Cumhuriyet Science Journal
CSJ

e-ISSN: 2587-246X
ISSN: 2587-2680

Cumhuriyet Sci. J., Vol.39-1(2018) 118-126

Shell Closure N = 82 nuclei interpretation by the Cranked Nilsson-Strutinsky Model: $^{136\text{-}138}$Ce

S. KAIM[1*], A. ABOUDI[1], R. BELGHARBI[1], A. LAALA[1]

[1] *Université des Frères Mentouri Constantine1, Laboratory of Mathematical and Subatomic Physics, Constantine, ALGERIA*

Received: 30.11.2017; Accepted: 26.01.2018   *http://dx.doi.org/10.17776/csj.359907*

Abstract: A theoretical studying of nuclei in the shell closure region (N =82), in particular cerium isotopes $^{136\text{-}138}$Ce, has been made in the frame of macroscopic-microscopic, Cranked Nilsson Strutinsky (CNS)model. Many nuclear properties have been calculated and discussed: the single particle energy, the total energy, the liquid drop energy, the moments of inertia, the deformation parameters, the nuclei shape, and potential energy surfaces (PES) have been presented. High spin rotational bands of these isotopes have been studied and interpreted based on fixed configurations in the frame of CNS code. A good agreement is found with the observed results for these nuclei in the recent works.

Keywords: Shell closure N=82, Cerium isotopes $^{136\text{-}138}$Ce, Cranked Nilsson Strutinsky (CNS), rotational bands, shape evolution, deformation parameters, PES.

Cranked Nilsson-Strutinsky Model ile N = 82 Çekirdeğinde Kabuk Kapanması Yorumu

Özet: Makroskopik-mikroskopik, Cranked Nilsson Strutinsky (CNS) modeli çerçevesinde, kabuk kapanma bölgesinde (N = 82), özellikle $^{136\text{-}138}$Ce seryum izotoplarında çekirdeklerin teorik bir incelemesi yapılmıştır. Birçok nükleer özellik hesaplanmış ve tartışılmıştır: tek parçacık enerjisi, toplam enerji, sıvı damlası enerjisi, eylemsizlik momentleri, deformasyon parametreleri, çekirdek şekli ve potansiyel enerji yüzeyleri (PES) sunulmuştur. Bu izotopların yüksek spin dönme bantları, CNS kodu çerçevesinde sabit konfigürasyonlara dayalı olarak çalışılmış ve yorumlanmıştır. Son çalışmalarda bu çekirdeklerin gözlenen sonuçları ile iyi bir uyum bulunmuştur.

Anahtar Kelimeler: Kabuk kapatması N = 82, Seryum izotopları $^{136\text{-}138}$Ce, Cranked Nilsson Strutinsky (CNS), dönme bantları, şekil evrimi, deformasyon parametreleri, PES.

1. INTRODUCTION

The shell closure N = 82 nuclei with few holes represent an interesting region for shape and deformation progress study. Indeed, in this region, valence protons and neutrons compete and induce important shape changes and shape co-existence phenomena. Although they are spherical or only moderately deformed in low spin states [1-4], the results of recent sensitive experiments for neighboring nuclei have confirmed, they can also get deformed at high spins. The investigation of high spin bands of cerium isotopes (Z = 58 and N = 78- 81) [5-8] is very important because they show interesting

* Corresponding author. *Email address: skaim3@yahoo.fr*

phenomena such as magnetic rotation, shears bands [9] and other phenomena.

The main motivation of these work is to study and investigate the structure of the observed high spin states of $^{136-138}$Ce nuclei [5-8] by suggesting several positive and negative parity configurations with different signatures of the dipole bands on the basis of Cranked Nilsson-Strutinsky (CNS) calculations where we have assigned the configurations having the best agreement with the experimental results of these nuclei.

This paper is structured as follows. First, a general background on the configuration that dependent the CNS approach is presented in Section 2. Section 3 is devoted to outline the obtained calculation results along with their discussion and the comparison with the recent observed results for these nuclei, before concluding in Section 4.

