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Modeling Long Memory Volatilities of Nigeria Selected Macro Economic Variables with Arfima and Arfima Figarch

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ABSTRACT

The research delved into analysing the stochastic characteristics of Nigeria's Real GDP, the exchange rate of the Naira to US Dollar, and the inflation rate employing Autoregressive fractionally integrated moving average (ARFIMA) and the Autoregressive Fractionally Integrated Moving Average Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) modelling approach. The ability of the hybrid formation of ARFIMA-FIGARCH model with Nigeria macroeconomic variables in modeling the periodicity of long memory volatilities was examined. ARIMA GARCH method of modeling was also employed in analyzing the volatilities of Nigeria selected macroeconomic variables to enrich the study. The efficiency of ARFIMA, ARFIMA FIGARCH and ARIMA GARCH models were evaluated with the forecast evaluation measurements. Results revealed that ARFIMA FIGARCH and ARIMA GARCH models are more adequate in modeling the Inflation rate and the exchange rate while ARFIMA present more adequacies in modeling the RGDP. This result revealed evidence of high volatilities in Nigeria Inflation and the exchange rate of Naira to US dollar.

Keywords: Volatilities, Long Memory, Macroeconomic variables, Arfima Figarch.

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Introduction

Improvements in econometrics methods have fetched different tools of exploring the theoretical features of economic variables over time. Analyzing and modelling applied times series from diverse area of application in econometric study is very essential due to their salient features. Recently, there is evolution of phenomenon of modelling and forecasting series with long memory behavior with time varying conditional variance,[1-3] introduced a widely adopted method for analyzing long memory time series data through the utilization of autoregressive fractionally integrated moving average (ARFIMA) processes. The perception of long memory characteristics relates to the interdependence among data points gathered over a period of time. In the study of [4,5], long-term memory descriptions were described as the gradual decrease occurring in the autocorrelation function's graphical representation within a dataset. This phenomenon led to suggestion of applying fractional differencing in mean models when long memory is being identified in the time series data.

Researchers have extensively worked on analyzing significant inferences of long memory returns of modelling financial economic series. Modern researches delved into modelling of long memory in econometrics model, prediction of prices of agricultural products and numerous financial series [3,6,1,7] among others.

However, ARFIMA model is based on the assumptions of linearity, stationarity and homoscedasticity of error variance, ARFIMA model is incompetent in handling dataset that exhibits presence of high volatility, most financial time series portrays features involving high volatilities in unstable phase succeeding stability periods. Autoregressive conditional heteroscedastic (ARCH) model was proposed by [8] to handle cases of volatility in Times series.

Nevertheless, ARCH model has the features of rapid decay in squared residuals of unconditional autocorrelation function when compared to a usual observed values unless there is large extreme lag. The generalized autoregressive conditional heteroskedasticity (GARCH) model was developed by [9,10] with the purpose of handling the evolving pattern of conditional variance, thus addressing the limitations of the ARCH model. The square volatility modelling was assumed to relate to its past values and errors in estimating the parameters involved, GARCH model are independent of one another. Several theoretical and empirical works has been established relating GARCH and its kind.[11,12]. Long memory procedure allows the integration of conditional heteroscedasticity which revealed the presence non-periodic cycles. [13-15] Employed a seasonal ARFIMA model with GARCH errors in analyzing PM concentration. [16,17] explored nonlinear time series with GARCH models and nonetheless, neither the Generalized Autoregressive Conditional Heteroscedasticity nor ARCH process can effectively capture the handling of the presence of long memory in model volatility. Addressing long-term memory effects in variations, incorporating fractional differencing into variance models.

Developed a new notable model named FIGARCH in the realm of long-term memory variance from Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH), a generalized family of GARCH that allows persistency in conditional variance. Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity (FIGARCH) model is mostly used for modeling long memory in time series with volatility. [18] FIGARCH model was introduced mainly to create a more flexible process of estimating and summarizing conditional variance that has dependencies in financial market volatility, the FIGARCH model permits gradual hyperbolic rate of decay for the lagged squared in the conditional variance function. Several studies had been conducted on modelling and forecasting times series model applying concept and relevance of FIGARCH model. FIGARCH model was employed by [2,19] in modeling and predicting the effect of long-range relationship long memory patterns in conditional variance. Applicability of FIGARCH in other field were well spelt out by researchers, [20,21] introduced a FIGARCH model with seasonality, the work gives room for examining both periodic patterns and long memory comportments in conditional variance.[22] ascertained that FIGARCH model operates in the opposite direction as that of ARFIMA in persistency, as the fractional parameters parameter approaches zero, the memory of the process rises. ARFIMA-FIGARCH model is a connection between ARFIMA model and FIGRACH model [23]. Several studies show the significant evidences of long memory model in series that exhibits volatilities through the use of ARFIMA-FIGARCH [24,25] among others.

A number of recent studies suggest that most macroeconomic variables exhibit long-range dependence followed by the periods of instability in volatility and should be modelled as a fractionally integrated process [26-28] among others. Although long-memory models have gained popularity in modeling and forecasting future series but there are limited studies explored the possibility of long-period dependence and volatilities of macro-economic variables in Nigeria. Modeling time series volatility aids in improving parameter estimation efficiency and forecast accuracy [29].

However, this study aims to leverage the combination of ARFIMA-FIGARCH models to capture both the mean and long memory aspects of volatility within combined data samples. The research endeavors to explore the empirical stochastic properties of the inflation rate, exchange rate, and Nigeria's Real GDP using both the autoregressive fractionally integrated moving average (ARFIMA) and the Autoregressive Fractionally Integrated Moving Average Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) modeling approaches.

