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Notes on Fuzzy Parametrized Soft Sets

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Abstract. In this paper, we introduced the notion of inverse Fp soft set and studied some properties of it. Moreover, by using this new concept we characterized the continuity of Fp soft mappings and continuity of fuzzy soft mapping.

Keywords: Fuzzy soft set, fuzzy parametrized soft set, fuzzy soft contiunity [2000] 03E72, 54C05, 54D30.

Bulanık Parametreli Esnek Kümeler Üzerine Notlar

Özet. Bu makalede devrik bulanık parametreli esnek küme kavramına giriş yapılarak, bu kümelerin bazı özellikleri çalışılacaktır. Ek olarak, bu yeni kavram yardımıyla Fp esnek dönüşümlerin ve bulanık esnek dönüşümlerin süreklilikleri karakterize edilecektir.

Anahtar Kelimeler: Bulanık esnek küme, Bulanık parametreli esnek küme, Bulanık esnek süreklilik [2000] 03E72, 54C05, 54D30.

1. INTRODUCTION

In 1965, following Zadeh's [1] generalization of the usual set notion with the introduction of fuzzy set, the fuzzy set became popular in the areas of general theories and applied to many real-life problems in uncertain, ambiguous environment. In 1968, the definition of fuzzy topology and many topological notions in the fuzzy setting are introduced by Chang [2].

Later, In 1999, Molodtsov [3] introduced the concept of soft set theory which is a completely new approach for modelling uncertainty and pointed out several directions for the applications of soft sets, such as game theory, perron integrations, and smoothness of functions. Also, this concept is applied to topological spaces (e.g.,[4, 5, 6]), group theory, ring theory (e.g.,[7]), and also decision-making problems (e.g., [8]) by many researchers.

Recently, fuzzy sets and soft sets have been combined to generalize the spaces and solve more complicated problems. In this way, researchers study many interesting applications of soft set theory. Maji et al. [9] gave the definition of fuzzy soft sets which is the first combination of fuzzy set and soft set. Then fuzzy soft set theory has been applied in various fields, such as topology (e.g.,[10, 11, 12]), various algebraic structures (e.g., [13]), and especially decision making (e.g.,[14]). Fuzzy parametrized soft set (for short FP-soft set) which is another combination of fuzzy sets and soft sets was given by Çağman et al. [15]. In their study, Çağman et al. ([16, 17]) defined operations on FP-soft sets and defined some decision making methods by using FP-soft sets.

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In this paper, we introduced the notion of inverse Fp soft set and studied some properties of it. Moreover, by using this new concept we characterized the continuity of Fp soft mappings and continuity of fuzzy soft mapping.

2. PRELIMINARIES

Throughout this paper X denotes initial universe, E denotes the set of all possible parameters which are attribut

Definition 1 *[1] A fuzzy set $\tilde{A}$ of a non-empty set X is characterized by a membership function $\mu_A: X \to [0,1]$ whose value $\mu_A(x)$ represents the "grade of membership" of x in $\tilde{A}$ for $x \in X$.*

Let I^X denote the family of all fuzzy sets on X. If $\tilde{A}, \tilde{B} \in I^X$, then some basic set operations for fuzzy sets are given by Zadeh [16] as follows:

(1) $\tilde{A} \leq \tilde{B} \Leftrightarrow \mu_A(x) \leq \mu_B(x)$, for all $x \in X$.

(2) $\tilde{A} = \tilde{B} \Leftrightarrow \mu_A(x) = \mu_B(x)$, for all $x \in X$.

(3) $\tilde{C} = \tilde{A} \vee \tilde{B} \Leftrightarrow \mu_C(x) = \mu_A(x) \vee \mu_B(x)$, for all $x \in X$.

(4) $\tilde{D} = \tilde{A} \wedge \tilde{B} \Leftrightarrow \mu_D(x) = \mu_A(x) \wedge \mu_B(x)$, for all $x \in X$.

(5) $\tilde{E} = \tilde{A}^C \Leftrightarrow \mu_E(x) = 1 - \mu_A(x)$, for all $x \in X$.

A fuzzy point in X, whose value is α $(0 < \alpha \leq 1)$ at the support $x \in X$, is denoted by x^α. Let $\tilde{A}$ is a fuzzy set in X. A fuzzy point $x^\alpha \in A$ if and only if $\alpha \leq \mu_A(x)$. The class of all fuzzy points will be denoted by $S(X)$.

