Cumhuriyet Science Journal
CSJ

e-ISSN: 2587-246X
ISSN: 2587-2680

Cumhuriyet Sci. J., Vol.39-2(2018) 389-398

Theoretical Description of Energy Spectra and Quadrupole Transition Probabilities of ^{190} Hg

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Received: 26.12.2017; Accepted: 05.03.2018 *http://dx.doi.org/10.17776/csj.371319*

Abstract. In this paper, we have considered the coexistence of two quite different structures, the deformed and spherical shapes in ^{190}Hg nucleus. To this aim, we have determined the energy spectra and quadrupole transition probabilities of this nucleus. A transitional Interacting Boson Model Hamiltonian which are based on affine $SU(1,1)$ lie algebra have been used to provide a new general technique for description of shape coexistence. Parameter free (up to overall scale factors) predictions for theoretical predictions are found to be in good agreement with experimental counterparts. Also, our results offer a combination of O(6) and U(5) dynamical symmetries for description of regular and intruder configurations, respectively.

Keywords: Shape coexistence, interacting boson model, affine lie algebra, deformed and spherical shape, energy spectra, quadrupole transition.

Enerji Spektrumları ve Quadrupole Geçiş olasılıklarının ^{190} Hg'nin Teorik Tanımı

Özet. Bu yazıda, 190Hg çekirdeğindeki iki farklı yapının, deforme ve küresel şekillerin bir arada varlığını düşündük. Bu amaçla, bu nükleusun enerji spektrumları ve kuadrupol geçiş olasılıklarını belirledik. Afin yalan cebirine dayanan geçişli bir etkileşimli Boson Modeli Hamiltoniyeni, şekil bir arada bulunmanın tanımlanması için yeni bir genel teknik sağlamak amacıyla kullanılmıştır. Parametrik olmayan (genel ölçek faktörlerine kadar) teorik tahminlere yönelik tahminlerin, deneysel eşlerle iyi bir uyum içinde olduğu bulunmuştur. Ayrıca, sonuçlarımız sırasıyla düzenli ve saldırgan konfigürasyonların tanımlanması için O (6) ve U (5) dinamik simetrilerin bir kombinasyonunu sunmaktadır.

Anahtar Kelimeler: Şekil bir arada var olma, etkileşimli bozon model, yalancı yalan cebiri, deforme ve küresel şekil, enerji spektrumları, dört kutuplu geçiş.

1. INTRODUCTION

Shape coexistence in atomic nuclei has become a very active field of research during the last decades. Clear signals of the existence of shape coexistence have been obtained at and near proton or neutron closed shells [1–3], more in particular. in the light nuclei with a closed neutron shell at N = 8, 20, 28 and 40 closed shells as well as in heavy nuclei such as the Sn and the Pb nuclei. The shape coexistence is associated with the presence of low-lying excited 0^+ states between the levels of ground band. To study shape coexistence, there are available several different approximations. In the

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first approach, mean-field methods such as the Hartree-Fock Bogolibov (HFB) type have been used for medium and heavy masses [4-7]. In a shell model picture, the excited 0^+ states are generated by multi-particle multi-hole (mp-mh) proton excitations across in mass regions near shell closure [8-15]. Many particle-many hole (mp-nh) excitations cannot be incorporated easily in full large scale shell model studies because of extremely large dimensions of the model spaces involved. These mp-nh excitations, however, can be handled within an algebraic framework of the interacting boson model (IBM).

The interacting boson model (IBM) describes the nuclear structure of even–even nuclei the was written from the beginning in second quantization form in terms of the generators of the U(6) unitary lie algebra [12-19]. The IBM-1, which does not distinguish between proton and neutron degrees of freedom, assumes that low-lying collective excitations of the nucleus can be described in terms of the number N of s and d bosons. The bosons correspond to pairs of nucleons in valance shell, coupled to angular momentum (*j*=0) s boson (*j*=2) d boson. The model presents three special limits that can be solved easily. IBM has four dynamical symmetries, e.g. $U(5)$, $SU(3)$, $\overline{SU}(3)$ and $O(6)$ limits which are correspond respectively, to vibrational nuclei with a spherical form, an axially symmetric prolate rotor with a minimum in the energy at $\gamma = 0$ and an axially symmetric oblate rotor with a minimum at $\gamma = 60$. The fourth symmetry is located in the middle of the $SU(3) \leftrightarrow \overline{SU}(3)$ transitional region and corresponds to a rotor with a flat potential in γ, e.g. O(6) limit [23-25].

