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Statistical Estimation for the Weibull Distribution Under Progressive Type-I Interval Censoring with Part-Time Operator

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Abstract: In this study, a new life testing method called "progressive type-I interval censoring with part time operator" is introduced. We obtained the maximum likelihood estimators for parameters of Weibull distribution. A simulation study is conducted to investigate the mean biases, mean variances and MSE (Mean Squared Error)'s of estimates. The results are compared for sampling progressive type-I interval censored data, part-time operator and complete data.

***Keywords*:** Interval censoring, Part-Time operator, Weibull distribution, Maximum likelihood estimation

Part-Time Operatörlü İlerleyen Tür Tip-I Aralık Sansürleme Altında Weibull Dağılımı için İstatistiksel Tahmin

Özet: Bu çalışmada yeni bir yaşam testi metodu olan yarı-zamanlı operatörlü ilerleyen tür tip-I aralık sansürleme tanıtılmıştır. Weibull dağılımı parametreleri için en çok olabilirlik tahminleri elde edilmiştir. Tahminlere ait ortalama yan değerlerini, ortalama varyanslarını ve hata kareler ortalamasını (MSE) araştırmak için simülasyon çalışması yapıldı. Elde edilen sonuçlar, ilerleyen tür tip-I aralık sansürlü örneklem, part-time operatörlü sansürlü örneklem ve tam örneklem durumları için karşılaştırılmıştır.

***Anahtar Kelimeler*:** Aralık sansürleme, Part-Time operatörü, Weibull dağılımı, En çok olabilirlik tahmini

1. INTRODUCTION

In the classical life testing, experimenter must stay on the test from beginning to end continuously. Unfortunately, this can be difficult in some life test for the experimenter. In this context, the progressive type-I interval censoring scheme has been introduced. Aggarwala [1], proposed a combination of type-I interval censoring and progressive censoring and called it as progressive type-I interval censoring. In this scheme, experimenter only record whether a test unit fails in an interval instead of measuring failure time exactly.

In some situations, it is often impossible continuously to observe the testing process and hence, the failure times of test units cannot be recorded exactly. That is, one can only record whether a test unit fails in a time interval instead of measuring lifetime exactly. Thus, the test units are inspected intermittently. This type of inspection is called interval-censoring. In the literature, interval-censored data have been studied

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by many researchers such as Cohen [2], Cheng and Chen [3], Tse and Yuen [4], Yang and Tse [5] and Lu and Tsai [6].

In recent years, progressive interval-censoring scheme has received the attention of many researchers. Some important literature can be found. Aggarwala [1] introduced progressive type-I interval censoring and obtained classical estimates for unknown parameter of exponential distribution. Xiang and Tse [7] discussed the maximum likelihood estimation of the model parameters and derive the corresponding asymptotic variances when survival times are assumed to be Weibull distributed. Wuand Changand Liao and Huang [8] provided a sampling plan for progressively type I interval censored life tests when the lifetime followed the exponential distribution. They used the maximum likelihood method to obtain the point estimation of the parameter of failure time distribution. Ng and Wang [9] used maximum likelihood method to obtained the parameters estimation for Weibull distribution under progressive type-I interval censoring. Chen and Lio [10] obtained parameter estimations for generalized exponential distribution under progressive type-I interval censoring. Statistical inference under progressive type-I interval censoring has been discussed by Yang and Tse [5] for planning accelerated life tests, Ashour and Afify [11] for exponentiated Weibull family, Yangand Fan and Tse [12] investigated optimal design of life tests for log-normal distribution, Singh and Tripathi [13], for inverse Weibull distribution. Bayesian inference and model selection has been studied by Singhand Singh and Kumar [14] for exponentiated gamma distribution, Lin and Lio [15] for generalized exponential and Weibull distributions, Peng and Yan [16] for generalized exponential distribution, Singhand Singh and Sharma [17] for generalized Lindley distribution, Kaushikand Singh and Singh [18] for Weibull distribution with beta-binomial removals.