Definition 2 *[18] For two fuzzy sets $\tilde{A}$ and $\tilde{B}$ in X, we write $\tilde{A}q\tilde{B}$ to mean that $\tilde{A}$ is quasi-coincident with $\tilde{B}$, i.e. there exists at least one point $x \in X$ such that $\mu_A(x) + \mu_B(x) > 1$. If $\tilde{A}$ is not quasi-coincident with $\tilde{B}$, then we write $\tilde{A}\bar{q}\tilde{B}$.*

Definition 3 *[19]Let $A \subseteq E$. A fuzzy soft set $\widetilde{f_A}$ over universe X is mapping from the parameter set E to I^X, i.e. $\widetilde{f_A}: E \to I^X$, where $\widetilde{f_A}(e) \neq 0_X$ if $e \in A \subset E$ and $\widetilde{f_A}(e) = 0_X$ if $e \notin A$, where 0_X denotes empty fuzzy set on X.*

Definition 4 *[19]Let $FS(X,E)$ denote the family of all fuzzy soft sets on X and $\widetilde{f_A}, \widetilde{g_B} \in FS(X,E)$.*

(1) The fuzzy soft set $\widetilde{f_A} \in FS(X,E)$ is called null fuzzy soft set if $\widetilde{f_A}(e) = 0_X$ for all $e \in E$ and denoted by $\tilde{0}_E$.

(2) The fuzzy soft set $\widetilde{f_A} \in FS(X,E)$ is called universal fuzzy soft set if $\widetilde{f_A}(e) = 1_X$ for all $e \in E$, *where 1_X denotes universal fuzzy set on X.*

and denoted by $\tilde{1}_E$.

(3) $\widetilde{f_A}$ is called a fuzzy soft subset of $\widetilde{g_B}$ if $\widetilde{f_A}(e) \leq \widetilde{g_B}(e)$ for all $e \in E$ and we denoted by $\widetilde{f_A} \sqsubseteq \widetilde{g_B}$.

(4) The union of $\widetilde{f_A}$ and $\widetilde{g_B}$ is also a fuzzy soft set $\widetilde{h_C}$, defined by $\widetilde{h_C}(e) = \widetilde{f_A}(e) \vee \widetilde{g_B}(e)$ for all $e \in E$, where $C = A \cup B$. Here we write $\widetilde{h_C} = \widetilde{f_A} \sqcup \widetilde{g_B}$.

(5) The intersection of $\widetilde{f_A}$ and $\tilde{g}_B$ is also a fuzzy soft set $\widetilde{h_C}$, defined by $\widetilde{h_C}(e) = \widetilde{f_A}(e) \wedge \widetilde{g_B}(e)$ for all $e \in E$, where $C = A \cap B$. Here we write $\widetilde{h_C} = \widetilde{f_A} \sqcap \widetilde{g_B}$.

Definition 5 *[11]Let $\widetilde{f_A} \in FS(X,E)$. The complement of $\widetilde{f_A}$, denoted by $\widetilde{f_A^c}$, is a fuzzy soft set defined by $\widetilde{f_A^c}(e) = 1_X - \widetilde{f_A}(e)$ for every $e \in E$.*

Clearly , $\tilde{1}_E^c = \tilde{0}_E$ and $\tilde{0}_E^c = \tilde{1}_E$.

Definition 6 *[20]Let $FS(X,E)$ and $FS(Y,K)$ be the families of all fuzzy soft sets over X and Y, respectively. Let $u: X \to Y$ and $p: E \to K$ be two functions. Then u_p is called a fuzzy soft mapping from X to Y and denoted by $u_p: FS(X,E) \to FS(Y,K)$.*

(1) Let $\widetilde{f_A} \in FS(X,E)$, then the image of $\widetilde{f_A}$ under the fuzzy soft mapping u_p is the fuzzy soft set over Y and defined by $u_p(\widetilde{f_A})$, where

$$u_p(\widetilde{f_A})(k)(y) = \begin{cases} \bigvee_{x\in u^{-1}(y)} \left(\bigvee_{e\in p^{-1}(k)\cap A} \widetilde{f_A}(e)\right)(x) & if\, u^{-1}(y) \neq \emptyset, p^{-1}(k) \cap A \neq \emptyset; \\ 0 & otherwise. \end{cases}$$

(2) Let $\widetilde{g_B} \in FS(Y,K)$, then the preimage of $\widetilde{g_B}$ under the fuzzy soft mapping u_p is the fuzzy soft set over X and defined by $u_p^{-1}(\widetilde{g_B})$, where

$$u_p^{-1}(\widetilde{g_B})(e)(x) = \begin{cases} \tilde{g}_B(p(e))(u(x)) & , for\ p(e) \in B; \\ 0 & , otherwise. \end{cases}$$

If u and p are injective then the fuzzy soft mapping u_p is said to be injective. If u and p are surjective then the fuzzy soft mapping u_p is said to be surjective. The fuzzy soft mapping u_p is called constant, if u and p are constant.

