lambda})$ is a fuzzy subring of $F((x_i)_{i \in \Lambda})$.

Theorem 28 Let $\{(f_i, E_i) | i \in \Lambda\}$ be a family of RP-T-fuzzy soft subrings of (F, A) . Then, $\bigcup_{i \in \Lambda} {}^r(f_i, E_i)$ is an RP-T-fuzzy soft subring of (F, A) if T is an infinitely \vee -distributive t-norm.

Proof. Let $\bigcup_{i \in \Lambda} {}^r(f_i, E_i) = (f, E)$. Then, $E = \bigcap_{i \in \Lambda} E_i \subseteq \bigcap_{i \in \Lambda} A = A$. Let $a, b \in F(x)$ for any $x \in E$.

$$\begin{aligned} f(x)(a - b) &= (\bigvee_{i \in \Lambda} f_i(x))(a - b) = \bigvee_{i \in \Lambda} f_i(x)(a - b) \geq \bigvee_{i \in \Lambda} (f_i(x)(a)Tf_i(x)(b)) \\ &= \bigvee_{i \in \Lambda} f_i(x)(a)T \bigvee_{i \in \Lambda} f_i(x)(b) = f(x)(a)Tf(x)(b). \end{aligned}$$

$$\begin{aligned} f(x)(ab) &= (\bigvee_{i \in \Lambda} f_i(x))(ab) = \bigvee_{i \in \Lambda} f_i(x)(ab) \geq \bigvee_{i \in \Lambda} (f_i(x)(a)Tf_i(x)(b)) \\ &= \bigvee_{i \in \Lambda} f_i(x)(a)T \bigvee_{i \in \Lambda} f_i(x)(b) = f(x)(a)Tf(x)(b). \end{aligned}$$

Thus, $f(x)$ is a T-fuzzy subring of $F(A)$. Therefore, $\bigcup_{i \in \Lambda} {}^r(f_i, E_i)$ is an RP-T-fuzzy soft subring of (F, A) .

Let $\{(f_i, E_i) | i \in \Lambda\}$ be a family of RP-T-fuzzy soft subring of (F, A) . The following example shows that $\bigvee_{i \in \Lambda} (f_i, E_i)$ is not an RP-T-fuzzy soft subring of $\bigvee_{i \in \Lambda} (F, A)$ in general.

Example 29 Let $R = \mathbb{Z}_4, \Lambda = \{1, 2\}, E_1 = \{e_1, e_2\}, E_2 = \{e_2, e_3\}, A = \{e_1, e_2, e_3\}, T = \wedge$. Let (F, A) be defined as $F(e_1) = \{\bar{0}\}, F(e_2) = \{\bar{0}, \bar{2}\}, F(e_3) = R$. Then, (F, A) is a soft ring over R . Let fuzzy soft sets (f_1, E_1) and (f_2, E_2) be defined by

$f_1(e_1)(\bar{0}) = 0.1, f_1(e_2)(\bar{0}) = 0.5, f_1(e_2)(\bar{2}) = 0.3,$
 $f_2(e_2)(\bar{0}) = 0.6, f_2(e_2)(\bar{2}) = 0.2, f_2(e_3)(\bar{0}) = 0.4, f_2(e_3)(\bar{1}) = 0.2, f_2(e_3)(\bar{2}) = 0.3, f_2(e_3)(\bar{3}) = 0.2$
 Then, $\{(f_i, E_i) | i \in \Lambda\}$ is a family of RP-T-fuzzy soft subring of (F, A) . Therefore, $\vee_{i \in \Lambda} (F, A) = (G, B)$ is obtained as $G(e_1, e_1) = F(e_1), G(e_1, e_2) = G(e_2, e_1) = G(e_2, e_2) = F(e_2), G(e_3, e_1) = G(e_1, e_3) = G(e_2, e_3) = G(e_3, e_2) = G(e_3, e_3) = R$. However, $\vee_{i \in \Lambda} (f_i, E_i) = (f_1, E_1) \vee (f_2, E_2) = (f, E)$ can not be obtained by the knowledge of the definitions above since $f(e_1, e_2)(\bar{2})$ is incalculable. If we have $f_1(e_1)(\bar{2}) = 1$, then $f(e_1, e_2)(\bar{0}) = 0.6, f(e_1, e_2)(\bar{2}) = 1$. Consequently, (f, E) is not RP-T-fuzzy soft subring of (G, B) since $f(e_1, e_2)(\bar{2} - \bar{2}) = 0.6 \not\geq 1 = f(e_1, e_2)(\bar{2}) \wedge f(e_1, e_2)(\bar{2})$.