Moreover, this work investigates the capability of the ARFIMA-FIGARCH model using Nigeria macroeconomic variables to estimate the long memory volatilities. The model efficiency is then evaluated with the forecast evaluation measurements.

Material and Method

ARFIMA Model Process

The Autoregressive Fractionally Integrated Moving Average (ARFIMA) process is stated as:

$$\gamma(U)\,(1-U)^d X_t = \vartheta(U)\varepsilon_t \tag{1}$$

U is defined as the lag operator such that

$$UX_t = UX_{t-1} \tag{2}$$

and the $(1U)^d$ fractional difference operator replaced the usual standard difference operator $(1-U)$ of a short memory ARIMA process, d is a non-integer parameter that represent the level of the fractional difference. ε_t is independently and identically distributed with mean 0 and variance σ^2, $\gamma(U)$ and $\vartheta(U)$ signify AR and MA components respectively. The method is covariance stationary for the interval of d lying between - 0.5 and 0.5; which involve mean reversion when d is less than 1. [29],[4] and [5]) generalized process of ARFIMA as the fractional white-noise process Where $\gamma(U)$ is established to equal to unity to further analyze the features of the method. Following the fact that many time series steadily exhibits slow decay autocorrelations process, the possibility virtue of exploiting ARFIMA process with decay hyperbolic autocorrelation patterns in financial time series modeling are numerous compared to modelling the process of ARMA model that have either geometric exponential decay.

$$\begin{aligned}(1-U)^d &= \sum_{k=0}^{\infty}(-1)^k\binom{d}{k}(U)^k \\ &= 1-dU \\ &+ \frac{d(d-1)}{2!}U^2\frac{d(d-1)(d-2)}{3!}U^3+,\dots,+\sum_{k=0}^{\infty}C_k(d)\end{aligned} \tag{3}$$

for any d>-1. When d = 0, equation (3) above reduces to the classical ARMA(p,q) model, following the expression in equation (3), obvious significance of the hyperbolic features is shown.

Long Memory Test

Testing whether the observed data series exhibits long memory behavior is a prior process to method of estimating ARFIMA models. The techniques of Hurst Exponent will be employed in checking whether the data conforms to long memory structures.

Hurst Exponent

The Hurst exponent is one of the time series long-memory family. The long memory structure happens when the values of H fall in the interval 0.5 < H < 1. The Hurst exponent estimation process uses the formula;

$$H = \frac{\log\left(\frac{R}{S}\right)}{\log(N)} \quad (4)$$

N signifies length of the sample data, R is the range, S is the standard deviation and $\frac{R}{S}$ is the matching value of the rescaled evaluation.

ARFIMA process estimation

The estimations of d are usually done in frequency domain. ARFIMA estimators of d are generally categorized into semi parametric and parametric forms.

This research employed the approaches of the Hurst exponent and semi-parametric approaches of Geweke and Porter–Hudak (GPH) methods to test and estimate long memory parameters, using the following regression,

$$\ln(w_k) = U - dln[4sin^2(w_k/2)] + n_k \quad (5)$$

where $w_k = \frac{2nk}{T}, k = 1,2,\ldots,n,\ n_k$ is the white noise term and w_k represent the Fourier frequencies.

The periodogram of a time series a_1 is $I(w_k)$ defined as;

$$I(w_k) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} a_1 e^{-w_k t}\right|^2 \quad (6)$$

GARCH Model

The variance equation of GARCH(u,v) model can expressed as;

$$\sigma_t^2 = \omega + \sum_{t=1}^{u} \alpha_i \varepsilon_{t-1}^2 + \sum_{t=1}^{v} \beta_j \sigma_{t-j}^2 \quad (7)$$

The σ_t^2 are the model parameter to be estimated according to GARCH(u,v) models where $\omega > 0$, for i= 1…,u and $\beta_j \geq 0$, for j = 1,…,v.

α_i represent the parameter determining the effect of previous residual ε_{t-1}^2while βj measures the effect of change in its lagged value σ_{t-j}^2..

From equation (7), conditional variance $\sigma_{t,}^2$, ε_t at time t dependent on the occurrence of the lagged squared errors in the preceding past periods and also on the conditional variance over the past periods.

In general, [7] has established that the stationarity of GARCH (*u, v*) if there is satisfactory of the following conditions,

$$E(\varepsilon_t) = 0 \quad (8)$$

$$Var(\varepsilon_t) = \frac{\omega}{(1-\alpha(1)-\beta(1))} \quad (9)$$

$Cov(\varepsilon_t, \varepsilon_s), t \neq s,\ provided\ \alpha(1) + \beta(1)$ is less than 1

The Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity (FIGARCH).

The FIGARCH (*u,D,v*); model was introduced by [2]) in depicting long memory in volatility. The effect of shocks on the volatility is not finite. This infinite idea leads to the process of fractional integration in volatility. Fractional difference parameter (*D*) is used to model the persistent behavior of volatility in the FIGARCH model, whereas short term volatility is being considered by usual ARCH and GARCH parameters.

Considering, the typical (GARCH) model defined as;

$$\sigma_t^2 = \omega + \alpha(U)\varepsilon_t^2 + \beta(U)\sigma_t^2 \quad (10)$$

where σ_t^2 and ε_t^2 are conditional and unconditional variances of ε_t respectively. U is the backshift operator and

$$\alpha(U) = \alpha_1 u + \alpha_2 u^2 +, \ldots, \alpha_v u^v$$

$$\beta(U) = \beta_1 u + \beta_2 u^2 +, \ldots, \beta_u u^u$$

To ascertain the stability and covariance stability of the ε_t process, all the roots of 1- $\alpha(U) - \beta(U)$and 1- $\beta(U)$ lies outside the unit circle.