In this paper, we introduce a new modification of progressive type-I interval censoring. This new scheme is constructed on the idea of part-time working. Suppose that n units are simultaneously placed on a life test at time $t_0 = 0$ and number of interval k is even. The experimenter will observe the exact times of the d_1 failed units until t_1 and will randomly remove r_1 surviving units from the test at time t_1. Experimenter will have a break to rest from t_1 to t_2. At the time t_2, the experimenter will come back to test and count the number of failed units (d_2) between (t_1, t_2) and will randomly remove r_2 surviving units from the test. The experimenter will observe the exact times of the d_3 failed units from t_2 to t_3 and will randomly remove r_3 surviving units from the test at time t_3. Experimenter will have a break to rest from t_3 to t_4. At the time t_4, the experimenter will come back to test and count the number of failed units (d_4) between (t_3, t_4) and will randomly remove r_4 surviving units from the test. And so on until t_k. Thus, $\{d_i, r_i, x_j;\ i = 1, 2, \ldots, k;\ j = 1, 2, ..., d_1 + d_3 + ... + d_{k-1}\}$ are the observed data. Note that the number of failed units d_i and the number of removed units r_i and x_j are random variables. In general, the values of r_i, $i = 1, 2, ..., k$, be computed by the pre-specified percentages of the remaining live units $p_1, p_{2,\ldots,} p_k$ (with $p_k = 1$). That is $r_i = [|(m_i - d_i)p_i|]$, where $m_i = n - \sum_{j=1}^{i-1} d_j - \sum_{j=1}^{i-1} r_j$, $i = 1, 2, ..., k$, are the number of non-surviving units at the beginning of the i th stage.

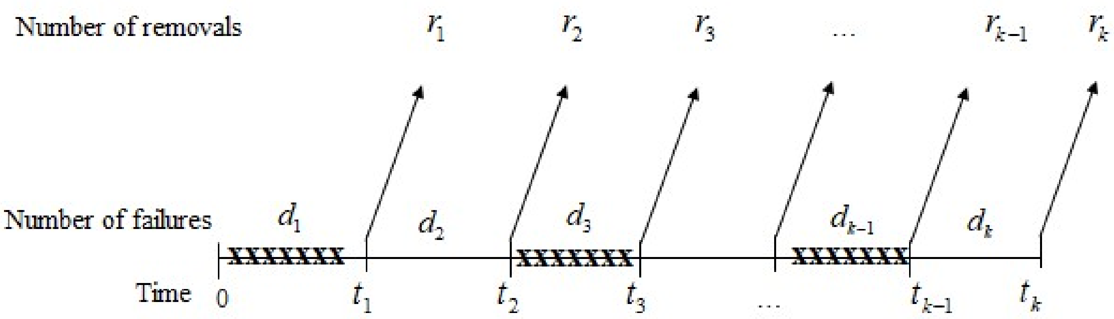

Figure 1. *k*-level progressive type-I interval censored sample with part time operator.

Let k is even. The likelihood function of θ under progressive type-I interval censoring sample with part time operator is given by

$$L(\boldsymbol{\theta})\ \alpha\ \underbrace{f(x_1)f(x_2)...f(x_{d_1})[1-F(t_1,\boldsymbol{\theta})]^{r_1}}_{inspection\ 1}\times\underbrace{[F(t_2,\boldsymbol{\theta})-F(t_1,\boldsymbol{\theta})]^{d_2}[1-F(t_2,\boldsymbol{\theta})]^{r_2}}_{inspection\ 2}$$

$$\times\underbrace{f(x_{d_1+1})f(x_{d_1+2})...f(x_{d_1+d_3})[1-F(t_3,\boldsymbol{\theta})]^{r_3}}_{inspection\ 3}\times\underbrace{[F(t_4,\boldsymbol{\theta})-F(t_3,\boldsymbol{\theta})]^{d_4}[1-F(t_4,\boldsymbol{\theta})]^{r_4}}_{inspection\ 4}$$

$$\times...\times\underbrace{[F(t_k,\boldsymbol{\theta})-F(t_{k-1},\boldsymbol{\theta})]^{d_k}[1-F(d_k,\boldsymbol{\theta})]^{r_k}}_{inspection\ k}$$