Theorem 30 Let (f, E) and (g, A) be RP-T-fuzzy soft subrings of (F, K) and (G, L) , respectively. Then, $(f, E) \cap_T (g, A)$ is RP-T-fuzzy soft subring of $(F, K) \cap_r (G, L)$.

Proof. Let $(f, E) \cap_T (g, A) = (h, C)$ and $(F, K) \cap_r (G, L) = (H, B)$. Then, $C = E \cap A \subseteq K \cap L = B$. Let $x \in C$. For all $u, v \in F(x) \cap G(x)$,

$$\begin{aligned}
 h(x)(u)Th(x)(v) &= (f(x)Tg(x))(u)T(f(x)Tg(x))(v) \\
 &= (f(x)(u)Tg(x)(u))T(f(x)(v)Tg(x)(v)) \\
 &= (f(x)(u)Tf(x)(v))T(g(x)(u)Tg(x)(v)) \\
 &\leq f(x)(u - v)Tg(x)(u - v) \\
 &= (f(x)Tg(x))(u - v) \\
 &= h(x)(u - v).
 \end{aligned}$$

$$\begin{aligned}
 h(x)(u)Th(x)(v) &= (f(x)Tg(x))(u)T(f(x)Tg(x))(v) \\
 &= (f(x)(u)Tg(x)(u))T(f(x)(v)Tg(x)(v)) \\
 &= (f(x)(u)Tf(x)(v))T(g(x)(u)Tg(x)(v)) \\
 &\leq f(x)(uv)Tg(x)(uv) \\
 &= (f(x)Tg(x))(uv) \\
 &= h(x)(uv).
 \end{aligned}$$

Therefore, $(f, E) \cap_T (g, A)$ is an RP-T-fuzzy soft subring of $(F, K) \cap_r (G, L)$.

Following examples show that $(f, E) \cap_l (g, A)$ and $(f, E) \cap_s (g, A)$ may not be RP-T-fuzzy soft subrings of $(F, K) \cap_r (G, L)$ in general.

Example 31 Let $R = \mathbb{Z}_4, \Lambda = \{1, 2\}, E = \{e_1, e_2\}, A = \{e_2, e_3\}, K = L = \{e_1, e_2, e_3\}$. Let $(F, K) = (G, L)$ be defined as $F(e_1) = \{\bar{0}\}, F(e_2) = \{\bar{0}, \bar{2}\}, F(e_3) = \mathbb{Z}_4$. Then, (F, K) and (G, L) are soft rings over R . Then, $(F, K) \cap_r (G, L) = (F, K) = (G, L)$.

1) Let $T = \wedge$ and I be the residual implication of T and let (f, E) and (g, A) be defined by

$$\begin{aligned}
 f(e_1)(\bar{0}) &= 1, f(e_2)(\bar{0}) = 0.5, f(e_2)(\bar{2}) = 0.3, \\
 g(e_2)(\bar{0}) &= 0.4, g(e_2)(\bar{2}) = 0.4, g(e_3)(\bar{0}) = 0.6, g(e_3)(\bar{1}) = 0.4, g(e_3)(\bar{2}) = 0.5, g(e_3)(\bar{3}) = 0.4.
 \end{aligned}$$

Then, (f, E) and (g, A) are RP-T-fuzzy soft subrings of $(F, K) = (G, L)$. Let $(f, E) \cap_l (g, A) = (h, C)$. Then, $C = E \cap A = \{e_2\}$. Thus, (h, C) is obtained as

$$h(e_2)(\bar{0}) = 0.4, h(e_2)(\bar{2}) = 1$$

Therefore, (h, C) is not RP-T-fuzzy soft subring of $(F, K) \cap_r (G, L)$ since $h(e_2)(\bar{2} - \bar{2}) = 0.4 \not\geq 1 = h(e_2)(\bar{2}) \wedge h(e_2)(\bar{2})$.