2. CNS CALCULATIONS

In CNS calculations, we have described our nuclei in terms of a closed core, the tin 132 (^{132}Sn) plus valence nucleons (particles and/or holes) belonging to one or many *j*-shells. In the case of the chosen nuclei, the valence nucleons consist of eight protons and two, three or four neutron holes in N = 82 shell closure. The valence protons occupy the orbitals $1g_{7/2}$, $2d_{5/2}$ and $1h_{11/2}$; and the neutron holes take up the orbitals $1g_{7/2}$, $2d_{5/2}$, $1h_{11/2}$, $2d_{3/2}$, $3s_{1/2}$, $f_{7/2}$, $1h_{9/2}$ and $1i_{13/2}$. Although the participation of the core to the nuclear spin is null, the valence particles have a maximum angular momentum $I_{max} = I_{p\ max} + I_{n\ max}$. The configurations can be specified by the number of particles in the specified orbitals of different parities and signatures. We have adapted the coupled proton and neutron configurations notation:

$$\pi\left[(d_{5/2}g_{7/2})^{p_1}_{\alpha_1}(h_{11/2})^{p_2}_{\alpha_2}\right]\otimes\nu\left[(d_{3/2}s_{1/2})^{-n_1}_{\alpha_3}(h_{11/2})^{-n_2}_{\alpha_4}(h_{9/2}f_{7/2})^{n_3}_{\alpha_5}(i_{13/2})^{n_4}_{\alpha_6}\right],$$

where: $p_{1\text{-}2}$ and $n_{3\text{-}4}$ represent the number of protons and neutrons respectively on the appropriate orbitals. $n_{1\text{-}2}$ are the neutrons holes' numbers. $\alpha_{1\text{-}6}$ is the signature of each orbital. In the plots representations, we will used the condensed notation : $[p_1p_2, n_1n_2\,(n_3n_4)]$.

Our calculation (see next section) show that the variation of energies values as function of spins curves are strongly dependent on the chosen configuration. The different notations are given in the legend boxes in the figures below).

With an aim to better explain and interpret the behavior of high spin bands of the studying nuclei, we have fixed and proposed several protonic and neutron configurations for negative and positive parities (see table 1 and the discussion in Section 3).

The high spin bands in which we are interested are:

- Dipole bands of negative and positive parities B3 and B6 respectively, of the observed level scheme of ^{136}Ce nucleus [6]. B3 band is built on the level of exciting energy 5645 keV and spin I^π = 14−, and extended up to the level 9228 keV (I^π = 23−). B6 band is constructed on the bandhead level 5643 keV (I^π = 16+) and extended to the level 8316 keV (I^π = 23+)

- ΔI =1 positive parities bands, band4 and band6 of ^{137}Ce of the experimental work are described as follow [7]: the band 4 starts from the level 4225 keV (15.5+) to reach the level 6459.0 keV (18.5+). The band 6 is developed on the 5379.1 keV (16.5 +) state and extended to 7660.3 keV (21.5+) level.

- The bands B1 and B2 of ^{138}Ce of the observed level scheme [8] are reported:The negative parity $\Delta I = 1$ band B1 has been defined to be built on the 15− state at 6536.3 keV and to be extended up to the 22− (8957.8 keV) state. The ground state band B2 is extended up to spin 23+ at 9430.8 keV.

3. CALCULATIONS RESULTS and DISCUSSION

A. ^{136}Ce

We have performed CNS calculations in order to understand the structure of the high spin bands, in particular Band3 and Band6. With a closed ^{132}Sn core, the nucleus ^{136}Ce can be described as having eight protons and four neutron holes (valence nucleons) in addition of this core.

Band B3:

The excitation energy E(I) calculated values as a function of spin I, is expressed as ($E\text{-}E_{rld}(def)$), i.e. with a rigid rotation reference subtracted (Fig.1). We suggest the most energetically favorable [12] ($E\text{-}E_{rld}(def) \approx$ -1MeV), the lowest lying negative parity configuration generated by coupling of one proton and two neutron holes in the orbital $h_{11/2}$: $\pi(dg)^7h^1_{11/2} \otimes \nu h^{-2}_{11/2}$ (i.e., with notation [7(34)1,02(00)], see Fig.1 below).

Our interpretation of this band is in good agreement with the suggestion of the four-quasiparticle configuration $\pi[g_{7/2}h_{11/2}] \otimes \nu[h_{11/2}]^2$ supported in the work [6].

This configuration is located in an energy minimum at ($\varepsilon_2 \sim 0.12$; $\gamma \sim 4.15°$) showing a nearly prolate shape (γ not very far from 0°) with a trend to have a triaxial behavior. Our prediction is in difference of the work [6] where one proposed a nearly oblate ($\gamma = 52°$) shape with an average tilt angle $\theta = 32°$ for this band.

For this configuration, the maximum spin is $I_{max} = 32^+$ ($I^{\pi}_{p\ max} = 22^+$ and $I^{\pi}_{n\ max} = 10^+$), which indicates a possibility of observed band extending. However, further measurements are necessary to confirm this interpretation.

The calculated values of the kinematic moment of inertia $J^{(1)}$ decrease as a function of rotational frequency from *80* to 6*0* $\hbar/MeV$. This suggests the expectation of triaxial deformation of this band. This, in addition, is confirmed by the value of triaxial parameter ($\gamma \sim 4.15°$) which is relatively different from zero.