Definition 7 *[21]The fuzzy soft set $\widetilde{f_A} \in FS(X,E)$ is called fuzzy soft point if $A = \{e\} \subseteq E$ and $\widetilde{f_A}(e)$ is a fuzzy point in X i.e. there exists $x \in X$ such that $\widetilde{f_A}(e)(x) = \alpha$ $(0 < \alpha \leq 1)$ and $\widetilde{f_A}(e)(y) = 0$ for all $y \in X - \{x\}$. We denote this fuzzy soft point $e_{x^\alpha} = \{(e, x^\alpha)\}$.*

Definition 8 *[21]Let $\widetilde{f_A}$, $\widetilde{g_B} \in FS(X,E)$. $\widetilde{f_A}$ is said to be fs-quasi-coincident with $\widetilde{g_B}$ and denoted by $\widetilde{f_A} q \widetilde{g_B}$ if there exist $e \in E$ and $x \in X$ such that $\widetilde{f_A}(e)(x) + \widetilde{g_B}(e)(x) > 1$.*

If $\widetilde{f_A}$ is not fs-quasi-coincident with $\widetilde{g_B}$, then we write $\widetilde{f_A} \bar{q} \widetilde{g_B}$.

Definition 9 *(see [11 19]) A fuzzy soft topological space is a pair (X,τ) where X is a nonempty set and τ is a family of fuzzy soft sets over X satisfying the following properties:*

(1) $\tilde{0}_E, \tilde{1}_E \in \tau$

(2) If $\widetilde{f_A}, \widetilde{g_B} \in \tau$, then $\widetilde{f_A} \sqcap \widetilde{g_B} \in \tau$

(3) If , $(\widetilde{f_A})_i \in \tau$, $\forall i \in J$, then $\sqcup_{i\in J} (\widetilde{f_A})_i \in \tau$.

Then τ is called a topology of fuzzy soft sets on X. Every member of τ is called fuzzy soft open. $\widetilde{g_B}$ is called fuzzy soft closed in (X,τ) if $\widetilde{g_B^c} \in \tau$.

Definition 10 *[21]Let (X,τ_1) and (Y,τ_2) be two fuzzy soft topological spaces. A fuzzy soft mapping $u_p: (X,\tau_1) \to (Y,\tau_2)$ is called fuzzy soft continuous if $u_p^{-1}(\widetilde{g_B}) \in \tau_1$ for all $\widetilde{g_B} \in \tau_2$.*

Example 11 *[22]Let (X,τ) be a fuzzy soft topological space. Then, for every $e \in E$, $\tau_e = \{f_A(e): f_A \in \tau\}$ is a fuzzy topology on X.*

Theorem 12 *[10]Let (X,τ) and (Y,υ) be two fuzzy soft topological spaces.Then $u_p: FS(X,E) \to FS(Y,K)$ is fuzzy soft continuous if and only if for all $e \in E$, $u: (X,\tau_e) \to (Y,\upsilon_{p(e)})$ are fuzzy continuous.*

Definition 13 *[15] Let $\tilde{A}$ be a fuzzy set over E. An FP-soft set $F_{\tilde{A}}$ on the universe X is defined as follows:*

$$F_{\tilde{A}} = \{(e^{\mu_A(e)}, f_A(e)): e \in E, f_A(e) \in P(X), \mu_A(e) \in [0,1]\},$$

where the function $f_A: E \to P(X)$ is called approximate function such that $f_A(e) = \emptyset$ if $\mu_A(e) = 0$.

From now on, the set of all FP-soft sets over X will be denoted by $FPS(X,E)$**.**

Definition 14 *[15] Let $F_{\tilde{A}}, G_{\tilde{B}} \in FPS(X,E)$.*

(1) $F_{\tilde{A}}$ is called the empty FP-soft set if $\mu_A(e) = 0$ for all every $e \in E$, denoted by F_{0_X}.

(2) $F_{\tilde{A}}$ is called universal FP-soft set if $\mu_A(e) = 1$ and $f_A(e) = X$ for all $e \in E$, denoted by F_{1_X}.

(3) $F_{\tilde{A}}$ is called a FP-soft subset of $F_{\tilde{B}}$ if $\tilde{A} \leq \tilde{B}$ and $f_A(e) \subseteq f_B(e)$ for every $e \in E$ and we write $F_{\tilde{A}} \tilde{\subseteq} F_{\tilde{B}}$.