When the numbers of protons (or neutrons) are modified, the energy levels and electromagnetic transition rates of atomic nuclei change too and suggest a transition from one kind of the collective behavior to another [20-34]. The transitional Hamiltonians are especially interesting cases occur when they describe critical points in the transitions from a given shape to another. The quantum shape phase transitions have been studied 25 years ago with using the classical limits of the IBM.

It is known, by using the predictions of Bohr-Mottelson Collective model, the Z(5) critical point symmetry in the prolate to oblate shape phase transitional region which involving large rigid triaxiality, is very close to the predictions of γ-soft models involving γ-fluctuation similar to O(6) dynamical symmetry. These mean nuclei such as ^{190}Hg which is located near the magic number Z = 82 and expect to have spherical shape, belong to the so-called transitional region. This means that they have O(6)-like structure for ground band and U(5) symmetry for the intruder states. These combination of these two different symmetries make it possible to introduce shape coexistence phenomena.

Three transitional regions of IBM, can consider by using the simple Hamiltonian [12-14]:

$$\hat{H}(N,\eta,\chi) = E_0 + \eta\hat{n}_d + \frac{\eta-1}{N}\hat{Q}_\chi.\hat{Q}_\chi + CL^2.$$

where $\hat{n}_d = d^\dagger.\tilde{d}$ is the $d-$boson number operator and $\hat{Q}_\chi = (s^\dagger\tilde{d} + d^\dagger s)^{(2)} + \chi(d^\dagger \times \tilde{d})^{(2)}$ represents the quadrupole operator and $N(= n_s + n_d)$ stands for the total number of bosons. Also, the η an χ quantities are regard as control parameters and can vary within the range $\eta \in [0,1]$ and $\chi \in [-\sqrt{7/2}, +\sqrt{7/2}]$. Some complicated numerical calculation must be used to diagonalize the considered Hamiltonian in these transitional regions and critical points. To avoid these problems, an algebraic solution has been proposed by Pan *et al* [23-24] which was based on the affine $SU(1,1)$ Lie algebra to exhibits the properties of nuclei which are located in the

$U(5)\leftrightarrow SO(6)$ transitional region. Although the results of this approach are somewhat different from those of the IBM, but as have presented in Refs.[23-24,30-32], a clear correspondence with the description of the geometrical model is obvious for this transitional region.

In this study, we focused on the ^{190}Hg nucleus with emphasis on the energy levels and quadrupole transition probabilities. We have used the transitional Hamiltonian [35] to consider the evolution of this nucleus between vibrational and gamma unstable dynamical symmetries.

2. THEORETICAL FRAMEWORK

2.1. Transitional Hamiltonian based on affine $SU(1,1)$ algebra

The $SU(1,1)$ Algebra has been described in detail in Refs.[23-24]. Here, we briefly outline the basic ansatz and summarize the results. The Lie algebra corresponds to the $SU(1,1)$ group is generated by S^{ν}, $\nu = 0$ and $\pm$, which satisfies the following commutation relations:

$$[S^0, S^{\pm}] = \pm S^{\pm} \quad , \quad [S^+, S^-] = -2S^0 \tag{1}$$

The Casimir operator of $SU(1,1)$ group can be written as:

$$\hat{C}_2 = S^0(S^0 - 1) - S^+S^- \quad , \tag{2}$$

Representations of $SU(1,1)$ are determined by a single number κ, thus the representation of Hilbert space is spanned by orthonormal basis $|\kappa\mu\rangle$ where κ can be any positive number and $\mu = \kappa, \kappa+1, \ldots$. Therefore,

$$\hat{C}_2(SU(1,1))|\kappa\mu\rangle = \kappa(\kappa-1)|\kappa\mu\rangle \quad , \quad S^0|\kappa\mu\rangle = \mu|\kappa\mu\rangle \tag{3}$$