(1.1)

We can simplify the equation (1.1) as follows:

$$L(\boldsymbol{\theta})\ \alpha\left[\prod_{i=1}^{d_1+d_3+...+d_{k-1}} f(x_i)\right]\left[\prod_{i=1}^{k/2}[F(t_{2i},\boldsymbol{\theta})-F(t_{2i-1},\boldsymbol{\theta})]^{d_{2i}}\right]\left[\prod_{i=1}^{k}[1-F(t_i,\boldsymbol{\theta})]^{r_i}\right] \quad (1.2)$$

2. MODEL DESCRIPTION and PARAMETER ESTIMATION

Let X be a random variable from Weibull distribution. The probability density function (pdf) and cumulative distribution function (cdf) are

$$f(x;\lambda,\beta)=\frac{\beta}{\lambda^{\beta}}x^{\beta-1}e^{-(x/\lambda)^{\beta}}, \qquad x>0$$

$$F(x;\lambda,\beta)=1-e^{-(x/\lambda)^{\beta}}, \qquad x>0 \tag{2.1}$$

respectively, where $\lambda>0$ *and* $\beta>0$ are parameters [18].

2.1. Progressive Type-I Interval Censoring

Given a progressively type-I interval censored sample, $[q_i,r_i,t_i]$, $i=1,2,...,k$, of size *n*, from a continuous lifetime distribution with distribution function, $F(t,\boldsymbol{\theta})$, where θ is the parameter vector, the likelihood function can be constructed as follows [1].

$$\begin{aligned} L(\boldsymbol{\theta})\ \alpha\ & \underbrace{[F(t_1;\boldsymbol{\theta})-F(t_0;\boldsymbol{\theta})]^{q_1}[1-F(t_1;\boldsymbol{\theta})]^{r_1}}_{inspection\ 1} \\ & \times\underbrace{[F(t_2;\boldsymbol{\theta})-F(t_1;\boldsymbol{\theta})]^{q_2}[1-F(t_2;\boldsymbol{\theta})]^{r_2}}_{inspection\ 2} \\ & \times\ldots\times\underbrace{[F(t_{k-1};\boldsymbol{\theta})-F(t_{k-2};\boldsymbol{\theta})]^{q_{k-1}}[1-F(t_{k-1};\boldsymbol{\theta})]^{r_{k-1}}}_{inspection\ (k-1)} \\ & \times\underbrace{[F(t_k;\boldsymbol{\theta})-F(t_{k-1};\boldsymbol{\theta})]^{q_k}[1-F(t_k;\boldsymbol{\theta})]^{r_k}}_{inspection\ k} \end{aligned} \tag{2.2}$$

We can simplify the equation (2.2) as follows:

$$L(\boldsymbol{\theta})\ \alpha\ \left[\prod_{i=1}^{k}[F(\tau_i,\boldsymbol{\theta})-F(t_{i-1},\boldsymbol{\theta})]^{n_i}\right][1-F(\tau_i,\boldsymbol{\theta})]^{r_i} \tag{2.3}$$