(4) The union of $F_{\tilde{A}}$ and $F_{\tilde{B}}$, denoted by $F_{\tilde{A}} \tilde{\cup} F_{\tilde{B}}$, is the FP-soft set, defined by the membership and approximate functions $\mu_{A\cup B}(e) = max\{\mu_A(e), \mu_B(e)\}$ and $f_{A\cup B}(e) = f_A(e) \cup g_B(e)$ for every $e \in E$, respectively.

(5) The intersection of $F_{\tilde{A}}$ and $F_{\tilde{B}}$, denoted by $F_{\tilde{A}} \tilde{\cap} G_{\tilde{B}}$, is the FP-soft set, defined by the membership and approximate functions $\mu_{A\cap B}(e) = min\{\mu_A(e), \mu_B(e)\}$ and $f_{A\cap B}(e) = f_A(e) \cap g_B(e)$ for every $e \in E$, respectively.

(6) The the complement of $F_{\tilde{A}}$, denoted by $F_{\tilde{A}}^c$, is the FP-soft set, defined by the membership and approximate functions $\mu_{A^c}(e) = 1 - \mu_A(e)$ and $f_A^c(e) = X - f_A(e)$ for every $e \in E$, respectively.

Definition 15 *[23]The FP-soft set $F_{\tilde{A}} \in FPS(X,E)$ is called FP-soft point if $\tilde{A}$ is fuzzy singleton and $f_A(e) \in P(X)$ for e ∈suppA. If $A = \{e\}$, $\mu_A(e) = \alpha \in (0,1]$, then we denote this FP-soft point by e_f^α.*

Definition 16 *[23]Let $F_{\tilde{A}}$, $G_{\tilde{B}} \in FPS(X,E)$. $F_{\tilde{A}}$ is said to be fp-quasi-coincident with $G_{\tilde{B}}$, denoted by $F_{\tilde{A}} q G_{\tilde{B}}$, if there exists $e \in E$ such that $\mu_A(e) + \mu_B(e) > 1$ or $f_A(e)$ is not subset of $g_B^c(e)$. If*

$F_{\tilde{A}}$ is not fp-quasi-coincident with $G_{\tilde{B}}$, then we write $F_{\tilde{A}}\bar{q}G_{\tilde{B}}$.

Definition 17 *[17] Let $u: X \to Y$ and $p: E \to K$ be two functions. Then a FP-soft mapping $u_p: FPS(X,E) \to FPS(Y,K)$ is defined as:*

(1) for $F_{\tilde{A}} \in FPS(X,E)$, the image of $F_{\tilde{A}}$ under the FP-soft mapping u_p is the FP-soft set $G_{\tilde{S}}$ over Y defined by the approximate function, $\forall k \in K$,

$$g_S(k) = \begin{cases} \bigcup_{e\in p^{-1}(k)} u(f_A(e)), & if\ p^{-1}(k) \neq \emptyset; \\ \emptyset \quad , & otherwise. \end{cases}$$

where $p(\tilde{A}) = \tilde{S}$ is fuzzy set in K.

(2) for $G_{\tilde{S}} \in FPS(Y,K)$, then the pre-image of $G_{\tilde{S}}$ under the FP-soft mapping u_p is the FP-soft set $F_{\tilde{A}}$ over X defined by the approximate function, $\forall e \in E$

$f_A(e) = u^{-1}(g_S(p(e)))$ where $p^{-1}(\tilde{S}) = \tilde{A}$ is fuzzy set in E.

If u and p is injective, then the FP-soft mapping u_p is said to be injective. If u and p is surjective, then the FP-soft mapping u_p is said to be surjective. The FP-soft mapping u_p is called constant, if u and p are constant.

Definition 18 *[23]A FP-soft topological space is a pair (X,τ) where X is a nonempty set and τ is a family of FP-soft sets over X satisfying the following properties:*

(T1) $F_{0_X}, F_{1_X} \in \tau$

(T2) If $F_{\tilde{A}}, G_{\tilde{B}} \in \tau$, then $F_{\tilde{A}} \tilde{\cap} G_{\tilde{B}} \in \tau$

(T3) If $(F_{\tilde{A}})_i \in \tau$,$\forall i \in J$, then $\tilde{\bigcup}_{i\in J} (F_{\tilde{A}})_i \in \tau$.

Every member of τ is called FP-soft open in (X,τ) and $F_{\tilde{A}}$ is called FP-soft closed in (X,τ) if $F_{\tilde{A}}^c \in \tau$.