In IBM, the generators of $d-$boson pairing algebra is created by:

$$S^+(d) = \frac{1}{2}(d^{\dagger}.\, d^{\dagger}) \quad , \quad S^-(d) = \frac{1}{2}(\tilde{d}.\, \tilde{d}) \quad , \quad S^0(d) = \frac{1}{4}\sum_{\nu}(d_{\nu}^{\dagger}d_{\nu} + d_{\nu}d_{\nu}^{\dagger}) \tag{4}$$

Similarly, $s-$boson pairing algebra forms another $SU^s(1,1)$ algebra which is generated by:

$$S^+(s) = \frac{1}{2}s^{\dagger 2} \quad , \quad S^-(s) = \frac{1}{2}s^2 \quad , \quad S^0(s) = \frac{1}{4}(s^{\dagger}s + ss^{\dagger}) \tag{5}$$

On the other hand, the infinite dimensional $SU(1,1)$ algebra is generated by using of [23-24]

$$S_n^{\pm} = c_s^{2n+1}S^{\pm}(s) + c_d^{2n+1}S^{\pm}(d) \quad , \quad S_n^0 = c_s^{2n}S^0(s) + c_d^{2n}S^0(d) \tag{6}$$

Where c_s and c_d are real parameters and n can be $0, \pm1, \pm2, \ldots$. These generators satisfy the commutation relations,

$$[S_m^0, S_n^{\pm}] = \pm S_{m+n}^{\pm} \quad , \quad [S_m^+, S_n^-] = -2S_{m+n+1}^0 \tag{7}$$

Then, $\{S_m^{\mu}, \mu = 0,+,-;\pm1,\pm2,...\}$ generates an affine Lie algebra $SU(1,1)$ without central extension. By employing the generators of $SU(1,1)$ Algebra, the following Hamiltonian is constructed for the transitional region between $U(5) \leftrightarrow SO(6)$ limits [23-24]:

$$\hat{H} = g\ S_0^+ S_0^- + \varepsilon\ S_1^0 + \gamma\ \hat{C}_2(SO(5)) + \delta\ \hat{C}_2(SO(3)) \tag{8}$$

g, ε, γ and δ are real parameters where $\hat{C}_2(SO(3))$ and $\hat{C}_2(SO(5))$ denote the Casimir operators of these groups. It can be seen that Hamiltonian (8) would be equivalent with $SO(6)$ Hamiltonian if $c_s = c_d$ and with $U(5)$ Hamiltonian when $c_s = 0$ & $c_d \neq 0$. Therefore, the $c_s \neq c_d \neq 0$ requirement just corresponds to the $U(5) \leftrightarrow SO(6)$ transitional region. In our calculation we take c_d (=1) constant value and c_s vary between 0 and c_d.

Eigenstates of Hamiltonian (8) can obtain with using the Fourier-Laurent expansion of eigenstates and $SU(1,1)$ generators in terms of unknown $c-$number parameters x_i with $i = 1,2,...,k$. It means, one can consider the eigenstates as [23-24]:

$$\left|k;\nu_s \nu n_\Delta LM\right\rangle = \sum_{n_i \in Z} a_{n_1} a_{n_2} ... a_{n_k} x_1^{n_1} x_2^{n_2} ... x_k^{n_k} S_{n_1}^+ S_{n_2}^+ ... S_{n_k}^+ \left|lw\right\rangle \quad , \tag{9}$$

Due to the analytical behavior of wave functions, it suffices to consider x_i near zero. With using the commutation relations between the generators of $SU(1,1)$ Algebra, i.e. Eq.(7), wave functions can be considered as:

$$\left|k;\nu_s \nu n_\Delta LM\right\rangle = N S_{x_1}^+ S_{x_2}^+ ... S_{x_k}^+ \left|lw\right\rangle \quad , \tag{10}$$

where N is the normalization factor and

$$S_{x_i}^+ = \frac{c_s}{1 - c_s^2 x_i} S^+(s) + \frac{c_d}{1 - c_d^2 x_i} S^+(d) \quad , \tag{11}$$