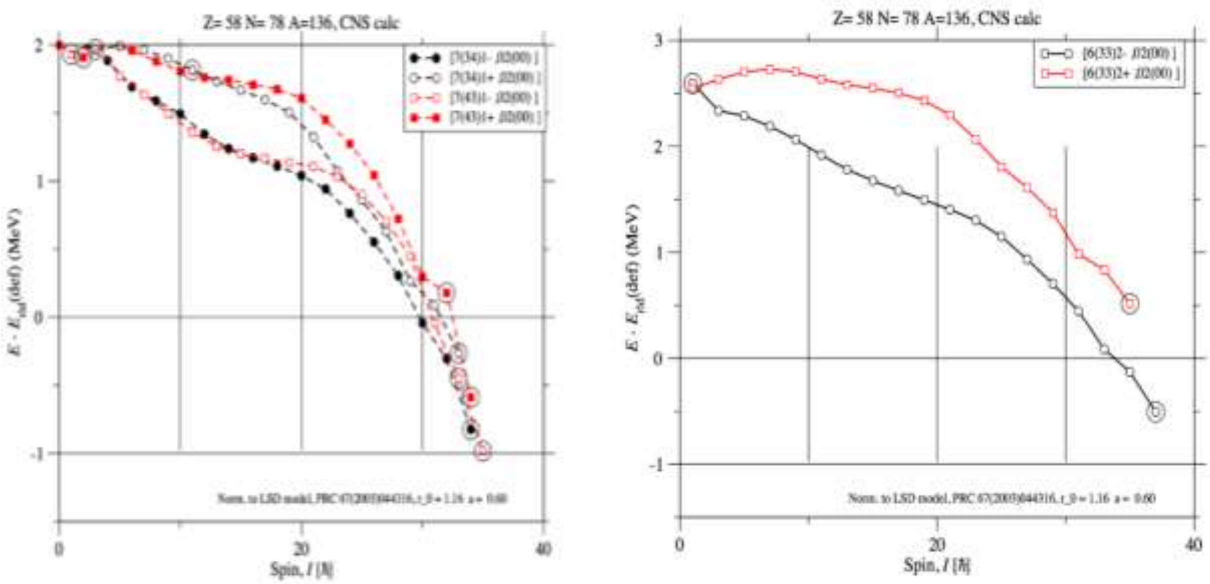

Figure 1. Calculated total energy values expressed relative to the deformation dependent rotating liquid drop energy [5] (*E-E$_{rld}$(def)*) vs nuclear spin of bands 'B3' and 'B6' on the left and on the right respectively of ^{136}Ce nucleus. Positive and negative parity configurations are drawn by full and dashed lines, respectively. Closed symbols are used for signature, α = +1/2 and open symbols for signature α = −1/2. The encircled symbols represent non-collective aligned states. The configurations are labeled by the condensed notation [p_1p_2,n_1n_2(n_3n_4)], as explained in the text.

<u>Band B6:</u>

To interpret this band, we propose the positive parity configuration generated by coupling two excited protons and two neutron holes in the orbitals $h_{11/2}$: $\pi(dg)^6h^2_{11/2} \otimes \nu h^{-2}_{11/2}$ (≡ the notation [6(33)2,02(00)] in Fig.1) showing a good agreement with the work [6] that proposes four-quasiparticle configuration $\pi[h_{11/2}{}^2]\otimes\nu[h_{11/2}{}^{-2}]$ for this band.

The total energy calculated relative to the the rotational liquid drop terminates (Fig. 1, right side) favorably in energy near I_{max} = 38$^+$, and *E-E$_{rld}$(def)* ≈ -0.5MeV. This value shows also a rotor rigid core behavior of this nuclei. And one can confirm that high spin bands of ^{136}Ce are interpreted by interchanges between the excitation particles and the collective configurations.

The minimization of total energy fixes the deformation parameters, (ε_2 ~ 0.12 ; γ ~ 0.16°) showing a nearly prolate shape which is in difference of the work [6] where oblate character is supported (ε_2 = 0.131, ε_4 = 0.015, γ = 52°) with an average tilt angle of θ = 40°.

B. ^{137}Ce

The nucleus of ^{137}Ce is described to have eight protons and three neutron holes in the valence space in addition of ^{132}Sn core. We consider in particular the high spin bands, Band4 and Band6.