Definition 19 *[23]Let (X,τ_1) and (Y,τ_2) be two FP-soft topological spaces. A FP-soft mapping $u_p: (X,\tau_1) \to (Y,\tau_2)$ is called FP-soft continuous if $u_p^{-1}(G_{\tilde{S}}) \in \tau_1$ for all $G_{\tilde{S}} \in \tau_2$.*

3. INVERSE FUZZY PARAMETRIZED SOFT SETS

Definition 20 *Let $F_{\tilde{A}} \in FPS(X,E)$. Then the fuzzy soft set $\widetilde{F_{\tilde{A}}}: X \to I^E$ defined as $\widetilde{F_{\tilde{A}}}(x) = \{e^{\mu_A(e)}: x \in f_A(e)\}$ is called the inverse of $F_{\tilde{A}}$.*

It is clear that this fuzzy soft set $\widetilde{F_{\tilde{A}}}$ can be expressed as set of pairs $\{(x, \widetilde{F_{\tilde{A}}}(x)): x \in X\}$.

Example 21 *Let $E = \{e_1,e_2,e_3\}$, $X = \{x_1,x_2,x_3\}$ and $F_{\tilde{A}} = \{(e_1^{0,4},\{x_1,x_3\}),\ (e_2^{0,7},\{x_1,x_2\}),(e_3^{0,5},\{x_2\})\}$. Then $\widetilde{F_{\tilde{A}}} = \{(x_1,\{e_1^{0,4},e_2^{0,7}\}),\ e_3^{0,5}\}, (x_3,\{e_1^{0,4}\})\}$.*

Theorem 22 *Let $e_f^{\alpha} \in FPS(X,E)$ be a fuzzy parametrized soft point. Then, $\widetilde{(e_f^{\alpha})}$ is a fuzzy soft point on E if and only if $f(e)$ is a singleton set.*

Proof. Let $e_f^\alpha = \{(e^\alpha, f(e))\} \in FPS(X,E)$ be a fuzzy parametrized soft point. Then $\widetilde{(e_f^\alpha)} = \{(x, \{e^\alpha : x \in f(e)\}) : x \in X\}$. It is obvious that $f(e)$ must be singleton set in order $\widetilde{(e_f^\alpha)}$ to be fuzzy soft point.

Conversely, let $f(e) = \{x\}$ be a singleton set for $x \in X$. Then $\widetilde{(e_f^\alpha)} = \{(x, \{e^\alpha : x \in f(e)\}) : x \in X\} = \{(x, e^\alpha)\}$. This completes the proof.

Theorem 23 *Let* $F_{\tilde{A}}, G_{\tilde{B}}, \{F_{\tilde{A}_i}\}_{i\in J} \in FPS(X,E)$. *Then the followings hold*

(1)$\widetilde{(F_{\tilde{A}} \tilde{\cap} G_{\tilde{B}})} = \widetilde{F_{\tilde{A}}} \tilde{\cap} \widetilde{G_{\tilde{B}}}$

(2) $\widetilde{(\tilde{\bigcup}_{i\in J} F_{\tilde{A}_i})} = \tilde{\bigcup}_{i\in J} \widetilde{F_{\tilde{A}_i}}$

(3) $F_{\tilde{A}} \tilde{\subseteq} G_{\tilde{B}} \Leftrightarrow \widetilde{F_{\tilde{A}}} \tilde{\subseteq} \widetilde{G_{\tilde{B}}}$

(4)$\widetilde{F_{0_X}} = \widetilde{0_E}$

(5) $\widetilde{F_{1_X}} = \widetilde{1_E}$

Proof.

(1)Let $F_{\tilde{A}} \tilde{\cap} G_{\tilde{B}} = H_{\tilde{C}}$. Then

$$\widetilde{F_{\tilde{A}} \tilde{\cap} G_{\tilde{B}}} = \widetilde{H_{\tilde{C}}}$$

$$= \{\{(x, \widetilde{H_{\tilde{C}}}(x))\} : x \in h_C(e) = f_A(e) \cap g_B(e)\}$$

$$= \{(x, \{e^{\{min\{\mu_A(e), \mu_B(e)\}\}}\}) : x \in f_A(e) \cap g_B(e)\}$$

$$= \{(x, \{e^{\{\mu_A(e)\}}\}) : x \in f_A(e)\} \wedge \{(x, \{e^{\{\mu_B(e)\}}\}) : x \in g_B(e)\}$$