The c-numbers x_i are determined through the following set of equations:

$$\frac{\grave{o}}{x_i} = \frac{g c_s{}^2 (\nu_s + \frac{1}{2})}{1 - c_s{}^2 x_i} + \frac{g c_d^2 (\nu + \frac{5}{2})}{1 - c_d^2 x_i} - \sum_{i \neq j} \frac{2}{x_i - x_j} \qquad \text{for i=1,2,...,k} \tag{12}$$

Eigenvalues of Hamiltonian (8), i.e. $E^{(k)}$, can be expressed as [23-24]:

$$E^{(k)} = h^{(k)} + \gamma\nu(\nu+3) + \delta L(L+1) + \varepsilon\Lambda_1^0 \quad , \quad \Lambda_1^0 = \frac{1}{2}[c_s^2(\nu_s + \frac{1}{2}) + c_d^2(\nu + \frac{5}{2})] \tag{13}$$

Which

$$h^{(k)} = \sum_{i=1}^{k} \frac{\varepsilon}{x_i} \quad , \tag{14}$$

The quantum number *k*, is related to total boson number *N*, by

$$N = 2k + \nu_s + \nu$$

To obtain the numerical results for $E^{(k)}$, we have followed the prescriptions have introduced in Refs.[23-24], namely a set of non-linear Bethe-Ansatz equations (BAE) with k – unknowns for k – pair excitations must be solved. To this aim we have changed the variables as:

$$\grave{o} = \frac{\varepsilon}{g} (g = 1 \ keV \ [23\text{-}24]) \qquad c = \frac{c_s}{c_d} \leq 1 \qquad y_i = c_d^2 x_i$$

so, the new form of Eq.(12) would be:

$$\frac{\grave{o}}{y_i} = \frac{c^2(\nu_s + \frac{1}{2})}{1 - c^2 y_i} + \frac{(\nu + \frac{5}{2})}{1 - y_i} - \sum_{i \neq j} \frac{2}{y_i - y_j} \qquad \text{for i=1,2,...,k} \tag{15}$$

We have solved Eq. (15) with definite values of c and ε for $i = 1$ to determine the roots of Beth-Ansatz equations (BAE) with specified values of ν_s and ν, similar to procedure which have done in Refs.[23-24]. Then, we have used the “Find root” in the Maple17 to get all $y_j^{'}$ s. We carry out this procedure with different values of c and ε to provide energy spectra (after inserting γ and δ) with minimum variation as compared to the experimental counterparts;

$$\sigma = \left(\frac{1}{N_{tot}} \sum_{i,\, tot} \left| E_{\exp}(i) - E_{cal}(i) \right|^2\right)^{1/2}$$

Which N_{tot} is the number of energy levels where are included in extraction processes. We have extracted the best set of Hamiltonian’s parameters, i.e. γ and δ, via the available experimental data [31] for excitation energies of selected states, $0_1^+, 2_1^+, 4_1^+, 0_2^+, 2_2^+, 4_2^+$ and *etc,* e.g. 12 levels up to 2_4^+, or two neutron separation energies. In summary, we have extracted γ and δ externally from empirical evidences and other quantities of Hamiltonian, e.g. c and ε would determine through the minimization of s . The results for different parameters of transitional Hamiltonian in the IBM-1 version are presented in Table 1 The results suggest a combination of the vibrational and gamma unstable symmetries must be used to explore the experimental energy spectra for ^{190}Hg nucleus.

2.2. $B(E2)$ Transition

Electric multiple moments are measures of the charge distribution of nuclear states and especially of their deviation from spherical. The reduced electric quadrupole transition probabilities, $B(E2)$, are considered as the observables which as well as quadrupole moment ratios within the low-lying state bands prepare more information about the nuclear structure. The *E2* transition operator must be a Hermitian tensor of rank two and consequently, number of bosons must be conserved. With these constraints, there are two operators possible in the lowest order, therefore the electric quadrupole transition operator employed in this study is defined as [7],

$$\hat{T}_{\mu}^{(E2)} = q_2 \left[[\hat{d}^{\dagger} \times \tilde{s} + \hat{s}^{\dagger} \times \tilde{d}]_{\mu}^{(2)} + p_2 \, [\hat{d}^{\dagger} \times \tilde{d}]_{\mu}^{(2)} \right] \quad , \qquad (16)$$

q_2 is the effective quadrupole charge, p_2 is a dimensionless coefficient and $s^{\dagger}(d^{\dagger})$ represent the creation operator of $s(d)$ boson. The reduced electric quadrupole transition rate between $I_i \rightarrow I_f$ states is given by:

$$B(E2; I_i \rightarrow I_f) = \frac{\left| \langle I_f \| T(E2) \| I_i \rangle \right|^2}{2I_i + 1} \quad , \qquad (17)$$