2) Let $T = \wedge$ and S be the nilpotent t-conorm S_N and let (f, E) and (g, A) be defined by

$$\begin{aligned}
 f(e_1)(\bar{0}) &= 1, f(e_2)(\bar{0}) = 0.5, f(e_2)(\bar{2}) = 0, \\
 g(e_2)(\bar{0}) &= 0.7, g(e_2)(\bar{2}) = 0.3, g(e_3)(\bar{0}) = 0.6, g(e_3)(\bar{1}) = 0.4, g(e_3)(\bar{2}) = 0.5, g(e_3)(\bar{3}) = 0.4.
 \end{aligned}$$

Then, (f, E) and (g, A) are RP-T-fuzzy soft subrings of $(F, K) = (G, L)$. Let $(f, E) \cap_s (g, A) = (l, B)$. Then, $B = E \cap A = \{e_2\}$. Thus, (l, B) is obtained as

$$l(e_2)(\bar{0}) = 0, l(e_2)(\bar{2}) = 0.3$$

Therefore, (l, B) is not RP-T-fuzzy soft subring of $(F, K) \cap_r (G, L)$ since $l(e_2)(\bar{2} - \bar{2}) = 0 \not\geq 0.3 = l(e_2)(\bar{2}) \wedge l(e_2)(\bar{2})$.

Theorem 32 Let (f, E) and (g, A) be RP-T-fuzzy soft subrings of (F, K) and (G, L) , respectively. Then, $(f, E) \cup_T (g, A)$ is RP-T-fuzzy soft subring of $(F, K) \cap_e (G, L)$.

Proof. Let $(f, E) \cup_T (g, A) = (h, C)$ and $(F, K) \cap_e (G, L) = (H, B)$. Then, $C = E \cup A \subseteq K \cup L = B$. The rest of the proof is analogue with the proof of Theorem 30.

Definition 33 Let $(f, E), (g, A) \in (\widetilde{R}, \widetilde{P})$ and $*$ be a binary operation on the unit interval $[0,1]$. Then, a mapping $fg_*: E \times A \rightarrow [0,1]^R$ is defined by $fg_*(e, a) = f(e) * g(a)$ for all $e \in E$ and $a \in A$. It is clear to see that $(fg_*, E \times A) \in (\widetilde{R}, \widetilde{P} \times \widetilde{P})$.

Theorem 34 Let (f, E) and (g, A) be RP-T-fuzzy soft subrings of (F, K) and (G, L) , respectively. Then, $(fg_T, E \times A)$ is RP-T-fuzzy soft subring of $(F, K) \wedge (G, L)$.

Proof. Let $(F, K) \wedge (G, L) = (H, B)$. Then, $E \times A \subseteq K \times L = B$. Let $(e, a) \in E \times A$ and let $u, v \in H(e, a) = F(e) \cap G(a)$. Then,

$$\begin{aligned} fg_T(e, a)(u)Tfg_T(e, a)(v) &= (f(e)Tg(a))(u)T(f(e)Tg(a))(v) \\ &= (f(e)(u)Tg(a)(u))T(f(e)(v)Tg(a)(v)) \\ &= (f(e)(u)Tf(e)(v))T(g(a)(u)Tg(a)(v)) \\ &\leq f(e)(u - v)Tg(a)(u - v) \\ &= fg_T(e, a)(u - v). \\ fg_T(e, a)(u)Tfg_T(e, a)(v) &= (f(e)Tg(a))(u)T(f(e)Tg(a))(v) \\ &= (f(e)(u)Tg(a)(u))T(f(e)(v)Tg(a)(v)) \\ &= (f(e)(u)Tf(e)(v))T(g(a)(u)Tg(a)(v)) \\ &\leq f(e)(uv)Tg(a)(uv) \\ &= fg_T(e, a)(uv). \end{aligned}$$

Hence, $(fg_T, E \times A)$ is an RP-T-fuzzy soft subring of $(F, K) \wedge (G, L)$.

Following examples show that $(fg_I, E \times A)$ and $(fg_S, E \times A)$ may not be RP-T-fuzzy soft subrings of $(F, K) \wedge (G, L)$ in general.

