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Consistent Empirical Physical Formula Construction for Gamma Ray Angular Distribution Coefficients by Layered Feedforward Neural Network

Nihat YILDIZ, Serkan AKKOYUN*, Hüseyin KAYA

Sivas Cumhuriyet University, Faculty of Science, Department of Physics, Sivas, TURKEY

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Abstract. Multipolarities of gamma rays and spins-parities of nuclear states are usually investigated by the angular distribution of gamma rays emitted from aligned states formed by nuclear reactions. For different multipolarities of the transitions, the distribution shows different characteristics. The distribution is obtained by using angular distribution formula which has literature tabulated coefficients for different spins and multipolarities. However, these coefficients involve r-fold tensor products and they are highly nonlinear in nature. Furthermore, as the calculation of these coefficients implicitly involves highly complicated integral quantities, they are very difficult to handle explicitly for larger r values. In this respect, as we theoretically proved in a previous paper, universal nonlinear function approximator layered feedforward neural network (LFNN) can be applied to construct consistent empirical physical formulas (EPFs) for nonlinear physical phenomena. In this paper, by concentrating on the integer spins of nuclear states and dipole and quadrupole type multipolarities of the transitions, we consistently estimated the coefficients by constructing suitable LFNNs. The LFNN-EPFs fitted the literature coefficient data very well. Moreover, magnificent LFNN test set forecastings over previously unseen data confirmed the consistent LFNN-EPFs for the determination of coefficients. In this sense, we can conclude that the LFNN consistently infers nonlinear physical laws governing the angular distribution of gamma rays, which are otherwise difficult to obtain by conventional coefficient calculation methods.

***Keywords*:** Angular distribution, multipolarity, nuclear spin, layered feed-forward neural network.

Katmanlı Beslemeli Sinir Ağı ile Gama Işını Açısal Dağılım Katsayıları için Tutarlı Ampirik Fiziksel Formül Eldesi

Özet. Gama ışınlarının multipolariteleri ve nükleer durumların spinleri, genellikle, nükleer reaksiyonlarla oluşturulan hizalanmış durumlardan yayılan gama ışınlarının açısal dağılımı ile incelenir. Geçişlerin farklı multipolarite değerleri için, dağılım farklı özellikler göstermektedir. Dağıtım, farklı spinler ve çok kutupluluklar için literatürdeki tablolanmış katsayılarve açısal dağılım formülü kullanılarak elde edilir. Bununla birlikte, bu katsayılar r katlı tensör çarpımları içerir ve yapıları oldukça doğrusal olmayan şekildedir. Dahası, bu katsayıların hesaplanması karmaşık integraller içerdiğinden, daha büyük r değerleri için açıkça ele alınması çok zordur. Bu bağlamda, daha önceki bir çalışmamızla teorik olarak ispatlandığımız gibi, doğrusal olmayan fiziksel fenomenler için, tutarlı, ampirik fiziksel formüller (EPF'ler) oluşturmak için, evrensel doğrusal olmayan bir katmanlı beslemeli sinir ağı (LFNN) kullanılabilir. Bu makalede, nükleer durumların tamsayı spinlerine ve geçişlerin dipol ve kuadrupol multipolaritelerine odaklanarak, uygun LFNN'leri inşa ederek katsayıları tutarlı bir şekilde tahmin ettik. LFNN-EPF'ler, literatür katsayısı verisini çok iyi bir şekilde fitledi. Ayrıca, daha önce görülmemiş veriler üzerinde yapılan LFNN test seti tahminleri, katsayıların belirlenmesi için tutarlı LFNN-EPF'leri doğrulamıştır. Bu bağlamda, LFNN'nin, gama ışınlarının açısal

* Corresponding author. *Email address: sakkoyun@cumhuriyet.edu.tr*

dağılımını yöneten doğrusal olmayan fiziksel yasalara tutarlı bir şekilde uyduğu sonucuna varabiliriz. Bu da, geleneksel katsayı hesaplama yöntemleri ile elde edilmesi zor olan bir sonuçtur.