To analyze the $B(E2)$ transition ratios for isotopic chain, we have calculated the matrix elements of $T(E2)$ operator between considered states, then with comparing the results with experimental counterparts[31], we can extract (q_2, p_2) quantities. To this aim and also to simplify the description, we have followed the method introduced in Refs.[23-24] and in the fitting procedures, these parameters would be described as a function of only, total boson number (N).

3. THEORETICAL RESULTS FOR ENERGY LEVELS AND TRANSITION PROBABILITIES OF ^{190}HG

Investigations of experimental energy spectra which have done in Refs.[9-20], propose the configuration mixing of spherical and axially deformed symmetries in the ^{190}Hg nucleus. As a results of other theoretical predictions and experimental evidences, one may expect a O(6)-like structure for Hg isotopic chain which are located in the oblate to prolate transitional region. The existence of some excited levels between the states of ground band, intruder states, suggest the effect of spherical symmetry together dominant O(6) structure. To this aim and consider the both shape coexistence and phase transition phenomena, we have used a transitional Hamiltonian which include both U(5) and SO(6) symmetries to describe ^{190}Hg nucleus.

If we consider $c_s = 1$ which comes from this idea that Hg isotopes are located in or near the critical point of oblate to the prolate transitional region and therefore correspond with SO(6) limit, and then extracted other parameters of Hamiltonian, the maximum deviation in comparison with experimental values are yield as have presented in Tables1. The results suggest a combination of the vibrational and gamma unstable symmetries must be used to explore the experimental energy spectra for ^{190}Hg nucleus. To get best combination of these symmetries, we have changed the c_s values in the 1 to 0.2 region and σ values define the best choice of the weight of each symmetry.

Table 1. Parameters of IBM-1 transitional Hamiltonian for ^{190}Hg. All quantities (except c_s) are in keV.

c_s	ε	δ	γ	σ
1	500	29.75	-0.95	449
0.9	500	28.65	-1.36	415
0.8	500	27.66	-1.73	390
0.7	500	26.78	-2.06	374
0.6	500	26.53	-2.34	367
0.5	500	25.39	-2.58	361
0.4	500	24.86	-2.78	360
0.3	500	24.45	-2.93	362
0.2	500	24.16	-3.04	363

The best agreement of theoretical predictions with experimental values achieved by $c_s \approx 0.4$. This result confirm our idea for the combination of symmetries and therefore, spherical and axial deformed shapes coexistence in ^{190}Hg.

On the other hand, the electromagnetic transition rates make it possible to test the wave functions which suggested by considered model together the accuracy of transition operators. We have considered quadrupole transition rates which have experimental counterpart. To this aim and by using Eqs.(16-17), we have calculated the expectation value of transition operator between selected states and then, we have extracted q_2 and p_2 quantities in comparison with empirical values. Theoretical predictions together their experimental counterparts are presented in Figure 2.

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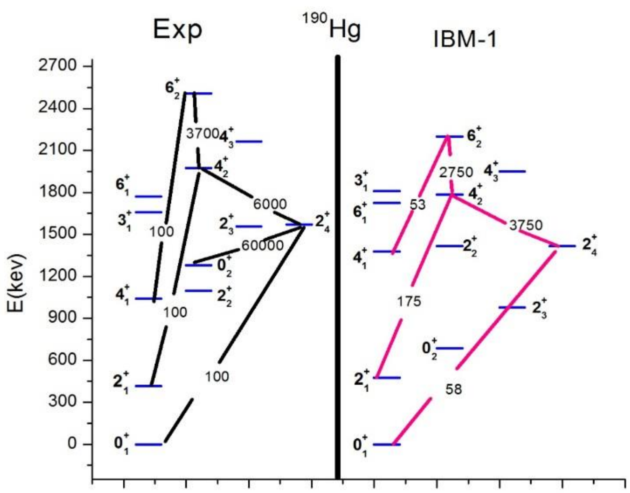

Figure 1. Energy spectra (in keV) and transition probabilities (in W.u.) of ^{190}Hg nucleus which are determined by for c_s=0.4 as the more exact condition. Also, the parameters of E2 transition operator are q_2 = 1.844 and p_2 = - 0.279 (in W.u.). Experimental values are taken from [31].