From GARCH (*u,v*) model, the conditional variance of σ_t^2, ε_t^2 depends on the squared innovations in the previous *u* periods, and the conditional variance in the previous *v* periods. Equation (10) can be expressed as;

$$\sigma_t^2 = \omega + \sum_{t=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{t=1}^{q} \beta_j \sigma_{t-j}^2 \quad (11)$$

where $\sigma_{t-j}^2$is the volatility at day $t - j$, ω > 0, αi ≥ 0 for i = 1, … , p, and βj ≥ 0 for j = 1, … , q, are parameters of the model to be estimated.

The GARCH (*u, v*) process in equation [10] can be simplified as an ARMA (r,s) process in ε_t^2 as follows;

$$1 - \alpha(U) - \beta(U)\varepsilon_t^2 = \omega + \beta(U)v_t \quad (12)$$

where r is equivalent to maximum of {u, v} and

$$v_t = \varepsilon_t^2 - \sigma_t^2 = (\epsilon_t^2 - 1)\ \sigma_t^2 \quad (13)$$

The$\{v_t\}$ is known to be the innovations for the conditional variance. $\epsilon_t{}'s$ are uncorrelated with E(ϵ_t)= 0 and var (ϵ_t) is one.

From the integrated GARCH (IGARCH) models of [30] whose unconditional variance does not occur represented in equation (14) below

$$\text{(IGARCH) (p,q)} = \emptyset(U)(1 - U)\varepsilon_t^2 = \omega + [1 - \beta(U)v_t] \quad (14)$$

where

$$(\emptyset(U) = 1\text{-}\ \alpha(U) - \beta(U)(1 - U)^{-1} \quad (15)$$

is of order r-1.

The Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity (FIGARCH) is achieved by substituting first difference in equation (15) above with fractional difference operator $(1-U)^d$ such that;

$$\emptyset(U)(1-U)^d\varepsilon_t^2 = \omega + [1-\beta(U)]v_t \qquad (16)$$

If d=0, the FIGARCH (u, D, v) process reduces to a GARCH (u, v) process and if d=1, the FIGARCH process becomes an integrated GARCH process. Rearranging the terms in Eq.(16), the FIGARCH model can be simplified as;

$$1-\beta(U)\sigma_t^2 = \omega + [1-\beta(U)]\,(1-U)^d\varepsilon_t^2 \qquad (17)$$

ARFIMA- FIGARCH Model

The ARFIMA-FIGARCH model is employed to simultaneously investigate the long memory and volatility characteristics of a time series. The ARFIMA (*u,d,v*)-FIGARCH (*U,D.V*) model is a conditional time-dependent variance of the process σ_t^2 specified by the FIGARCH model defined in equation (16). The FIGARCH model propose enhanced flexibility in modeling volatility by enforcing an ARFIMA structure on ε_t^2 yielding a hybrid of ARFIMA FIGARCH model. In this hybrid forms, the two fractional integration parameters *d* and *D* will account for the long-term dynamics of the volatility of the series.

The ARFIMA (*u,d,v*)-FIGARCH (*U,D.V*) model follow this polynomial form

$$\emptyset(1-U)^d\,(r_{t-}\mu) = \theta(U)\varepsilon_t \qquad (18)$$

$$1-\beta(U)(1-U)^D\varepsilon_t^2 = \omega + [1-\alpha(U)v_t] \qquad (19)$$
$$\varepsilon_t = z_t\sigma_t\ z_t \sim N(0.1)$$

μ is an unconditional mean, u and v representing the AR and MA lag orders addressing the short memory, $d\epsilon(0,1)$ represents the long memory in the series; ε_t is a white noise process;

$$\emptyset(U) = 1-\emptyset_1U - \emptyset_2U^2 -,\dots,\emptyset_pU^u \qquad (20)$$

And

$$\theta(U) = 1+\theta_1U + \theta_2U^2 +,\dots,\theta_qU^v \qquad (21)$$

are the AR and MA polynomials, $D\epsilon(0,1)$ measures the degree of volatility persistence; where $\omega$is a constant,

$$\alpha(U) = \alpha_1U+\alpha_2U^2+,\dots,\alpha_qU^v \qquad (22)$$

$$\beta(U) = \beta_1U+\beta_2U^2+,\dots,\beta_pU^u \qquad (23)$$

are the ARCH and GARCH polynomials; v_t represents serially uncorrelated, zero-mean residuals, measured by

$$v_t = \varepsilon_t^2 - \sigma_t^2 \qquad (24)$$

The methodology involved in the work involved the iterative steps of [31] which include identification of the adequate model, parameter estimations of the parameters involved, model diagnostic and forecasting.

THE ARIMA-GARCH Model

The ARIMA-GARCH model is employed to examine trend and volatility of a time series concurrently. ARIMA (p,d,q) and GARCH(u,v) is generally defined as

$$\emptyset(B)(1-U)^dY_t = \emptyset(U)\varepsilon_t \qquad (25)$$

$$\varepsilon_t\,\big|Y_{t-1} \sim N(\mu,\sigma_t^2) \qquad (26)$$

The ARIMA-GARCH method has been established to handle the serial correlated residuals encountered in ARIMA models. ARIMA-GARCH model permits concurrent modeling of both the conditional means and the volatility of the series. Moreover, this method of modeling times series yields more precise estimate values and higher forecast performance compared to ARIMA models.