$$= \widetilde{F_{\tilde{A}}} \tilde{\cap} \widetilde{G_{\tilde{B}}}$$

(2) Let $\tilde{\bigcup}_{i\in J} F_{\tilde{A}_i} = H_{\tilde{C}}$. Then

$$\tilde{\bigcup}_{i\in J} F_{\tilde{A}_i} = H_{\tilde{C}}$$

$$= \{(\mathrm{x}, \{e^{\{\mu_C(e)\}}\}) : \mathrm{x} \in \mathrm{h}_C(\mathrm{e}) = \bigcup_{i\in J} f_{A_i}(\mathrm{e})\}$$

$$= \{(\mathrm{x}, \{e^{sup_{i\in J}\{\mu_{A_i}(e)\}}\}) : \mathrm{x} \in \bigcup_{i\in J} f_{A_i}(\mathrm{e})\}$$

$$= \tilde{\bigcup}_{i\in J} \{(\mathrm{x}, \{e^{\mu_{A_i}(e)}\}) : \mathrm{x} \in f_{A_i}(\mathrm{e})\}$$

$$= \tilde{\bigcup}_{i\in J} \widetilde{F_{\tilde{A}_i}}$$

.

(3) Let $F_{\tilde{A}} \tilde{\sqsubseteq} G_{\tilde{B}}$. Suppose that $\widetilde{F_{\tilde{A}}}$ is not fuzzy soft subset of $\widetilde{G_{\tilde{B}}}$. Then there exist a $x \in X$ and $e \in E$ such that $\widetilde{F_{\tilde{A}}}(x)(e) > \widetilde{G_{\tilde{B}}}(x)(e)$. Thus $\mu_A(e) > \mu_B(e)$ is obtained which contradicts with $F_{\tilde{A}} \tilde{\sqsubseteq} G_{\tilde{B}}$. Then it should be $\widetilde{F_{\tilde{A}}} \tilde{\sqsubseteq} \widetilde{G_{\tilde{B}}}$.

Conversely, let $\widetilde{F_{\tilde{A}}} \tilde{\sqsubseteq} \widetilde{G_{\tilde{B}}}$. Then $\widetilde{F_{\tilde{A}}}(x) \leq \widetilde{G_{\tilde{B}}}(x)$ for all $x \in X$. If $F_{\tilde{A}}$ is not subset $G_{\tilde{B}}$, then there exist $e \in E$ such that $\mu_A(e) > \mu_B(e)$ or $f_A(e) \nsubseteq g_B(e)$

If $\mu_A(e) > \mu_B(e)$, then there exists a $x \in X$ such that $\widetilde{F_{\tilde{A}}}(x)(e) \leq \widetilde{G_{\tilde{B}}}(x)(e)$. This is contradiction with $\widetilde{F_{\tilde{A}}}(x) \leq \widetilde{G_{\tilde{B}}}(x)$ for all $x \in X$.

If $f_A(e) \nsubseteq g_B(e)$, then there exists a $x \in X$ such that $x \in f_A(e)$, $x \notin g_B(e)$. Hence $\widetilde{G_{\tilde{B}}}(x)(e) = 0$ and $\widetilde{F_{\tilde{A}}}(x)(e) = \alpha$, for an $\alpha \in (0,1]$. This is contradiction with $\widetilde{F_{\tilde{A}}}(x) \leq \widetilde{G_{\tilde{B}}}(x)$ for all $x \in X$.

(4)

$$\begin{aligned}
\widetilde{F_{0_X}} &= \{(x, \widetilde{F_{0_X}}(x)) : x \in X, \widetilde{F_{0_X}}(x) = \{e^{\mu_A(e)} : x \in f_A(e)\}\} \\
&= \{(x, \widetilde{F_{0_X}}(x)) : x \in X, \widetilde{F_{0_X}}(x) = 0_E\} \\
&= \{(x, 0_E) : x \in X\} \\
&= \widetilde{0_E}
\end{aligned}$$

(5)

$$\begin{aligned}
\widetilde{F_{1_X}} &= \{(x, \widetilde{F_{1_X}}(x)) : x \in X, \widetilde{F_{1_X}}(x) = \{e^{\mu_A(e)} : x \in f_A(e)\}\} \\
&= \{(x, \widetilde{F_{1_X}}(x)) : x \in X, \widetilde{F_{1_X}}(x) = 1_E\} \\
&= \{(x, 1_E) : x \in X\} \\
&= \widetilde{1_E}
\end{aligned}$$

Proposition 24 *Let $F_{\tilde{A}}$, $G_{\tilde{B}} \in FPS(X, E)$. If the $\widetilde{F_{\tilde{A}}}$ fs-quasi-coincident with $\widetilde{G_{\tilde{B}}}$, then $F_{\tilde{A}}$ fp-quasi-coincident with $G_{\tilde{B}}$.*

Proof. Let $\widetilde{F_{\tilde{A}}}$ fs-quasi coincident with $\widetilde{G_{\tilde{B}}}$. Then for a $x \in X$ and an $e \in E$, $\widetilde{F_{\tilde{A}}}(x)(e) + \widetilde{G_{\tilde{B}}}(x)(e) > 1$. This shows $\mu_A(e) + \mu_B(e) > 1$. Therefore $F_{\tilde{A}}$ fp-quasi-coincident with $G_{\tilde{B}}$.