The IBM-1 results for different energy levels and transition probabilities are in good agreement with experimental evidences. Also, deviation from the first approximation which suggest c_s=1, e.g. completely correspond with SO(6) dynamical symmetry limit, confirm the shape coexistence and therefore, symmetry combination in this nucleus. the Hg isotopic chain expect to be located in and near the critical point of prolate to oblate transitional region. The results which offer the role of spherical symmetry in this isotopic chain and consequently the combination of these two symmetries, not absolutely but likely, may suggest a shape coexistence-like meaning. Also, if we consider that, interplay between the stabilizing effect of a closed shell on one hand and the residual interactions between protons and neutrons outside closed shells on the other hand, leads to the concept of 'shape coexistence', where normal near-spherical and deformed structures coexist at low energy, our result show the similar competition between these two interactions[32-36].

On the other hand, the great uncertainty of theoretical prediction in the excited energy levels force us to consider extended version of IBM. As have expressed in different lectures such [20-30], we expect 2p-2h configuration in the Hg isotopic chain and in such conditions, the IBM-2 version are usual method which we would consider in the next studies.

4. CONCLUSIONS

A $SU(1,1)$-based transitional Hamiltonian in the interacting boson model has been used to determine the energy spectra and quadrupole transitional rates of ^{190}Hg nucleus. Parameters of Hamiltonian and transition operator are determined by Bethe-Ansatz technique and experimental data. The control parameter of model describes the effect of each symmetry and suggest a combination of spherical shape together deformed one in this nucleus. This formalism would extend in future studied for Hg isotopic chain via other versions of IBM to consider the effect of other degrees of freedom. Also, obtained results in this study confirm that this technique is worth extending for investigating the nuclear structure of other nuclei existing around the mass of A ~ 200.

REFERENCES

[1]. Garcia-Ramos G.and Heyde K.,Comparing the Interacting Boson Model With Configuration Mixing and the Extended Consistent-Qformalism, Nucl. Phys.A., 82 (2009) 39.

[2]. Garcia-Ramos G.and Hellemans V.and Heyde K., Concealed Configuration mixing and shape coexistence, Phys.Rev.C., 84 (2011) 014331.

[3]. Dracoulis G D.,Shape coexistence in very neutron deficient Pt isotopes, Nucl.phys.G.,12 (1986) 97-103.

[4]. Fossion R.and Hellemans V., Shape coexistence in the Lead isotopes using Algebraic Models,Acta.Phsica. Polonica.B.,36 (2005) 4.

[5]. Garcia-Ramos G E.and Heyde K.,Nuclear Shape Coexistence a study of the even-even Hg isotopes,Phy .Rev.C., 89 (2014) 014306.

[6]. Poves A.,Shape Coexistence the Shell Model view, J.Phys.G.Nucl.part.Phys., 43 (2016) 024010.

[7]. Stachel G.and Isacker p.and Heyde K., Interpretation of the A=90 transitional region in the framework of the interacting Boson Model,Phys.Rev.C., 25 (1982) 1.

[8]. Larabee A J.and Carpenter M P.and Riedinger L ., Shape coexistence and alignment processes in the Pt and Au region,Phys.Lett.B.,169 (1986) 1.

[9]. Heyde K.and Meyer R A.,Monopole Strength and Shape coexistence, Phys.Rev.C., 42 (1990) 2.

[10]. Doung H T.and Pinard G.and Liberman S.,Shape transition in ne utron deficient Pt isotopes, Phys.Lett.B., 217 (1989) 4.

[11]. King S L.and Simpson G., First observation of excited state in the neutron deficient nuclei, Phys. Letters. B., 443 (1998) 82-88

[12]. Heyde K.and Meyer R A.,Monopole Strength as a measure of nuclear Shape mixing , Phys.Rev.C., 37 (1988) 5.