Test Statistics

Model Identification**:** stationarity and fractional integration modelling of the data involved were evaluated with the Autocorrelation function, partial autocorrelation function, ADF and KPSS at 0.05 level of significant level.

Augmented Dickey Fuller Test of Stationarity: ADF test model is expressed as;

$$\Delta X_t = \alpha X_{t-p} + Y_t\varphi + \beta_1\Delta X_{t-1} + \beta_2\Delta X_{t-2}+,\dots,\beta_p\Delta X_{t-p} \qquad (27)$$

where,

ΔX_t denotes the differenced series

ΔX_{t-p} denotes the immediate past observations.

Y_t signifies the optional exogenous regressor which can be constant or be represented as constant trend

α and φ are parameters needed to be estimated.

$\beta_1,\dots,\beta_p$ signifies the coefficients of the lagged terms.

The ADF test statistic is denoted by

$$t_\alpha = \frac{\hat{\hat{\alpha}}}{S_e(\hat{\hat{\alpha}})} \qquad (28)$$

The test of hypothesis involves;

$H_0{:}\alpha = 0$, it implies that the series contains unit roots

$H_1{:}\alpha < 0$, it implies that the series contains no unit roots.

Decision rule: Reject H_0: if t_α is less than asymptotic critical value

Kwiatkowski-Philips-Schmidt-Shin(KPSS)Test

Considering the following DGP with no linear trend that assumes the null hypothesis of stationarity;

$$y_t = x_t + z_t \tag{29}$$

where

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} +, \dots, \alpha_p x_{t-p} + u_t \tag{30}$$

$u_t \sim iid(0.\sigma^2)$ and z_t is assume to follow a stationary process.

KPSS test statistic is expressed as;

$$KPSS = \frac{1}{N^2}\sum_{n=1}^{N}\frac{s_t^2}{\sigma^2\infty} \tag{31}$$

Where $s_t = \sum_{j=1}^{t}\widehat{m}_j$ with $\widehat{m}_t = x_t - x$ and $\hat{\sigma}_\infty{}^2$ is a long run variance estimator of the stationary Process z_t.

Model selection

The model selection was accomplished implementing the optimum selection criteria by choosing the model with minimum Akaike Information Criteria (AIC), Schwarz Information Criterion (SIC) and Hannan-Quinn Information Criteria (HQIC).

Model estimation

After identification of the best tentative model, Quasi Maximum Likelihood (QMLE) method of estimation will be adopted to estimate the ARFIMA-FIGARCH model that is normally distributed based with the following log-likelihood function:

$$Log\ (\varepsilon_t\ \theta) = -\frac{1}{2}\log N(2\pi) - \frac{1}{2}\sum_{n=1}^{N}\left[\log\sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2}\right] \tag{32}$$

where $\theta' = (\alpha_0, d, \beta_1, \dots, \beta_p, \emptyset_1, \dots, \emptyset_q)$

Model Diagnostics

The white noise, serial correlation and the heteroscedasticity test was examined applying the residual normality test, the Portmanteau test and Autoregressive Conditional Heteroscedasticity Lagrange Multiplier (ARCH-LM) test respectively to validate the adequacy of the selected ARFIMA models. It is actualized by examining the test of the hypothesis of white noise residuals that assumed to be independently distributed.

Employing the methods of [29] the variance of autocorrelation is defined as

$\text{Var}(\rho_k(\varepsilon) = \frac{1}{N(N-2)}$ (N-K), k= 1,2,…, K

And

$$\left(\sqrt{\frac{N-K}{N(N+2)}}\right)^{-1} \qquad \rho_k(\varepsilon) \approx N(0,1)$$

$$Q_{LB} = \left(\left(\sqrt{\frac{N-K}{N(N+2)}}\right)^{-1} \quad \rho_k(\varepsilon)\right)^2 \tag{33}$$

$$= N(N+2)\sum_{k=1}^{K}\frac{[\rho_k(\varepsilon)]^2}{N-K} \quad \approx \chi^2\ (K-1) \tag{34}$$

where K-1 = k-p-q and there is no inclusion of constant term in p+q, N is the sample size and ρ symbolize the autocorrelation coefficient.

Autoregressive Conditional Heteroscedastic-Lagrange Multiplier (ARCH –LM) Test

[8] proposed ARCH-LM test that allows issues of conditional heteroscedasticity in squared residuals, it has the null hypothesis that there is no heteroscedasticity in the model residuals. The test statistic is given by;

$$Q = B(B+2)\sum_{i=1}^{M}\frac{\rho_i}{(M-1)'} \tag{35}$$

the Q statistic is an asymptotic χ^2distribution that has n degrees of freedom with uncorrelated squared residuals, B is the number of observation and the sample correlation coefficient between squared residuals $\hat{\varepsilon}_t^2$ and $\hat{\varepsilon}_{t-1}^2$is denoted by ρ_i.

Model Forecasting and Performance Evaluation

Validation criterion such as; Akaike Information criteria (AIC), Schwarz Information Criterion (SIC) and Hannan-Quinn Information Criteria (HQIC) were employed for examining and comparing the predicting performances of the selected models

$$AIC = 2T - m \tag{36}$$

$$SIC = 2Tlogn - logm \tag{37}$$

$$HQIC = -2logm + 2Tlogn \tag{38}$$

where T symbolizes the total of estimable parameters, m denotes the maximum likelihood and n is the digits of samples. Moreover, the forecasts accuracy of fitted ARFIMA and ARFIMA FIGARCH model are evaluated in terms of Root Mean Square Error (RMSE), the Mean Absolute Error (MAE) and the Relative Mean Absolute Percentage Error (MAPE) respectively.