Converse of the above proposition is not generally true.

Example 25 *Let $E = \{e_1, e_2\}$, $X = \{x_1, x_2\}$, $F_{\tilde{A}} = \{(e_1^{0,7}, \{x_1\})\}$ and $G_{\tilde{B}} = \{(e_1^{0,5}, \{x_2\})\}$. Then $F_{\tilde{A}}$ fp-quasi coincident with $G_{\tilde{B}}$, but the fuzzy soft sets $\widetilde{F_{\tilde{A}}} = \{(x_1, \{e_1^{0,7}\})\}$ and $\widetilde{G_{\tilde{B}}} = \{(x_2, \{e_1^{0,5}\})\}$ are not fs-quasi-coincident.*

Theorem 26 *Let (X, τ) be a fuzzy parametrized soft topological spaces. Then the family $\tau^{\sim} = \{\widetilde{F_{\tilde{A}}} : F_{\tilde{A}} \in \tau\}$ is a fuzzy soft topology on E.*

Proof. It is obvious from Theorem 23.

Corollary 27 *Let (X,τ) be a fuzzy parametrized soft topological spaces. Then for all $\in X$, the family $\tau_x^{\sim} = \{\widetilde{F_{\tilde{A}}}(x): F_{\tilde{A}} \in \tau\}$ is fuzzy topology on E by Example 11.*

Proposition 28 *Let $u_p: FPS(X,E) \to FPS(Y,K)$ be a fuzzy parametrized soft function and $G_{\tilde{B}} \in FPS(Y,K)$. Then $u(x) \in g_B(p(e))$ if and only if $p(e) \in \widetilde{G_{\tilde{B}}}(u(x))$.*

Proof. It is clear.

Proposition 29 *Let $u_p: FPS(X,E) \to FPS(Y,K)$ be a fuzzy parametrized soft function. Then $(\widetilde{u_p^{-1}(G_{\tilde{B}})}) = p_u^{-1}(\widetilde{G_{\tilde{B}}})$ for all $G_{\tilde{B}} \in FPS(Y,K)$.*

Proof. It is clear that

$$\left(\widetilde{u_p^{-1}(G_{\tilde{B}})}\right)(x)(e) = \begin{cases} \mu_B(p(e)) & ; u(x) \in g_B(p(e)) \\ 0 & ; u(x) \notin g_B(p(e)) \end{cases}$$

and

$$\left(p_u^{-1}(\widetilde{G_{\tilde{B}}})\right)(x)(e) = \begin{cases} \mu_B(p(e)) & ; p(e) \in \widetilde{G_{\tilde{B}}}(u(x)) \\ 0 & ; p(e) \notin \widetilde{G_{\tilde{B}}}(u(x)) \end{cases}$$

for all $e \in E$. This equality shows that $(\widetilde{u_p^{-1}(G_{\tilde{B}})})(x) = (p_u^{-1}(\widetilde{G_{\tilde{B}}}))(x)$ for all $x \in X$. Thus the proof is completed.

Theorem 30 *Let (X,τ) and (Y,υ) be two fuzzy parameterized soft topological spaces. The function $u_p: FPS(X,E) \to FPS(Y,K)$ is fuzzy parametrized soft continuous if and only if the function $p_u: FS(E,X) \to FS(K,Y)$ is fuzzy soft continuous for fuzzy soft topologies $(E,\tau^{\sim})$ and $(K,\upsilon^{\sim})$.*

Proof. Let $u_p: FPS(X,E) \to FPS(Y,K)$ be fuzzy soft parametrized continuous and $\widetilde{G_{\tilde{B}}} \in \upsilon^{\sim}$. Then $G_{\tilde{B}} \in \upsilon$. Since u_p fuzzy parametrized soft continuous, $u_p^{-1}(G_{\tilde{B}}) \in \tau$. Hence $(\widetilde{u_p^{-1}(G_{\tilde{B}})}) \in \tau^{\sim}$. Then $p_u^{-1}(\widetilde{G_{\tilde{B}}}) \in \tau^{\sim}$ by the Proposition 29. Thus the proof is completed.