[13]. Frank A.and Van Isacker P.andVargas C E., Shape coexistence and phase transition in the Platinum isotope, Phys. Rev. C., 69 (2004) 024303.

[14]. Andreyev A N.and Huyse M.,The discovery of a prolate oblate Spherical Shape triple of spin $^{+}0$ state,Nucl.Phys.A ., 682 (2001) 482.

[15]. Morales I O.and Vargas C E.and Frank A.,Shape coexistence in the neutron deficient Pt isotopes in a configuration mixing IBM, Nucl.Phys., 73 (2004) 54.

[16]. Xu Y.and Krane K S.and Gummin M A.,shape coexistence and Eletric Monopole Transition in Pt, Phys.Rev.Lett., 68 (1992) 26.

[17]. Otsuka T.and Tsunoda Y.,The role of shell evolution in shape coexistence, J. Phys. G.Nucl.Part.Phys.,43 (2016) 024.

[18]. Neyens G., Shape coexistence in the N=19isotones,J.Phys.G.Nucl.Part.Phys ., 43 (2016) 024007

[19]. Li Z P.and Niksic T.and Vretenar D.,Coexistenc of nuclear Shapes, J.Phys.G.Nucl.Part.Phys.,43 (2016)22.

[20]. Gorgen A.and Korten W.,Coulomb excitation studies of shape coexistence in atomic nuclei J. Phys. G. Nucl. Part. Phys., 43 (2016) 18.

[21]. Garrett P E., Shape coexistence at low spin in the Z=50 region ,Phys.G.Nucl.Part.Phys.,43 (2016) 36.

[22]. Lipska K W.and Gaffney L P., Unique and complementary information on shape coexistence in the neutron deficient pb region from coulomb excitation, J. Phys. G. Nucl. Part. Phys., 43 (2016) 024012.

[23]. Feng Pan G P.and Draayer G., New algebraic solution for SO(6)-U(5) transitional nuclei in the interacting Boson model,Nucl. Phys. A., 636 (1998) 156.

[24]. Feng Pan G P.and Draayer G., Algebraic solution of SL-Boson system in U(2L+1)-O(2L+2) transitional region,Phys. A. Math. Gen.,35(2002) 7173

[25]. Soramel F.and Bednarczyk P., Study of very neutron deficient nuclei , Eur.Phys.J.A.,4 (1999) 17

[26]. Oros A M.and Heyde K., Shape coexistence in the light Po isotopes, Nucl.Phys.A., 645 (1999) 107

[27]. Heyde K.and Van Isaker P., A shell model interpretation of intruder states and the onset of deformation in even – even nuclei , phys.Lett.B.,155 (1985) 5

[28]. Bijnens N.and Decrock P., Intruder states and the onset of deformation in the neutron deficient even-even polonium isotopes, Phys.Rev.Lett., 75 (1995) 25

[29]. Fossion R.and Heyde K.,Intruder bands and configuration mixing in Lead isotopes, Phys. Rev. C., 67 (2003) 024306

[30]. Hinohara N.and Nakatsukasa T Microscopic derivation of the quadrupole collective Hamiltonian for shape coexistence mixing dynamics, Phys.Rev.C., 80 (2009) 014305

[31]. ,B.Singh.,Nuclear Data sheets for A=190, Nucl.Data Sheets .,99 (2003) 275

[32]. Duval P D.and Barrett B R., configuration mixing in the interacting

Boson model, Phys.Lett.B., 100 (1980) 3.

[33]. Dickmann F.and Dietrich K., Coexistence and mixing of Spherical and deformed states, Z. Physik., 271 (1974) 417.

[34]. Beraud R.and Meyer M., Band crossing in ^{186}Hg,Nucl.Phys.A., 284(1977)221.

[35]. Nomura K.and Otsuka T., Shape coexistence in the microscopically guided interacting Boson model, Jurnal of Physics G. 43 (2016) 2

[36]. Duval P D.and Barrett B R.,Quantitative description of configuration mixing in the interacting Boson model, Nucl.Phys.A., 376 (1982) 213.