2.2. Maximum Likelihood Method

Given a progressively type-I interval with part time operator censored sample, $Z=\left\{d_i,r_i,x_j,t_i;\ i=1,2,...,k;\ j=1,2,...,(d_1+d_3+...+d_{k-1})\right\}$ of size *n,* from a Weibull distribution with parameter λ and β. The likelihood is given Equation (1.2). Progressive type-I interval censored sample with part time operator sample log-likelihood function can be written as

$$\ell(\boldsymbol{\theta})=\left[(d_1+d_3+\ldots+d_{k-1})\ln\left(\frac{\beta}{\lambda}\right)\right]+\left[\left(\sum_{i=1}^{d_1+d_3+\ldots+d_{k-1}}(\beta-1)\ln\left(\frac{x_i}{\lambda}\right)-\left(\frac{x_i}{\lambda}\right)^{\beta}\right)\right]$$
$$+\left[\sum_{i=1}^{2}d_{2i}\ln\left(e^{-\left(\frac{t_{2i-1}}{\lambda}\right)^{\beta}}-e^{-\left(\frac{t_{2i}}{\lambda}\right)^{\beta}}\right)^{d_{2i}}\right]-\left[\left(\sum_{i=1}^{k}r_i\left(\frac{t_i}{\lambda}\right)^{\beta}\right)\right] \quad (2.4)$$

The MLEs of λ and β can be obtained respectively.

$$\frac{\partial\log\ell(\boldsymbol{\theta})}{\partial\lambda}=\left[-\frac{(\mathrm{d}_1+\mathrm{d}_3+\ldots+\mathrm{d}_{\mathrm{k}-1})}{\lambda}-\frac{(\mathrm{d}_1+\mathrm{d}_3+\ldots+\mathrm{d}_{\mathrm{k}-1})(\beta-1)}{\lambda}\right.$$
$$-\sum_{\mathrm{i}=1}^{\mathrm{d}_1+\mathrm{d}_3+\ldots+\mathrm{d}_{\mathrm{k}-1}}\left(-\frac{\left(\frac{\mathrm{x_i}}{\lambda}\right)^{\beta}}{\lambda}\right)+\sum_{\mathrm{i}=1}^{\mathrm{k}/2}\mathrm{d}_{2\mathrm{i}}\frac{\left(\frac{\mathrm{t}_{2\mathrm{i}-1}}{\lambda}\right)^{\beta}\frac{\beta}{\lambda}\mathrm{e}^{-\left(\frac{\mathrm{t}_{2\mathrm{i}-1}}{\lambda}\right)^{\beta}}-\left(\frac{\mathrm{t}_{2\mathrm{i}}}{\lambda}\right)^{\beta}\frac{\beta}{\lambda}\mathrm{e}^{-\left(\frac{\mathrm{t}_{2\mathrm{i}}}{\lambda}\right)^{\beta}}}{\mathrm{e}^{-\left(\frac{\mathrm{t}_{2\mathrm{i}-1}}{\lambda}\right)^{\beta}}-\mathrm{e}^{-\left(\frac{\mathrm{t}_{2\mathrm{i}}}{\lambda}\right)^{\beta}}}=0 \quad (2.5)$$
$$\left.+\sum_{\mathrm{i}=1}^{\mathrm{k}}\mathrm{r_i}\left(\frac{\mathrm{t_i}}{\lambda}\right)^{\beta}\left(\frac{\beta}{\lambda}\right)\right]$$

and

$$\frac{\partial\log\ell(\boldsymbol{\theta})}{\partial\beta}=\left[\frac{(d_1+d_3+\ldots+d_{k-1})}{\beta}+\sum_{i=1}^{d_1+d_3+\ldots+d_{k-1}}\ln\left(\frac{x_i}{\lambda}\right)-\left(\frac{x_i}{\lambda}\right)^{\beta}\ln\left(\frac{x_i}{\lambda}\right)\right.$$
$$+\sum_{i=1}^{k/2}d_{2i}\frac{-\left(\frac{t_{2i-1}}{\lambda}\right)^{\beta}\ln\left(\frac{t_{2i-1}}{\lambda}\right)e^{-\left(\frac{t_{2i-1}}{\lambda}\right)^{\beta}}+\left(\frac{t_{2i}}{\lambda}\right)^{\beta}\ln\left(\frac{t_{2i}}{\lambda}\right)e^{-\left(\frac{t_{2i}}{\lambda}\right)^{\beta}}}{e^{-\left(\frac{t_{2i-1}}{\lambda}\right)^{\beta}}-e^{-\left(\frac{t_{2i}}{\lambda}\right)^{\beta}}}=0 \quad (2.6)$$
$$\left.+\sum_{i=1}^{k}r_i\left(\frac{t_i}{\lambda}\right)^{\beta}\ln\left(\frac{t_i}{\lambda}\right)\right]$$