<u>Band4:</u>

The positive parity configuration that we propose and which represents the lowest lying calculated energy of the Band4 is the configuration $\pi(dg)^6h^2_{11/2}$U$\nu(ds)^{-1}$ with the notation [62,10(00)], i.e., the configuration of two protons excited to the orbital $h^2_{11/2}$ and one neutron hole in the sub-shell *(ds)*. Particularly the configurations: [6(33)1+1-,10-(00)] and [6(33)1+1-,10+(00)], where the signs + and – indicate positive and negative signature values of the specific orbital. The double numbers (33) show the distribution of the protons in the active orbitals of the sub-shell ($d_{5/2}g_{7/2}$).

Our low lying proposed configuration is in a good accord with the suggested quasiparticle structure ($\pi[g_{7/2}\ d_{5/2}\ h^2_{11/2}\ ] \otimes \nu[s_{1/2}\ ]$) proposed in the work [7].

The maximum spin in the respective proton and neutron configurations are $I^{\pi}_{p\ max}$ = 22^+ and $I^{\pi}_{n\ max}$ $=3^+/2$. The maximum spin in this configuration is then, I_{max} = $47^+/2$, a value which is greater than the highest spin value observed experimentally. This predicts the possibility of extending this band.

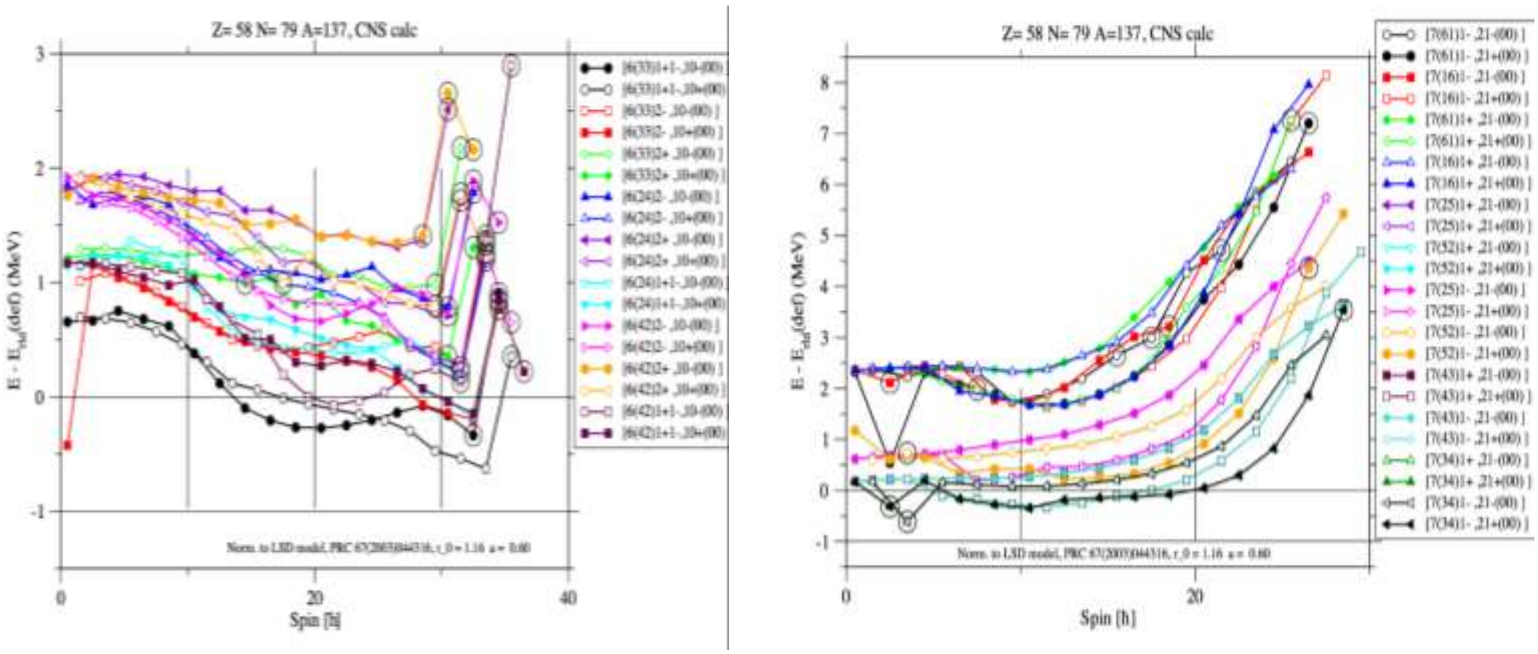

Figure 2. Calculated energy vs spin plots for several negative and positive parity candidate configurations of high spin bands of ^{137}Ce, Band4 and Band6 on the left and on the right respectively.