MAE is the absolute value of the difference between the forecasted value and the actual value. It calculates the average absolute deviation of predicted values from real values. MAE is estimated as follow:

$$HQIC = -2logm + 2Tlogn \tag{38}$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|\hat{y}_f - y_t\right| \tag{39}$$

RMAPE is projected as the mean absolute percent error for each time period minus real values divided by real values. It computes the percentage of mean absolute error occurred in the model formation. It is stated as follows;

$$RMAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{\hat{y}_f - y_t}{y_t}\right| \times 100\% \quad (40)$$

RMSE illustrate the absolute fit of the model to the observed data, it is computed as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{y}_f - y_t)^2} \quad (41)$$

Where: $\hat{y}_f$ and y_t are the estimated and the real values respectively; n is the sample size. Model with lesser value is likely to have the best precision power of forecast.

Data Collection and Description

The data for this study is a secondary monthly data set from 1970 to 2023 obtained from Nigeria CBN bulletin, it comprised figures of Real Gross Domestic Product (RGDP) per capital, Inflation rate and Exchange rate of Naira-US Dollar.

Results and Discussion

Table 1. Summary Statistics

	RGDP	Inflation Rate	Exchange rate
Mean	173.8996	0.181853	91.78458
Median	73.48	0.1301	21.89526
Maximum	574.18	0.7284	425.9792
Minimum	9.18	0.0346	0.546781
Std. Dev.	175.9127	0.151777	115.8329
Skewness	0.893138	2.010273	1.302508
Kurtosis	2.190781	6.343058	3.872064
Jarque-Bera	8.492408	6.3776556	16.66542
P- value	0.014318	0.0000000	0.000241
Sum Sq. Dev.	1609155	1.197882	697698
Observations	648	648	648

NB: RGDP is Real Gross Domestic Product per capital.

Table 1. gives the summary statistics of monthly average macro-economic data for this study, the series ranges from 1970 to 2023.The total observation is 648, which is large enough for modelling Autoregressive Fractionally Integrated Moving Average models. The series are normally distributed as revealed by the high p-value and low Jarque-Bera test values.

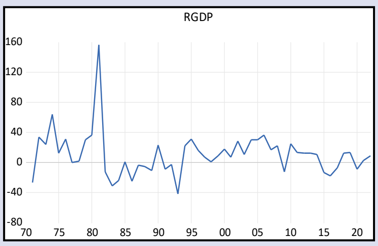


Figure 1. Times series plot of RGDP

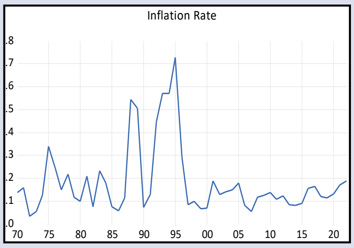


Figure 2. Times series plot of INF. RT

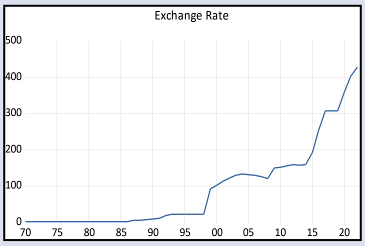


Figure 3. Times series plot of EXC. RT.

Fig 1, 2 and 3 displays times series plots of the average annual series of Nigeria Real Gross Domestic Product (RGDP) per capital, Inflation rate and exchange rate of Naira-US dollar respectively. The plot shows the direction of the series over time.

Table 2. Stationarity test results at level.

	ADF		KPPS	
Variables	ADF Test Stat	Prob.	KPSS Test Stat	Prob.
RGDP	-4.2670	0.0278	0.5196	0.0341
INFLATION RATE	0.1953	0.1374	0.3301	0.0922
EXCHANGE RATE	-2.3203	0.0732	0.6282	0.0638

Table 3. Stationarity test results at First Difference.

	ADF		KPPS	
Variables	ADF Test Stat	Prob.	KPSS Test Stat	Prob.
RGDP	-2.1023	0.008	0.6318	0.000
INFLATION RATE	0.2310	0.002	0.6027	0.036
EXCHANGE RATE	-4.1203	0.000	0.8924	0.091

Table 2 and 3 shows the result of ADF and KPSS test for unit root of Nigeria Real Gross Domestic Product (RGDP) per capital, Inflation rate and exchange rate of Nigeria Naira to US Dollar using the lag length of 12. The outcome of the unit root test for ADF-test of RGDP shows that the variable is stationary at level of 5% level of significant, which implies that RGDP is integrated of order zero i.e., 1(0) while the results of ADF of the inflation and exchange rate indicate that the time series data integrated at I (1). The KPSS tests at both level and first difference are greater than 5% critical values which shows that the series is neither I(0) nor I(1), this warrant needs to carry out fractional difference on the data.

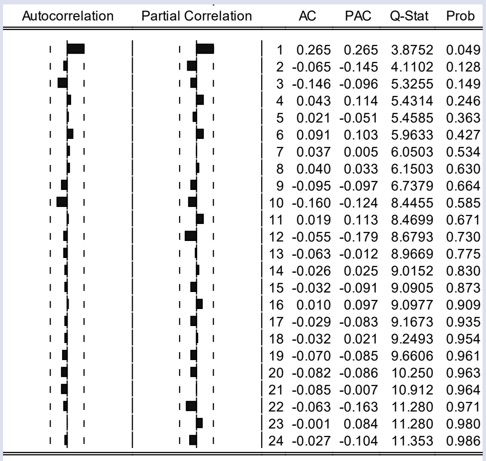
Table 4: Long Memory tests of the ARFIMA models.