By settings the derivates of the log-likelihood function with respective to λ and β to zero, the solutions for MLEs of λ and β can be obtained as follows. But there is no closed form of the solution and the MLEs of λ and β must be derived numerically. Newton-Raphson algorithm is one of the standard methods to determine the MLEs of the parameters.

3. SIMULATION STUDY

In this section, maximum likelihood estimation method simulation study results are given.

3.1. Maximum Likelihood Estimation Method

In this section, three schemes are compared, the first scheme is complete data, the second scheme is progressive type-I interval censoring data and the last one that is our new scheme is progressive type-I interval censored sampling with part time operator. Some simulations are made to get the bias, variance and MSEs of the estimates for Weibull parameters based on under three schemes.

Table 1. Biases, Variances and MSE's for the parameters with complete, part-time operator and interval censoring data.

$\lambda=2,\beta=1$				**p=0.1 and 10,000 repetitions**							
				Mean Estimations		Mean Biases		Mean Var.		MSE	
n	k	t	Methods	$\hat{\lambda}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\beta}$
50	4	0.25	Complete	1.9983	1.0294	-0.0017	0.0294	0.0860	0.0140	0.0860	0.0149
			Part-Time	2.0898	1.0630	0.0898	0.0630	0.4723	0.0597	0.4804	0.0637
			Interval	2.2238	1.0474	0.2238	0.0474	0.9613	0.0813	1.0113	0.0835
50	6	0.25	Complete	2.0019	1.0277	0.0019	0.0277	0.0865	0.0136	0.0864	0.0144
			Part-Time	2.0772	1.0400	0.0772	0.0400	0.3005	0.0414	0.3064	0.0430
			Interval	2.1264	1.0291	0.1264	0.0291	0.4260	0.0530	0.4419	0.0538
50	4	0.5	Complete	2.0100	1.0316	0.0100	0.0316	0.0888	0.0142	0.0888	0.0152
			Part-Time	2.0613	1.0329	0.0613	0.0329	0.1936	0.0315	0.1974	0.0326
			Interval	2.0883	1.0222	0.0883	0.0222	0.2425	0.0483	0.2502	0.0487
50	6	0.5	Complete	2.0127	1.0291	0.0127	0.0291	0.0899	0.0139	0.0900	0.0147
			Part-Time	2.0405	1.0280	0.0405	0.0280	0.1374	0.0241	0.1391	0.0248
			Interval	2.0443	1.0230	0.0443	0.0230	0.1454	0.0332	0.1473	0.0337
50	4	0.75	Complete	2.0062	1.0295	0.0062	0.0295	0.0889	0.0137	0.0889	0.0146
			Part-Time	2.0258	1.0304	0.0258	0.0304	0.1225	0.0233	0.1231	0.0242
			Interval	2.0279	1.0253	0.0279	0.0253	0.1308	0.0385	0.1316	0.0392
50	6	0.75	Complete	2.0081	1.0291	0.0081	0.0291	0.0864	0.0138	0.0865	0.0146
			Part-Time	2.0206	1.0255	0.0206	0.0255	0.1045	0.0187	0.1049	0.0193
			Interval	2.0165	1.0234	0.0165	0.0234	0.1079	0.0285	0.1082	0.0290
100	4	0.25	Complete	2.0036	1.0129	0.0036	0.0129	0.0444	0.0064	0.0444	0.0065
			Part-Time	2.0634	1.0275	0.0634	0.0275	0.2535	0.0270	0.2575	0.0277
			Interval	2.1246	1.0190	0.1246	0.0190	0.4043	0.0380	0.4198	0.0384
100	6	0.25	Complete	2.0032	1.0137	0.0032	0.0137	0.0441	0.0065	0.0441	0.0067
			Part-Time	2.0423	1.0210	0.0423	0.0210	0.1494	0.0200	0.1511	0.0204
			Interval	2.0608	1.0164	0.0608	0.0164	0.1764	0.0255	0.1801	0.0258