The inverse of the proof can be done similarly.

The following corollary is a consequence of Theorems 12 and 30.

Corollary 31 *Let (X,τ) and (Y,υ) be two fuzzy parameterized soft topological spaces. The function $u_p: FPS(X,E) \to FPS(Y,K)$ is fuzzy parametrized soft continuous if and only if the function $p: (E,\tau_x^{\sim}) \to (K,\upsilon_{u(x)}^{\sim})$ is fuzzy continuous for all $x \in X$.*

Example 32 *Let $X = \{x_1,x_2,x_3\}$,$Y = \{y_1,y_2\}$ be initial universes and $E = \{e_1,e_2\}$,$K = \{k_1,k_2,k_3\}$ be sets of parameters and $F_{\tilde{A}} = \{(e_1^{0,7},X), (e_2^{0,4},\{x_2,x_3\})\}$, $G_{\tilde{B}} = \{(k_1^{0,4},\{y_1\}), (k_2^{0,7},Y\}),$ $(k_3^{0,8},\{y_2\})\}$. Let us define $\tau = \{F_{0_X},F_{1_X},F_{\tilde{A}}\}$ and $\upsilon = \{F_{0_Y},F_{1_Y},G_{\tilde{B}}\}$ as two topological spaces on X and Y ,respectively. If $u: X \to Y$ defined as $u(x_1) = y_2, u(x_2) = y_1$, $u(x_3) = y_1$ and $p: E \to K$ defined* *as* $p(e_1) = k_2$, $p(e_2) = k_1$, *then* $u_p: FPS(X,E) \to FPS(Y,K)$ *is a fuzzy parametrized soft function. On the otherhand,* $u_p^{-1}(G_{\tilde{B}})$ *is fuzzy parametrized soft set on* X *and for* $p^{-1}(\tilde{A}) = \tilde{B}$,

$$u_p^{-1}(G_{\tilde{B}})(e_1) = u^{-1}(g_B(p(e_1))) = u^{-1}(g_B(k_2)) = u^{-1}(Y) = X$$
$$u_p^{-1}(G_{\tilde{B}})(e_2) = u^{-1}(g_B(p(e_2))) = u^{-1}(g_B(k_1)) = u^{-1}(\{y_1\}) = \{x_2, x_3\}$$

Then $u_p^{-1}(G_{\tilde{B}}) = \{(e_1^{0,7}, X), (e_2^{0,4}, \{x_2, x_3\})\} = F_{\tilde{A}}$ which shows that u_p is a fuzzy parametrized soft continuous function. Moreover the fuzzy soft set families

$$\tau_{\tilde{x}_1} \quad \{0_E, 1_E, \widetilde{F_{\tilde{A}}}(x_1) = \{e_1^{0,7}\}\}$$
$$\tau_{\tilde{x}_2} \quad \{0_E, 1_E, \widetilde{F_{\tilde{A}}}(x_2) = \{e_1^{0,7}, e_2^{0,4}\}\}$$
$$\tau_{\tilde{x}_3} \quad \{0_E, 1_E, \widetilde{F_{\tilde{A}}}(x_3) = \{e_1^{0,7}, e_2^{0,4}\}\}$$
$$\upsilon_{\tilde{y}_1} \quad \{0_K, 1_K, \widetilde{G_{\tilde{B}}}(y_1) = \{k_1^{0,4}, k_2^{0,7}\}\}$$
$$\upsilon_{\tilde{y}_2} \quad \{0_K, 1_K, \widetilde{G_{\tilde{B}}}(y_2) = \{k_2^{0,7}, k_3^{0,8}\}\}$$

are topologies on X and Y. It is obvious that $p^{-1}(\widetilde{G_{\tilde{B}}}(y_1)) = \widetilde{F_{\tilde{A}}}(x_2) = \widetilde{F_{\tilde{A}}}(x_3)$, $p^{-1}(\widetilde{G_{\tilde{B}}}(y_2)) = \widetilde{F_{\tilde{A}}}(x_1)$. This shows that $p: (E, \tau_{\tilde{x}_1}) \to (K, \upsilon_{\tilde{y}_2})$, $p: (X, \tau_{\tilde{x}_2}) \to (Y, \upsilon_{\tilde{y}_1})$ and $p: (X, \tau_{\tilde{x}_3}) \to (Y, \upsilon_{\tilde{y}_1})$ fuzzy soft continuous functions.

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