	Macroeconomics Variables		
	RGDP	INFLATION RATE	EXCHANGE RATE
HURST.E/RS	0.6302 (0.002)	0.7210 (0.000)	0.6664 (0.008)

Hurst. E/ RS is the Hurst Exponent Rescaled Range.

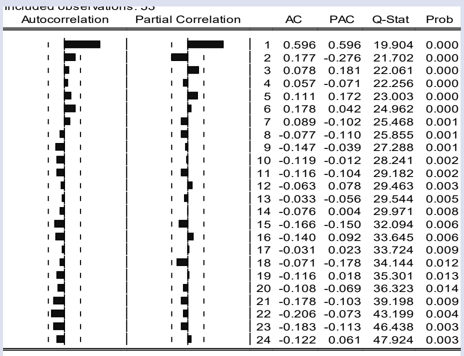
Table 4. above shows the Hurst exponent values of Nigeria RGDP per capital, Inflation and Exchange rate of Nigeria Naira to US Dollar data using the rescaled Range, Table 4 confirmed the existence of long memory of the series under study. Also, the Hurst exponent test gives values in the range of 0 < d < 1.

Fig 4,5 and 6 above displayed the correlogram of Nigeria RGDP per capital, Inflation and Exchange rate of Nigeria Naira to US Dollar respectively. From the correlogram of the series presented in fig 4,5 and 6 above, several speculative ARFIMA models for the variables were fitted to the series.



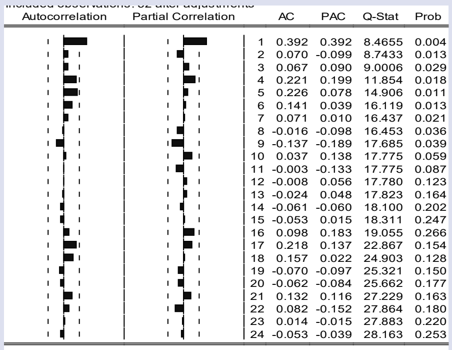
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.265	0.265	3.8752	0.049
		2	-0.065	-0.145	4.1102	0.128
		3	-0.146	-0.096	5.3255	0.149
		4	0.043	0.114	5.4314	0.246
		5	0.021	-0.051	5.4585	0.363
		6	0.091	0.103	5.9633	0.427
		7	0.037	0.005	6.0503	0.534
		8	0.040	0.033	6.1503	0.630
		9	-0.095	-0.097	6.7379	0.664
		10	-0.160	-0.124	8.4455	0.585
		11	0.019	0.113	8.4699	0.671
		12	-0.055	-0.179	8.6793	0.730
		13	-0.063	-0.012	8.9669	0.775
		14	-0.026	0.025	9.0152	0.830
		15	-0.032	-0.091	9.0905	0.873
		16	0.010	0.097	9.0977	0.909
		17	-0.029	-0.083	9.1673	0.935
		18	-0.032	0.021	9.2493	0.954
		19	-0.070	-0.085	9.6606	0.961
		20	-0.082	-0.086	10.250	0.963
		21	-0.085	-0.007	10.912	0.964
		22	-0.063	-0.163	11.280	0.971
		23	-0.001	0.084	11.280	0.980
		24	-0.027	-0.104	11.353	0.986

Figure 4. Correlogram of RGDP



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.596	0.596	19.904	0.000
		2	0.177	-0.276	21.702	0.000
		3	0.078	0.181	22.061	0.000
		4	0.057	-0.071	22.256	0.000
		5	0.111	0.172	23.003	0.000
		6	0.178	0.042	24.962	0.000
		7	0.089	-0.102	25.468	0.001
		8	-0.077	-0.110	25.855	0.001
		9	-0.147	-0.039	27.288	0.001
		10	-0.119	-0.012	28.241	0.002
		11	-0.116	-0.104	29.182	0.002
		12	-0.063	0.078	29.463	0.003
		13	-0.033	-0.056	29.544	0.005
		14	-0.076	0.004	29.971	0.008
		15	-0.166	-0.150	32.094	0.006
		16	-0.140	0.092	33.645	0.006
		17	-0.031	0.023	33.724	0.009
		18	-0.071	-0.178	34.144	0.012
		19	-0.116	0.018	35.301	0.013
		20	-0.108	-0.069	36.323	0.014
		21	-0.178	-0.103	39.198	0.009
		22	-0.206	-0.073	43.199	0.004
		23	-0.183	-0.113	46.438	0.003
		24	-0.122	0.061	47.924	0.003

Figure 5. Correlogram of INF.RT



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.392	0.392	8.4655	0.004
		2	0.070	-0.099	8.7433	0.013
		3	0.067	0.090	9.0006	0.029
		4	0.221	0.199	11.854	0.018
		5	0.226	0.078	14.906	0.011
		6	0.141	0.039	16.119	0.013
		7	0.071	0.010	16.437	0.021
		8	-0.016	-0.098	16.453	0.036
		9	-0.137	-0.189	17.685	0.039
		10	0.037	0.138	17.775	0.059
		11	-0.003	-0.133	17.775	0.087
		12	-0.008	0.056	17.780	0.123
		13	-0.024	0.048	17.823	0.164
		14	-0.061	-0.060	18.100	0.202
		15	-0.053	0.015	18.311	0.247
		16	0.098	0.183	19.055	0.266
		17	0.218	0.137	22.867	0.154
		18	0.157	0.022	24.903	0.128
		19	-0.070	-0.097	25.321	0.150
		20	-0.062	-0.084	25.662	0.177
		21	0.132	0.116	27.229	0.163
		22	0.082	-0.152	27.864	0.180
		23	0.014	-0.015	27.883	0.220
		24	-0.053	-0.039	28.163	0.253

Figure 6. Correlogram of EXC

(RGDP) per capital, INF. RATE and EXCH. RATE is the inflation rate and exchange rate of Nigeria Naira to US Dollar respectively.