100	4	0.5	Complete	2.0030	1.0130	0.0030	0.0130	0.0443	0.0064	0.0443	0.0066
			Part-Time	2.0271	1.0144	0.0271	0.0144	0.0882	0.0148	0.0889	0.0150
			Interval	2.0398	1.0080	0.0398	0.0080	0.0998	0.0232	0.1013	0.0232
100	6	0.5	Complete	2.0054	1.0131	0.0054	0.0131	0.0443	0.0065	0.0444	0.0067
			Part-Time	2.0203	1.0121	0.0203	0.0121	0.0659	0.0117	0.0663	0.0118
			Interval	2.0218	1.0088	0.0218	0.0088	0.0675	0.0161	0.0680	0.0162
100	4	0.75	Complete	2.0032	1.0140	0.0032	0.0140	0.0454	0.0065	0.0454	0.0067
			Part-Time	2.0122	1.0141	0.0122	0.0141	0.0602	0.0110	0.0604	0.0112
			Interval	2.0122	1.0115	0.0122	0.0115	0.0619	0.0187	0.0621	0.0188
100	6	0.75	Complete	2.0019	1.0149	0.0019	0.0149	0.0444	0.0064	0.0444	0.0066
			Part-Time	2.0075	1.0140	0.0075	0.0140	0.0528	0.0089	0.0529	0.0090
			Interval	2.0055	1.0132	0.0055	0.0132	0.0540	0.0137	0.0540	0.0139
250	4	0.25	Complete	2.0035	1.0055	0.0035	0.0055	0.0174	0.0025	0.0174	0.0025
			Part-Time	2.0301	1.0105	0.0301	0.0105	0.0900	0.0103	0.0909	0.0104
			Interval	2.0523	1.0067	0.0523	0.0067	0.1195	0.0151	0.1222	0.0151
250	6	0.25	Complete	2.0014	1.0057	0.0014	0.0057	0.0182	0.0025	0.0182	0.0025
			Part-Time	2.0212	1.0065	0.0212	0.0065	0.0537	0.0075	0.0541	0.0076
			Interval	2.0304	1.0030	0.0304	0.0030	0.0602	0.0100	0.0611	0.0100
250	4	0.5	Complete	2.0015	1.0064	0.0015	0.0064	0.0174	0.0025	0.0174	0.0025
			Part-Time	2.0102	1.0070	0.0102	0.0070	0.0317	0.0056	0.0318	0.0056
			Interval	2.0135	1.0055	0.0135	0.0055	0.0334	0.0088	0.0336	0.0089
250	6	0.5	Complete	2.0008	1.0064	0.0008	0.0064	0.0178	0.0025	0.0178	0.0025
			Part-Time	2.0057	1.0068	0.0057	0.0068	0.0258	0.0045	0.0258	0.0046
			Interval	2.0065	1.0051	0.0065	0.0051	0.0261	0.0064	0.0262	0.0064
250	4	0.75	Complete	2.0028	1.0064	0.0028	0.0064	0.0177	0.0025	0.0177	0.0025
			Part-Time	2.0078	1.0062	0.0078	0.0062	0.0232	0.0042	0.0233	0.0042
			Interval	2.0081	1.0048	0.0081	0.0048	0.0235	0.0071	0.0236	0.0071
250	6	0.75	Complete	2.0042	1.0059	0.0042	0.0059	0.0181	0.0025	0.0181	0.0025
			Part-Time	2.0058	1.0058	0.0058	0.0058	0.0209	0.0035	0.0209	0.0036
			Interval	2.0053	1.0057	0.0053	0.0057	0.0213	0.0054	0.0213	0.0055
500	4	0.25	Complete	2.0024	1.0029	0.0024	0.0029	0.0089	0.0012	0.0089	0.0012
			Part-Time	2.0121	1.0065	0.0121	0.0065	0.0423	0.0051	0.0425	0.0051
			Interval	2.0220	1.0048	0.0220	0.0048	0.0525	0.0074	0.0529	0.0075
500	6	0.25	Complete	2.0002	1.0031	0.0002	0.0031	0.0090	0.0012	0.0090	0.0012
			Part-Time	2.0073	1.0044	0.0073	0.0044	0.0259	0.0038	0.0260	0.0038
			Interval	2.0106	1.0035	0.0106	0.0035	0.0283	0.0050	0.0284	0.0050
500	4	0.5	Complete	2.0009	1.0028	0.0009	0.0028	0.0089	0.0012	0.0089	0.0012
			Part-Time	2.0053	1.0032	0.0053	0.0032	0.0159	0.0028	0.0159	0.0028