Example 35 Let $T = T_M$ and I be R-implication (residual implication) of T and let $P = \{e_1, e_2, e_3\}, E = \{e_1\}, A = \{e_1, e_2\}$ and \mathbb{R} be the real numbers with the known multiplication and addition operations. Let (F, E) and (G, A) be defined by $F(e_1) = 2\mathbb{Z} = G(e_2), G(e_1) = \mathbb{R}$. Thus, (F, E) and (G, A) are soft subrings of \mathbb{R} . If (f, E) and (g, A) are defined as follows

$$\begin{aligned} f(e_1)(x) &= \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases} \\ g(e_1)(x) &= \begin{cases} \frac{1}{2} & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases} \\ g(e_2)(x) &= \begin{cases} 1 & \text{if } x = 0 \\ \frac{1}{2} & \text{if } x \neq 0, \end{cases} \end{aligned}$$

then (f, E) and (g, A) are RP-fuzzy soft subrings of (F, E) and (G, A) , respectively. Hence, $(fg_I, E \times A)$ is not an RP-fuzzy soft subring of $(F, E) \wedge (G, A)$ since $fg_I(e_1, e_1)(2)Tfg_I(e_1, e_1)(2) = 1 \not\leq \frac{1}{2} = fg_I(e_1, e_1)(2 - 2)$.

Theorem 36 Let T be an infinitely \vee -distributive t-norm and (f, E) be an RP-T-fuzzy soft subring of (F, A) . If (φ, ψ) is a fuzzy soft homomorphism, then $(\varphi, \psi)(f, E)$ is a ZP_2 -T-fuzzy soft subring of $(\varphi(F), \psi(A))$.

Proof. Let $y_1, y_2 \in \varphi(F)(b)$ for any $b \in \psi(E)$. Suppose that there exist $x_1, x_2 \in R$ such that $\varphi(x_1) = y_1$ and $\varphi(x_2) = y_2$. Then, the inequality

$$\varphi(f)(b)(y_1y_2) \geq \varphi(f)(b)(y_1)T\varphi(f)(b)(y_2)$$

is verified by the similar way in the proof of Theorem 3.8 in [17] with the use of the infinitely \vee -distributivity of T . Thus, the proof of Theorem 3.8 in [17] completes the proof.

Theorem 37 Let (g, B) be a ZP_2 -T-fuzzy soft subring of (G, K) . If (φ, ψ) is a fuzzy soft homomorphism, then $(\varphi, \psi)^{-1}(g, B)$ is an RP_1 -T-fuzzy soft subring of $(\varphi^{-1}(G(\psi)), \psi^{-1}(K))$.

Proof. Let $x_1, x_2 \in \varphi^{-1}(G)(a)$ for all $a \in \psi^{-1}(B)$. Then, the inequality

$$\varphi^{-1}(a)(x_1x_2) \geq \varphi^{-1}(g)(a)(x_1) \wedge \varphi^{-1}(g)(a)(x_2)$$

is verified by the similar way in the proof of Theorem 3.9 in [17]. Thus, $(\varphi, \psi)^{-1}(g, B)$ is an RP_1 -T-fuzzy soft subring of $(\varphi^{-1}(G(\psi)), \psi^{-1}(K))$ by the proof of Theorem 3.9 in [17].

Definition 38 Let R be a ring and let (F, A) be a soft ideal in a soft class (R, P) and (f, E) be a fuzzy soft set in the fuzzy soft class $(\overline{R}, \overline{P})$. (f, E) is called an RP -T-fuzzy soft left (right) ideal of (F, A) if $E \subseteq A$ and $f(x)$ is a T-fuzzy left (right) ideal of $F(x)$ for all $x \in E$.

Theorem 39 Let (F, A) be a soft ring over R and (f, E) be an RP -T-fuzzy soft subset of (F, A) . Then, (f, E) is an RP -T-fuzzy soft left (right) ideal of (F, A) if (F_α, E) is soft left (right) ideal over R for all $\alpha \in [0, 1]$.

Proof. Let (F_α, E) be a soft left (right) ideal over R for all $\alpha \in [0, 1]$ and let $\alpha = f(x)(b)$ for any $b \in F(x)$. Thus, $f(x)(b) \geq \alpha$. So, $b \in F_\alpha(x)$. Thus, $rb \in F_\alpha(x)$ ($br \in F_\alpha(x)$) for all $r \in R$ since $F_\alpha(x)$ is a left (right) ideal of R for all $x \in E$. So, $f(x)(rb) \geq \alpha$ ($f(x)(br) \geq \alpha$), i.e. $f(x)(rb) \geq f(x)(b)$ ($f(x)(br) \geq f(x)(b)$). Hence, $f(x): F(x) \rightarrow [0, 1]$ is fuzzy left (right) ideal for all $x \in E$ accordance with Theorem 25 and Proposition 26. Therefore, (f, E) is an RP -T-fuzzy soft left (right) ideal of (F, A) .