Table 5 report the estimates of the fractional difference of Nigeria RGDP per capital, inflation and exchange rate series employing an automatic commencement of integration employing approaches of Geweke and Porter-Hundlak log-periodogram. The competitive estimated models of each series and their respective values for the selection criteria are as tabulated in Table 5. The optimum model for each series is in bold print and asterisk mark for easier identification.

Table 5. ARFIMA Model Identification

MODEL	ARFIMA (p,d,q)	D	AIC	SIC	HQIC
		RGDP			
MODEL 1	ARFIMA 1, d, 1	0.5117	6.393*	6.937*	6.842*
MODEL 2	ARFIMA 1, d, 2	0.1956	7.220	7.479	6.926
MODEL 3	ARFIMA 2, d, 1	0.8945	6.451	6.952	6.970
		INF. RATE			
MODEL 1	ARFIMA 1, d, 1	0.5339	12.002	10.573	10.208
MODEL 2	ARFIMA 1, d, 2	0.7167	13.342	9.175	9.944
MODEL 3	ARFIMA 2, d, 2	0.6802	9.035*	8.884*	9.240*
		EXCH. RATE			
MODEL 1	ARFIMA 1, d, 1	0.6930	4.720*	4.717*	4.509*
MODEL 2	ARFIMA 2, d, 2	0.7760	4.939	4.958	4.820

NB: RGDP is the of Nigeria Real Gross Domestic Product

Table 6: Parameter Estimates of the ARFIMA fitted model.

Parameter	Coefficient	Standard Error	Prob.
		RGDP	
D	0.5117	0.0324	0.0001
γ_1	0.1972	0.0208	0.0000
ϑ_1	0.8287	0.0450	0.0009
		Inflation Rate	
D	0.6802	0.5810	0.0057
γ_1	-0.9567	0.3981	0.0002
γ_2	0.8326	0.3106	0.0028
ϑ_1	0.2798	0.2851	0.0000
ϑ_2	0.2089	0.0495	0.0005
		Exchange Rate	
D	0.6930	0.8314	0.0049
γ_1	1.0038	0.0348	0.0000
ϑ_1	-0.9822	0.3119	0.0000

Diagnostics checking of ARFIMA Models.

Table 7: Statistical tests of the residuals of selected ARFIMA models.

Times series	ARFIMA(p,fd,q)	Autocorrelation Test		Heteroskedacity Test		Normality test	
	Model	Lung Box Q	Portmanteau	Breusch Pagan	White	Jarque Test	Bera Shapiro Wiki
		p- value	p- value	p-value	p. value	p-value	p-value
RGDP	ARFIMA(1,0.5117,1)	0.1923	0.1011	0.1422	0.1721	0.2431	0.2142
Inflation RT	ARFIMA(2, 0.6802,2)	0.2831	0.1829	0.2160	0.3230	0.2206	0.3339
Exchange RT	ARFIMA(1, 0.6930,1)	0.2925	0.2026	0.2542	0.4951	0.3534	0.1474
α		0.05	0.05	0.05	0.05	0.05	0.05

Table 7 gives the outcome of the autocorrelation, Heteroskedacity and the normality check and the respective p values for each selected ARFIMA models for the variables. The normality tests revealed that the residuals generated from the selected ARFIMA models are normally distributed, the Ljung-Box and the Portmanteau value for all the variables are greater than the significant level which inferred that there is no autocorrelation among the residual of the model's forecast errors, moreso, the results of heteroscedasticity tests of residuals for the variables revealed homoscedasticity nature of the residuals.

FIGARCH Model Estimation.

Heteroscedasticity Test

Table 8: Results of test for ARCH effect on the series.

	RGDP		Inflation Rate		Exchange Rate	
	Test statistic value	Prob.	Test statistic value	Prob.	Test statistic value	Prob.
F Statistic	134.4516	0.0000**	89.3780	0.0001**	158.2819	0.0000**
Observed R^2	140.1827	0.0000**	94.2268	0.000**	147.2072	0.0000**

**1% level

To model the volatility of a time series variable, it is mandatory to test for the presence of ARCH Effect in the residuals of the series.

The result of the ARCH-LM test in Table 8 revealed the presence heteroscedasticity in the series. The macro-economic variables reveal presence of conditional volatility from the result which can well be captured by fitting a FIGARCH model to the series.

Table 9: FIGARCH 1, D,1 Models

	RGDP	Inflatıon	Exchange
ω	1.3269 (1.7219)	1.4649 (1.0172)	1.2895 (1.0170)
α_1	0.0496 (0.1206)	-0.0427(0.192)	-0.3102(0.282)
β_1	0.3314(0.225)	0.3509 (0.422)	0.5819 (0.014)
D	0.3842(0.162)	0.4682(0.332)	0.3936(0.171)
In (L)	1928.816	1048.619	2734.307
AIC	8.3694	6.2791	4.9164
SIC	8.2001	6.2640	.49387
Q(5)	7.8854	12.1552	5.3458
ARCH 5	0.6852 (0.773)	0.5293 (0.1193)	0.2204 (0.7898)
Skewness	-0.0295	0.0074	0.00367
Jarque Bera	43.39	68.06	92.18
Kurtosis	1.4438	1.3941	1.4140

Table 9 above presented the estimates of FIGARCH models, the standard errors are reported in parenthesis, the model selection is based on the selecting model that has the lowest selection criteria and passes Q-test simultaneously using AIC, SIC and Ljung-Box Q-statistics.