			Interval	2.0071	1.0023	0.0071	0.0023	0.0166	0.0043	0.0166	0.0043
500	6	0.5	Complete	2.0009	1.0023	0.0009	0.0023	0.0090	0.0012	0.0090	0.0012
			Part-Time	2.0039	1.0019	0.0039	0.0019	0.0125	0.0022	0.0126	0.0022
			Interval	2.0042	1.0012	0.0042	0.0012	0.0126	0.0031	0.0126	0.0031
500	4	0.75	Complete	2.0012	1.0026	0.0012	0.0026	0.0088	0.0012	0.0088	0.0012
			Part-Time	2.0037	1.0025	0.0037	0.0025	0.0117	0.0021	0.0117	0.0021
			Interval	2.0037	1.0008	0.0037	0.0008	0.0118	0.0037	0.0119	0.0037
500	6	0.75	Complete	2.0020	1.0033	0.0020	0.0033	0.0091	0.0012	0.0091	0.0012
			Part-Time	2.0036	1.0028	0.0036	0.0028	0.0107	0.0017	0.0107	0.0018
			Interval	2.0031	1.0025	0.0031	0.0025	0.0109	0.0027	0.0109	0.0027

According to Table 1, biases, variances and MSE's of estimates are decreased when sample size (n) is increase in all sampling schemes. In progressive type-I interval censoring with part time operator and progressive type-I interval censoring variances and MSE's of estimates are also decreased when t is increase. When k increases, the variances and MSE's of estimates are decreased in part-time operator and interval censoring sample. The important simulation results are progressive type-I interval censored sampling with part time operator's estimator biases, variances and MSE's generally smaller than progressive type-I interval censored estimators for all sampling size, number of interval and interval range.

4. CONCLUSION

In this paper, we have introduced a new life testing method that is called "progressive type-I interval censoring with part time operator". This method is a new modification of progressive interval censoring and new scheme is constructed on the idea of part-time working. We have obtained maximum likelihood estimators for Weibull distribution under progressive type-I interval censoring, progressive type-I interval censoring with part-time operator and complete data. A simulation study has conducted to investigate the bias, variance and MSE of estimates. According to the simulation results, progressive type-I interval censored sampling with part time operator's estimator's biases, variances and MSE's smaller than progressive type-I interval censored estimators. According to these results, progressive type-I interval censoring with part time operator's performances better than progressive type-I interval censoring. At the same time simulation study indicated that progressive type-I interval censored sampling with part time operator's estimator's mean biases, mean variances and MSE's have taken very close values to complete data. Thus, under appropriate conditions it is better to use this new model.

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