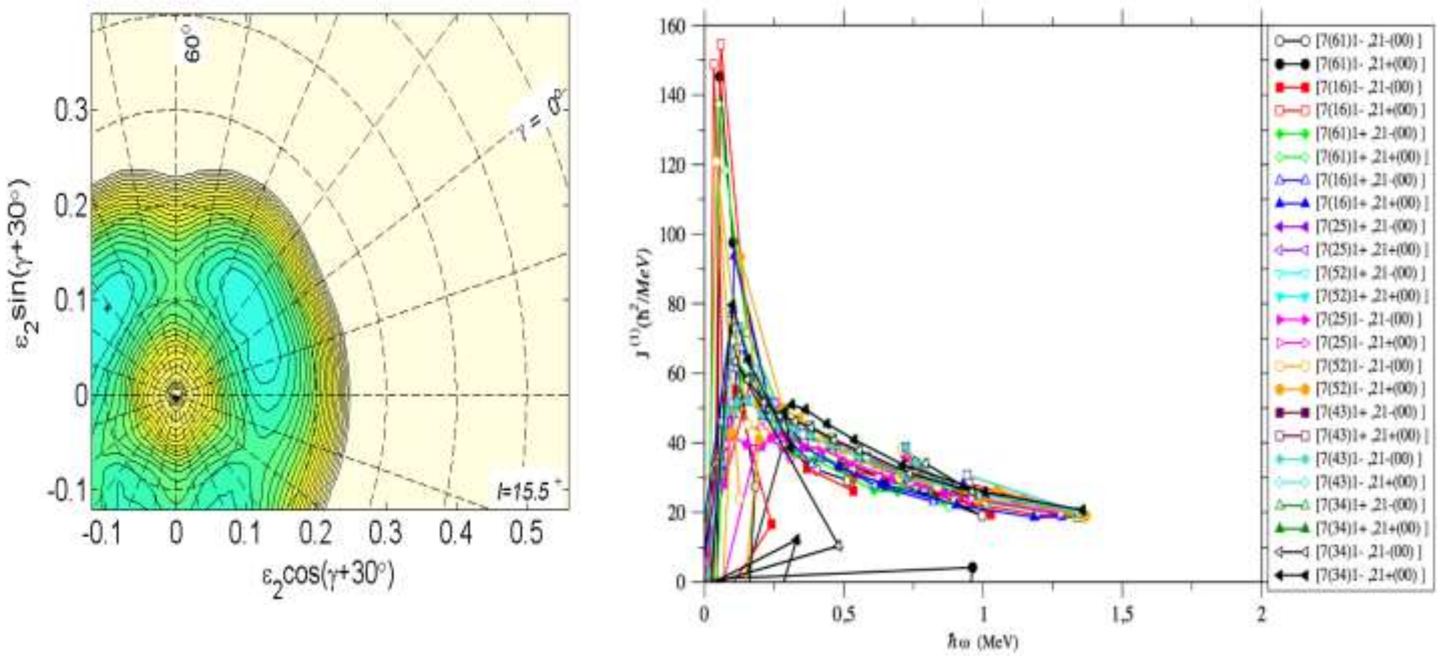

Figure 3. a. (On the left) The potential energy surface (PES) plotted in the (ε_2-γ) plane for the suggested configuration of band 4 of ^{137}Ce. b. (On the right) Kinematic moments of inertia ($J^{(1)}$) for band 6 of ^{137}Ce represented as a function of rotational frequency. See text for details.

In Fig. 2 the energies of the suggested configurations are calculated as a function of nuclear spin. The nucleus behavior is the same as rigid rotor body until spin 10ħ (E-$E_{rld}(def)$ close to zero), then the energy decreases somewhat to terminates slightly favorable (E-$E_{rld}(def)$ ≈ -0.5MeV) [12]. This band displays an obvious competition between collective and single excitation particles (holes) behavior.

The potential energy surfaces (PES in Fig. 3 a) have been performed for this band (Bnad4) with the proposed configuration [62,10(00)] for the bandhead (I = $15^{+}.5$). We have deduced deformation parameters ($\boldsymbol{\varepsilon_2}$ ~ 0.13 ; $\boldsymbol{\gamma}$ ~ 9.02°) showing a prolate deformation with a triaxial deformation trend.

Band6:

The ΔI = 1 magnetic band6 of the bandhead of 33+/2 is conjectured to have the positive parity configuration of one proton in the orbital $h_{11/2}$ and three neutron holes in orbitals *(ds)* and $h_{11/2}$, the configuration, $\pi(dg)^7h^1{}_{11/2} \otimes \nu(ds)^{-2}h^{-1}{}_{11/2}$ with deformation parameters ($\boldsymbol{\varepsilon_2}$ ~ 0.12 ; $\boldsymbol{\gamma}$ ~ -60°) which is reasonable oblate in shape.