Anahtar Kelimeler: Açısal dağılım, çok kutupluluk, nükleer spin, katmanlı iletimli sinir ağı.

1. INTRODUCTION

The angular distribution coefficients are used in the interpretation of measured gamma ray angular distributions. In heavy ion reactions excited states of nuclei are aligned relative to the beam axis. The gamma rays from these states exhibit characteristic angular distributions. The distributions depend on the multipolarities of the radiations emitted and the spins of the nuclear states. From oriented nuclei, the angular distribution of emitted gamma ray has no longer spherical symmetry. The type of the multipolarity of the transition is investigated by the distribution. It is possible to have electric and magnetic transition mixture, if more than one multipolarities in the transition are allowed by the angular momentum and parity section rules. For instance, it is common as E2 transitions are enhanced and compete with M1 transitions. The contributions of the different multipolarities to the transition are found by the angular distribution function. In this function the coefficients (A_k) depend on the nuclear spins of the states, angular momentum of the gamma rays in the transition and multipole mixing ratios.

Nevertheless, these coefficients involve r-fold tensor products and they are highly nonlinear in nature. Furthermore, as the calculation of these coefficients implicitly involves highly complicated integral quantities, they are very difficult to handle explicitly for larger r values. In this respect, as we theoretically proved in a previous paper [1], universal nonlinear function approximator layered feedforward neural network (LFNN) can be applied to construct consistent empirical physical formulas (EPFs) for nonlinear physical phenomena. Before going further, note that in recent years, neural network (NN) method has been used in many fields of nuclear physics [2-8]. In this paper, by concentrating on the integer spins of nuclear states and dipole and quadrupole type multipolarities of the transitions, we consistently estimated the literature data coefficients by constructing suitable LFNNs. The estimation of angular distribution coefficients for the angular distribution of gamma rays is useful for the analysis of experimental results. The LFNN-EPFs fitted the literature coefficient data very well. Moreover, magnificent LFNN test set forecastings over previously unseen data confirmed the consistent LFNN-EPFs for the determination of coefficients. In this sense, we can conclude that the LFNN consistently infers nonlinear physical laws governing the angular distribution of gamma rays, which are otherwise difficult to obtain by conventional coefficient calculation methods.

2. BRIEF THEORY FOR ANGULAR DISTRIBUTION COEFFICIENTS

Determination of angular distributions of gamma-rays from nuclear levels is a useful tool for the assignment of multipolarities of emitted gamma-rays and related spin and parity of nuclear levels. This method has been used in the nuclear spectroscopy for many years [9,10]. The distribution depends on the multipolarities and the spin sequences. In the angular distribution formula, table of coefficients are tabulated for the analyses of the experimental data in comparison with the theoretical one [10-12]. The angular distribution function for the transition from initial state (J_i) to final state (J_f) is given by Eq.(1)

$$W(\theta) = 1 + A_2P_2(cos\theta) + A_4P_4(cos\theta) \qquad (1)$$

Where A_k and P_k are angular distribution coefficients and Legendre polynomials, respectively. The sum in this equation involves only even values of k (to conserve parity) and extends to twice the lowest multipolarities in the transition. The polar angle θ is measured with respect to the axis of alignment which, in the case of heavy ion reaction, is along the direction of an incoming beam. For the complete alignment of the nuclear states, A_k coefficients are given in Eq.(2)

$$A_k(J_f L_1 L_2 J_i) = \frac{1}{1+\delta^2}[B_k(J_i)F_k(J_f L_1 L_1 J_i) + 2\delta B_k(J_i)F_k(J_f L_1 L_2 J_i) + \delta^2 B_k(J_i)F_k(J_f L_2 L_2 J_i)] \quad (2)$$

The B_k and F_k terms depend on J values, Clebsh-Gordan and Racah coefficients. The three parts in the expression correspond to first type $(L_1 L_1)$, mixed type $(L_1 L_2)$, and second type $(L_2 L_2)$ of multipolarity in transition. L_1 and L_2 are positive integers or zero. One can use experimental A_k values and calculated B_2F_2 values in Eq.(2) in order to extract multipole mixing ratios in the transitions. Therefore, the estimation of B_2F_2 values is of the important issue in experimental nuclear structure studies.