Theorem 40 Let $\{(f_i, A_i) | i \in \Lambda\}$ be a family of the RP -T-fuzzy soft left (right) ideals of (F_i, A_i) for all $i \in \Lambda$.

- (i) $\bigcap_{i \in \Lambda} {}^r(f_i, E_i)$ is RP -T-fuzzy soft left (right) ideal of $(\bigcap_{i \in \Lambda} (F_i, A_i))$ if $\bigcap_{i \in \Lambda} E_i \neq \emptyset$.
- (ii) $\bigcap_{i \in \Lambda} {}^e(f_i, E_i)$ is RP -T-fuzzy soft left (right) ideal of $(\bigcap_{i \in \Lambda} (F_i, A_i))$.
- (iii) $\bigcap_{i \in \Lambda} {}^r(f_i, E_i)$ is RP -T-fuzzy soft left (right) ideal of $(\bigcap_{i \in \Lambda} (F_i, A_i))$ if $\bigcap_{i \in \Lambda} E_i \neq \emptyset$.
- (iv) $\bigwedge_{i \in \Lambda} (f_i, E_i)$ is RP -T-fuzzy soft left (right) ideal of $\bigwedge_{i \in \Lambda} (F_i, A_i)$.
- (v) $\prod_{i \in \Lambda} (f_i, E_i)$ is RP -T-fuzzy soft left (right) ideal of $\prod_{i \in \Lambda} (F_i, A_i)$.

Proof. It is straightforward.

Theorem 41 Let $\{(f_i, E_i) | i \in \Lambda\}$ be a family of RP -T-fuzzy soft left (right) ideals of (F, A) . Then $\bigcup_{i \in \Lambda} {}^r(f_i, E_i)$ is an RP -T-fuzzy soft left (right) ideal of (F, A) if T is an infinitely \vee -distributive t-norm.

Proof. Let $\bigcup_{i \in \Lambda} {}^r(f_i, E_i) = (f, E)$. Then, $E = \bigcup_{i \in \Lambda} E_i \subseteq \bigcup_{i \in \Lambda} A = A$. Let $a, b \in F(x)$ for any $x \in E$.

$$\begin{aligned} f(x)(ab) &= \vee_{i \in \Lambda} (f_i(x))(ab) = \vee_{i \in \Lambda} f_i(x)(ab) \geq \vee_{i \in \Lambda} (f_i(x)(b)) \\ &= (\vee_{i \in \Lambda} f_i(x))(b) = f(x)(b). \end{aligned}$$

Therefore, $\bigcup_{i \in \Lambda} {}^r(f_i, E_i)$ is an RP -T-fuzzy soft left ideal of (F, A) by Theorem 28. The proof is similar for RP -T-fuzzy soft right ideals.

Theorem 42 Let (f, E) and (g, A) be RP -T-fuzzy soft left (right) ideals of (F, K) and (G, L) , respectively. Then, $(f, E) \cap_T (g, A)$ is RP -T-fuzzy soft left (right) ideal of $(F, K) \cap_r (G, L)$.

Proof. Let $(f, E) \cap_T (g, A) = (h, C)$ and $(F, K) \cap_r (G, L) = (H, B)$. Then, $C = E \cap A \subseteq K \cap L = B$. Let $x \in C$. For all $u, v \in F(x) \cap G(x)$,

$$\begin{aligned} h(x)(uv) &= (f(x)Tg(x))(uv) = f(x)(uv)Tg(x)(uv) \\ &\geq f(x)(v)Tg(x)(v) = h(x)(v). \end{aligned}$$

Therefore, $(f, E) \cap_T (g, A)$ is an RP -T-fuzzy soft left ideal of $(F, K) \cap_r (G, L)$ by Theorem 30. The proof is similar for RP -T-fuzzy soft right ideals.