The kinematic moment of inertia $J^{(1)}$ values for band 6 have been calculated and plotted with respect to the rotational frequency in Fig. 3(b). The clear sloping trend of $J^{(1)}$ in particular for the most favorable configuration [7(34)1,21(00)] (from *80* to *40ħ/MeV*), in addition of the relatively small and regular value of dynamic moment of inertia $J^{(2)} \approx 25\ \hbar^2/MeV$ calculated in the rotational frequency interval of this band, $\hbar\omega = 0.3 - 1.25MeV$, are in excellent agreement with the obtained results of the work [7]. This signifies that our calculations also confirm the expectation of triaxial deformation of this band.

C. ^{138}Ce

The high spin bands of negative and positive parity B1 and B2 are found to have the same protons behavior as respectively in B3 and B6 bands of ^{136}Ce.

They are explained by the most probable (the low lying) configurations. Indeed, B1 band structure is suggested to be: $\pi(dg)^7h^1_{11/2} \otimes \nu h^{-2}_{11/2}$ ([7(34)1,02(00)]) with deformation parameters $\varepsilon_2 \sim 0.097$; $\gamma \sim -120°$ (prolate shape). The band B2 is interpreted by the configuration: $\pi(dg)^6h^2_{11/2} \otimes \nu h^{-2}_{11/2}$ ([6(33)2,02(00)]), where among it [6(33)1+1-,02(00)] and [6(33)1+1-,02-(00)] (Fig.4) are the most favorable energetically. The proposed configurations are both in good acceptance with the results exhibited in the work [8].

The deduced deformation parameters of band B2 are ($\varepsilon_2 \sim 0.11$; $\gamma \sim 6.1°$) showing a triaxial trend of this nuclei.

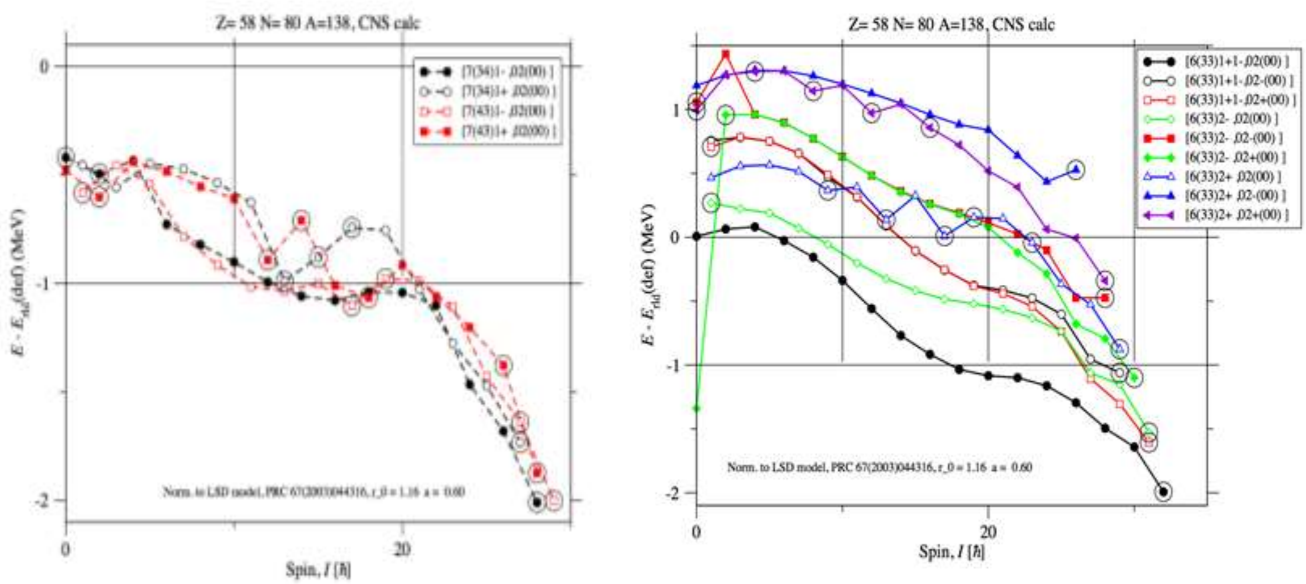

Figure 4. Calculated high spin states of bands 'B1' and 'B2' on the left and on the right respectively of ^{138}Ce nucleus.

In the following table1, we summarize the suggested configuration and the calculated deformation parameters for $^{136-138}$Ce nuclei.