For the partial alignment, the functions given above are multiplied by the attenuation coefficients which depend on J and m substates distributions [11]. For detail information about the formulation, we refer readers to [10,11].

3. SHORT LFNN FUNDAMENTALS AND EPF FORMATİON BY A LFNN

Both Clebsh-Gordan and Racah coefficients involve r-fold tensor products, and these coefficients are very difficult to compute even for small r values, and such formulas are not even known for larger values of r. They are highly nonlinear in nature. Furthermore, as the calculation of these coefficients implicitly involves highly complicated integral quantities, they are very difficult to handle explicitly. To overcome this obstacle, we built up definitive LFNN-EPFs to estimate the B_2F_2 functions of Eq.(2). As usual, the LFNN-EPFs found good agreements with both the actually trained literature data and also test data points. More details about LFNN-EPF formations can be found in our previous novel comprehensive papers [1,2]. Still, we reproduce the brief details of a LFNN-EPFs here.

3.1 Artificial neural networks (ANNs) and LFNN-EPFs in brief

Artificial neural network (ANN) [13] mimics the brain functionality. It consists of artificial neurons which have adaptive synaptic weights. By a proper modification of the weights, ANN finally learns the information embedded in data. LFNN is a particular kind of ANN with one input, many intermediate (hidden) and one output layer device. All layers are connected to each other by adaptable weights (Fig. 1). Theoretically speaking, a single hidden layer LFNN is sufficient for excellent nonlinear function approximation [14]. With enough sample train data, the ultimate aim of the LFNN is to find a set of final weights to minimize the error metric $\|f-g\|$ via a suitable weight adaptation algorithm. Here, $f: R^p \rightarrow R^r$ is the LFNN transformation function and $g: R^p \rightarrow R^r$ is the function to be approximated by the LFNN. In this paper, as clearly shown in Fig.1, p being the number input layer neurons and r number of output layer neurons ($p=4, r=1$), $g: R^p \rightarrow R^r$ is the B_2F_2 function of J values. By using the final weights after the train stage of LFNN, the performance of the network is tested over a previously unseen test data set. If test data predictions are good enough, the LFNN is considered to have consistently learned the functional relationship between input and output data. Let w^* be vector of final weights (25 adaptable weights in Fig.1), $f(w^*)$ can be taken the desired EPF for the physical data which has been trained and tested by the LFNN. Therefore, we say that we have a consistent LFNN-EPF, consistency simply being the accurate estimations of previously unseen test set data by the final LFNN.

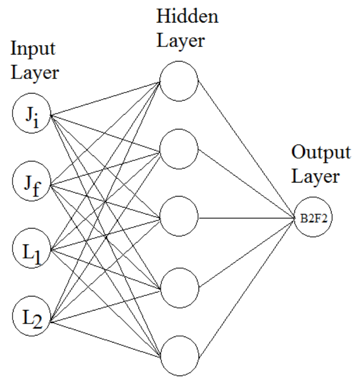

Figure 1. The LFNN used with (4, 5, 1) topology. There are $4x5+5x1=25$ adaptable weights.

3.2 Materials and methods: the details of LFNN application to EPFs

The literature data to produce LFNN-EPFs, the gamma angular distribution coefficient B_2F_2 (the output of the LFNN) versus initial and final spins of the nuclei (J_i and J_f) and first and second type of multipolarities L_1 and L_2 [the input of the LFNN] belonging to the transitions, was borrowed from [11]. The neural network software used was NeuroSolutions V5.06. The LFNN was the single hidden layer (with optimally 5 hidden neurons), four input layer neuron ($p = 4$) and with varying hidden layer neuron numbers (h = 5, 7 and 9) and one output layer neuron ($r = 1$). Although, in Fig.1, only 25 adaptable weights are shown, actually there were $25, 33$ and 41 adaptable weights for hidden layer neuron number h values of 5, 7 and 9, respectively. The activation functions in Fig.1, were, respectively, hyperbolic tangent $\tanh = (e^x - e^{-x})/(e^x + e^{-x})$ for hidden and linear for output layer. The LFNN weight adapting algorithm was back-propagation with Levenberg-Marquardt. For all LFNN processing cases, the angular distribution coefficients data were uniformly partitioned into two separate sets (80% and 20%) to use as LFNN training set for fitting and test set for prediction, respectively. The error function measuring the difference between desired and neural network outputs was the mean square error (*MSE*). The best final LFNN approximation errors for h=5, 7, 9 were $0.01, 0.002, 0.06$ (for train data) and 0.01, 0.04, 0.05 (for test data).