Theorem 43 Let (f, E) and (g, A) be RP-T-fuzzy soft left (right) ideals of (F, K) and (G, L) , respectively. Then, $(f, E) \cup_T (g, A)$ is RP-T-fuzzy soft left (right) ideal of $(F, K) \cap_e (G, L)$.

Proof. Let $(f, E) \cup_T (g, A) = (h, C)$ and $(F, K) \cap_e (G, L) = (H, B)$. Then, $C = E \cup A \subseteq K \cup L = B$. The rest of the steps are similar to the proof of Theorem 42.

Theorem 44 Let (f, E) and (g, A) be RP-T-fuzzy soft left (right) ideals of (F, K) and (G, L) , respectively. Then, $(fg_T, E \times A)$ is RP-T-fuzzy soft left (right) ideal of $(F, K) \wedge (G, L)$.

Proof. Let $(F, K) \wedge (G, L) = (H, B)$. Then, $E \times A \subseteq K \times L = B$. Let $(e, a) \in E \times A$ and let $u, v \in H(e, a) = F(e) \cap G(a)$. Then,

$$fg_T(e, a)(uv) = f(e)(uv)Tg(a)(uv) \geq f(e)(v)Tg(a)(v) = fg_T(e, a)(v)$$

Hence, $(fg_T, E \times A)$ is an RP-T-fuzzy soft left ideal of $(F, K) \wedge (G, L)$ by Theorem 34. The proof is similar for RP-T-fuzzy soft right ideals.

Theorem 45 Let T be an infinitely \vee -distributive t-norm and (f, E) be an RP-T-fuzzy soft left (right) ideal of (F, A) . If (φ, ψ) is a fuzzy soft homomorphism, then $(\varphi, \psi)(f, E)$ is a ZP_2 -T-fuzzy soft left (right) ideal of $(\varphi(F), \psi(A))$.

Proof. Let (f, E) be an RP-T-fuzzy soft left ideal of (F, A) and let $y_1, y_2 \in \varphi(F)(b)$ for any $b \in \psi(E)$. Suppose that there exist $x_1, x_2 \in R$ such that $\varphi(x_1) = y_1$ and $\varphi(x_2) = y_2$. Then the inequality

$$\varphi(f)(b)(y_1y_2) \geq \bigvee_{\varphi(x_2)=y_2} \bigvee_{\psi(a)=b} f(a)(x_2) = \varphi(f)(b)(y_2)$$

is verified since the inequality

$$\varphi(f)(b)(y_1y_2) = \bigvee_{\varphi(x)=y_1y_2} \bigvee_{\psi(a)=b} f(a)(x) \geq \bigvee_{\psi(a)=b} f(a)(x_1x_2) \geq \bigvee_{\psi(a)=b} (f(a)(x_2))$$

is satisfied for each $x_1, x_2 \in R$ such that $\varphi(x_1) = y_1$ and $\varphi(x_2) = y_2$. Thus, $(\varphi, \psi)(f, E)$ is a ZP_2 -T-fuzzy soft left ideal of $(\varphi(F), \psi(A))$ by Theorem 36. The proof is similar for right ideals.

Theorem 46 Let (g, B) be a ZP_2 -T-fuzzy soft left (right) ideal of (G, K) . If (φ, ψ) is a fuzzy soft homomorphism, then $(\varphi, \psi)^{-1}(g, B)$ is an RP_1 -T-fuzzy soft left (right) ideal of $(\varphi^{-1}(G(\psi)), \psi^{-1}(K))$.

Proof. Let (g, B) be a ZP_2 -T-fuzzy soft left ideal of (G, K) and let $x_1, x_2 \in \varphi^{-1}(G)(a)$ for all $a \in \psi^{-1}(B)$. Then,

$$\begin{aligned} \varphi^{-1}(g)(a)(x_1x_2) &= g(\psi(a))(\varphi(x_1x_2)) = g(\psi(a))(\varphi(x_1)\varphi(x_2)) \\ &\geq g(\psi(a))(\varphi(x_2)) = \varphi^{-1}(g)(a)(x_2). \end{aligned}$$

Thus, $(\varphi, \psi)^{-1}(g, B)$ is an RP_1 -T-fuzzy soft left ideal of $(\varphi^{-1}(T(\psi)), \psi^{-1}(K))$ by Theorem 37. The proof is similar for right ideals.

Conflicts of interest

The authors state that did not have conflict of interests.

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