Table 1. The suggested configurations and deformation parameters, quadru-polar deformation ε_2 and triaxiality γ parameters, obtained from CNS calculations.

Nuclei	Bands	Configurations	ε_2 =	$\gamma°$ =
^{136}Ce	B3	$\pi(dg)^7h^1_{11/2} \otimes \nu h^{-2}_{11/2}$	0.12	4,15
	B6	$\pi(dg)^6h^2_{11/2} \otimes \nu h^{-2}_{11/2}$	0.12	0.16
^{137}Ce	Band4	$\pi(dg)^6h^2_{11/2} \otimes \nu(ds)^{-1}$	0.13	9,02
	Band6	$\pi(dg)^7h^1_{11/2} \otimes \nu(ds)^{-2}h^{-1}_{11/2}$	0.12	-57.20
^{138}Ce	B1	$\pi(dg)^7h^1_{11/2} \otimes \nu h^{-2}_{11/2}$	0.097	-120
	B2	$\pi(dg)^6h^2_{11/2} \otimes \nu h^{-2}_{11/2}$	0.11	6,1

4. CONCLUSION

High spin states of $^{136-138}$Ce isotopes are successfully studied and interpreted by CNS codes. The obtained results are presented and the total energy values are expressed relative to the deformation dependent rotating liquid drop energy ($E-E_{rld}(def)$) vs a large range of nuclear spin I, kinematic moments of inertia as a function of rotational frequency $J^{(1)}(\omega)$, and PES (the potential energy surfaces) presented in the deformation space (ε_2 , γ).

The deduced deformation parameters values for $^{136-138}$Ce nuclei are : $\varepsilon_2 \approx 0.1$, $\gamma \approx 0°, -60°, -120°$ showing small deformation with an axial prolate ($\gamma \approx 0°$, 120°) or oblate shape ($\gamma \approx -60°$) with a slight inclination to triaxial shape for band B3 of

^{136}Ce ($\gamma \approx 4°$), band4 of ^{137}Ce ($\gamma \approx 9°$) and band B2 of ^{138}Ce ($\gamma \approx 6°$).

On the other hand, from the obtained triaxial parameter values (γ values in table1), we can evidence the coexisting shape of bands for these nuclei. Bands B3 and B6 of ^{136}Ce have triaxial trend and prolate shapes respectively. Band4 and band6 of ^{137}Ce have respectively triaxial trend and oblate shapes with a possible tendency to be triaxial because of the clear sloping inclination of $J^{(1)}$ (from 80 to $40\hbar/MeV$) and the rather small and regular value of dynamic moment of inertia ($J^{(2)} \approx 25\ \hbar^2/MeV$). This is in excellent agreement and confirm the obtained results of the work [7]. The B1 and B2 bands of ^{138}Ce with this order are prolate and of triaxial shape.

The high-spin states are explained by positive or negative parities configurations formed by a combined contribution on 2, 3 and 4 neutron holes in $h_{11/2}$ orbitals and/or in (ds) sub-shells coupled with 1 or 2 protons excitations to the $h_{11/2}$ orbitals (Table1). Where we have interpreted B3 and B6 of ^{136}Ce by respectively one and two protons excitation in $h_{11/2}$ orbital coupled with 2 neutron holes in $h_{11/2}$ orbitals ($\pi(dg)^7h^1_{11/2} \otimes \nu h^{-2}_{11/2}$ and $\pi(dg)^6h^2_{11/2} \otimes \nu h^{-2}_{11/2}$). Band4 and band6 of ^{137}Ce are explained by the positive parities configurations, $\pi(dg)^6h^2_{11/2} \otimes \nu(ds)^{-1}$ and $\pi(dg)^7h^1_{11/2} \otimes \nu(ds)^{-2}h^{-1}_{11/2}$ respectively, i.e. the configuration of 2 protons in $h_{11/2}$ orbitals coupled with a neutron hole in (ds) sub-shells and the configuration of one proton in $h_{11/2}$ orbital coupled with two neutron holes in (ds) sub-shells and one hole in $h_{11/2}$ orbital. B1 and B2 bands of ^{138}Ce are found to have the same protons behavior as B3 and B6 respectively of ^{136}Ce. The calculations have established that the negative and positive parities configurations $\pi(dg)^7h^1_{11/2} \otimes \nu h^{-2}_{11/2}$ and $\pi(dg)^6h^2_{11/2} \otimes \nu h^{-2}_{11/2}$ are the lowest lying configurations for B1 and B2 bands respectively.

All the proposed low lying configurations are in an good agreement with the nominated structures in the works [6,7,8], this can affirm the excellent reliability of the calculations method in CNS model.