4. RESULTS AND DISCUSSION

Many of the transitions between nuclear states have mixed character of different multipoles. In this study, we concentrated on dipole and quadrupole radiations and integer spin values of nuclear states only. As already noted both in the text and also in Fig.1, we estimated the output B_2F_2 terms in Eq.(2) for $k = 2$, with the inputs J_i, J_f, L_1 and L_2 values. Input of initial spin J_i was taken from 1 to 20 in the train set. From J_i to final spin J_f, only dipole and quadrupole transitions have been considered in this study. Therefore for $J_i = 1$, J_f was taken only as 0, 1, 2 and 3 in the calculations. For illustration, the transition to 0 is pure dipole and the transitions to $1, 2$ and 3 are dipole-quadrupole mixture. In other illustration for $J_i = 2$, the J_f was taken only as $0, 1, 2, 3$ and 4. The transitions to 0 and 4 are pure quadrupole and the transitions to $1, 2$ and 3 are dipole-quadrupole mixture and so on.

In Figs. 2 and 3, literature data and LFNN output values for B_2F_2 against the J_i values up to 20 were shown. The minimum absolute value errors for $h = 5, 7$ and 9 were $3.8x10^{-5}$, $9.4x10^{-6}$, and $1.5x10^{-4}$, respectively. The maximum absolute errors were 0.91, 0.28 and 0.50. All the error levels were taken as acceptable for the estimations. The results are consistent with the tabulated values [10] . The best estimation result was obtained with hidden neuron number $h = 7$.

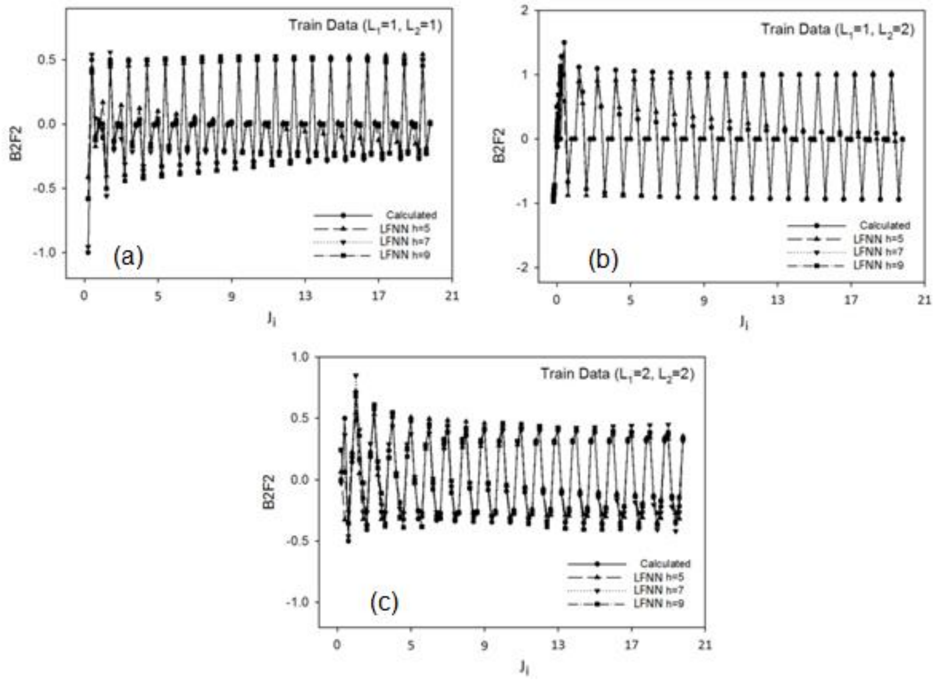

Figure 2. Calculated and LFNN B_2F_2 values in train set versus the initial spin values of transitions a) for pure dipole, b) dipole-quadrupole mixture and c) pure quadrupole transition.