5- KNOWLEDGMENTS

We express our deep gratitude for the authors of CNS model, in particular Prof. I. Ragnarsson for giving us the possibility to use the codes.

REFERENCES

[1]. Ludziejewski J., Arnold H., High-spin levels in ^{137,139}Ce and ^{139,141}Nd evidence for hole-core coupling, Z. Phys. A, 281 (1977) 287- 308.

[2]. Möller P., Nix J. R., Myers W. D., and Swiatecki W. J., Nuclear ground-state masses and deformations, At. Data and Nucl. Data Tables ,59 (1995) 185- 381.

[3]. Chanda S., Bhattacharjee T., Bhattacharyya S., Mukherjee A., Basu S.K., Ragnarsson I., Bhowmik R.K., Muralithar S., Singh R.P., Ghugre S.S., Datta Pramanik U., Seven-quasiparticle bands in ^{139}Ce, Phys. Rev. C, 79 (2009) 054332 (11 pages).

[4]. Kaim S., Petrache C. M., Gargano A., Itaco N., Zerrouki T., Leguillon R., Astier A., Deloncle I., Konstantinopoulos T., Régis J. M., Wilmsen D., Melon B., Nannini A., Ducoin C., Guinet D., and T. Bhattacharjee, High-spin spectroscopy of ^{139}Ce, Phys. Rev. C, 91 (2015) 024318 (8 pages).

[5]. Carlsson B. G., Ragnarsson I., Calculating the nuclear mass in the very high angular momentum regime, Phys. Rev., C 74 (2006) 011302(R) (5 pages).

[6]. Lakshmi S., Jain H.C., Joshi P.K., Mazumdar I., Palit R., Jain A.K., Malik S.S., High spin structure of ^{136}Ce, Nuclear Physics A, 761 (2005) 1-21.

[7]. Bhattacharjee T., Chanda S., Mukherjee A., Bhattacharyya S., and Basu S. K., Multi-quasiparticle bands in ^{137}Ce,

PHYSICAL REVIEW C, 78 (2008) 024304 (11 pages).

[8]. Bhattacharjee T., Chanda S., Bhattacharyya S., Basu S.K., Bhowmik R.K., Das J.J., Datta Pramanik U., Ghugre S.S., Madhavan N., Mukherjee A., Mukherjee G., Muralithar S., Singh R.P., Band structure in near spherical ^{138}Ce, Nuclear Physics A, 825 (2009) 16-38.

[9]. Chmel S., Frauendorf S., Hübel H., Deformation parameters and transition probabilities for shears bands in Pb isotopes, Phys. Rev., C 75 (2007) 044309 (19 pages).

[10]. Bengtsson T., Ragnarsson I., Rotational bands and particle-hole excitations at very high spin, Nucl. Phys. A, 436 (1985) 14-82.

[11]. Afanasjev A. V., Ragnarsson I., Gradual loss of collectivity in rotational bands in the $A \sim 110$ ($Z \geqslant 50$, $N \sim 60$) mass region, Nucl. Phys., A 591 (1995) 387-420.

[12]. Afanasjev A.V., Fossan D.B., Lane G.J. and Ragnarsson I., Termination of rotational bands: disappearance of quantum many-body collectivity, Phys. Rep., 322 (1999) 1-124.

[13]. Ragnarsson I., Configuration-constrained cranked Nilsson-Strutinsky description of the band structure at very hign spin, Proceedings of the 2nd International Balkan School on Nuclear Physics, Bodrum, Sept. 12-19, 2000, Balkan Physics Letters, (Special Issue/2000) 1-22.

[14]. Ragnarsson I., Carlsson G., Calculating high-spin states using the configuration-dependent cranked Nilsson-Strutinsky (CNS) formalism, Division of Mathematical Physics, Lund Institute of Technology, Lund, Sweden, (2012) 1- 47.

[15]. Ragnarsson I., Carlsson G., Plotting observed high-spin states using the spinmass program, Division of Mathematical Physics, Lund Institute of Technology, Lund, Sweden, (2012) 1- 13.

[16]. Hill D.L., Wheeler J.A., Nuclear constitution and the interpretation of fission phenomena, Phys. Rev., 89 (1953) 1102.

[17]. Nilsson S.G., Binding States of Individual Nucleons in Strongly Deformed Nuclei, Det KongeligeDanske Videnskabernes Selskab Matematisk-fysiske Meddelelser, 29- 16 (1955) 1- 69.

[18]. Ring P. and Schuck P.,The nuclear many body problem. ed. New York: Spring-Verlag, 1980; pp 1- 95.

[19]. Nilsson S. G., Ragnarsson I., Shapes and shells in nuclear structures. Cambridge University Press, 2005.