In the test set, initial spin J_i input had varying values from 20 to 26. We again note that only dipole and quadrupole transitions were estimated in test set. In Fig. 3, we showed the literature and neural network output values for B_2F_2 values versus the J_i values up to 20. The minimum absolute errors for $h = 5, 7$ and 9 were, respectively, $1.9x10^{-4}$, $5.3x10^{-4}$ and $2.6x10^{-4}$, while the maximums were, respectively, 0.32, 0.20 and 0.88. All the error values were accepted as reasonable estimations. The results are consistent with the tabulated values [10].

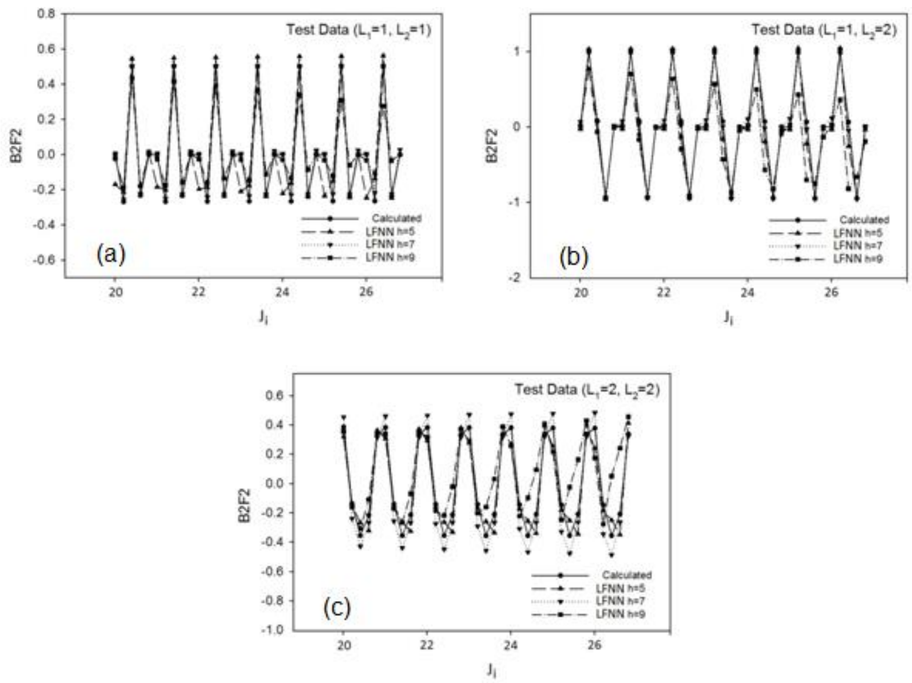

Figure 3. Calculated and LFNN B_2F_2 values in test set versus the initial spin values of transitions a) for pure dipole, b) dipole-quadrupole mixture and c) pure quadrupole (middle) transition.

5. CONCLUSION

Gamma ray angular distribution coefficients are useful to determine the nuclear spins and multipolarities of transitions between states. These coefficients are very difficult to compute even for small r values, and such formulas are not even known for larger values of r. Moreover, they are highly nonlinear in nature. To overcome these obstacles, as a novel approach, in this paper we used suitable LFFNs with train sets to obtain consistent LFNN-EPFs with test sets. The results are in agreement with the literature values. The major conclusions of this paper are as follows.

1. The LFNN can be safely used to determine spins of nuclear states and multipolarities of the gamma ray transitions.

2. The coefficients in the gamma ray angular distribution function which are not tabulated in the literature can also be accurately estimated by the LFNN.

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