The model fitting specifications are given in Table 9, Estimate of long memory parameter 'D' from the FIGARCH model above is shown to be significantly different from zero and falls within theoretical range. the revealing that the volatility exhibits a long memory process in the macro-economic variables under study. This justifies the significance of modeling persistence behavior in volatility and hence there is need for the dual long memory test. This justification brings about the examination of blended ARFIMA- FIGARCH model in investigating the structure of long memory and volatility simultaneously in the series.

Table 10: Estimates of ARFIMA FIGARCH Model.

Series	RGDP	Inflation	Exchange Rate
Mean Equation			
Constant (μ)	0.024^{**} (0.020)	0.037^{**} (0.016)	0.042^{**} (0.028)
AR 1	-0.168^{**} (0.221)	-0.721 (0.217)	-0.150 (0.4 20)
AR 2		0.135^{**}(0.024)	
MA 1	-0.354^{**} (0.095)	-0.779^{**} (0.010)	0.241 (0.193)
MA 2		-0.372^{**}(0.005)	
D	0.257^{**} (0.040)	0.450^{**} (0.000)	0.665^{**} (0.012)
Variance Equation			
Constant (ω)	0.025^{**} (0.001)	0.016^{**} (0.062)	0.033^{**} (0.031)
α_1	0.326^{**} (0.000)	0.064^{**} (0.000)	0.283^{**} (0.000)
β_1	0.532^{**} (0.005)	0.715^{**}(0.021)	0.867^{**} (0.041)
D	0.696^{**} (0.019)	0.305^{**} (0.001)	0.518^{**} (0.02)
In (L)	-6403.186	-3949.247	-7351.233
AIC	8.278	6.125	9.223
SIC	8.279	6.532	.9246
Q(5)	7.304	4.365	2.294
ARCH(5)	3.680	1.526	3.389

Notes: Table 10 gives the estimates of the quasi-maximum likelihood estimation of the hybrid of ARFIMA-FIGARCH model for the monthly data of Nigeria macroeconomic variables.

ARFIMA FIGARCH model combines the ARFIMA model that considered the mean behavior of the time series and the FIGARCH model which is employed to model the variance behavior (ARCH effect).

Table 11: Estimate of ARIMA GARCH.

	Coefficient	Std. Error	t- value	Prob.
		RGDP		
θ	0.3203	0.2936	-1.3144	0.0144
ω	0.0016	0.0026	2.0270	0.0039
α	0.1231	0.0182	2.2033	0.0000
β	0.2849	0.0315	4.3184	0.0007
		INFLATION RATE		
θ	0.2701	0.0631	0.3164	-2.284
ω	0.0019	0.0000	1.2012	0.0022
α	0.3861	0.0742	2.5433	0.0000
β	0.3849	0.0201	5.2934	0.0001
		EXCHANGE RATE		
θ	0.3031	0.1333	-1.8344	0.0038
ω	0.0029	0.0018	0.3712	0.0000
α	0.2171	0.0182	0.3733	0.0050
β	0.3749	0.0171	2.1634	0.0000

Applying the residual series obtained from the fitted ARFIMA models, suitable FIGARCH models were built. Table 10 above present results of the combined ARFIMA-FIGARCH model fitted to the macro-economic variables series and parameter estimates with their respective standard error in bracket, the best model is selected on the basis of the selection criteria. From Table 10, the fractional parameters for all the variables at 5% level of significance revealed significances.

ARIMA-GARCH model combines the ARIMA model that considered the mean behavior of the time series and the GARCH model which is employed to model the variance behavior (ARCH effect).

Table 12: Diagnostics check for the ARIMA GARCH Model.

Times series	Model	Portmanteau Test	ARCH LM Test
RGDP	ARIMA(1,1,1) GARCH(1,1)	(1623.258) 0.0122	(11.6128) 0.4721
INF RATE	ARIMA(1,1,1) GARCH(1,1)	(2102.1671) 0.3190	(9.336) 0.2775
EXCH RATE	ARIMA(1,1,1) GARCH(1,1)	(1918.2052) 0.1884	(11.3688) 0.0326

Applying the residual series obtained from the fitted ARIMA models, suitable GARCH models were built. The results of the combined models are presented in Table 11 above while Table 12 presents the serial correlation and heteroscedascity diagnostics for the ARIMA-GARCH models.

Table 13: Evaluation of selected ARFIMA, ARFIMA FIGARCH and ARIMA GARCH Models forecast Accuracy.

	ARFIMA			ARFIMA FIGARCH			ARIMA GARCH		
	RGDP	INF RATE	EXCH RATE	RGDP	INF RATE	EXCH RATE	RGDP	INF RATE	EXCH RATE
RMSE	0.0312	0.5051	0.3347	0.5147	0.0212	0.0112	0.0172	0.2934	0.0311
MAE	0.0235	0.4637	0.2619	0.1196	0.0257	0.0214	0.0219	0.4826	0.0292
MAPE	0.1947	0.6853	0.4417	0.2226	0.0051	0.0092	0.0039	0.5044	0.0018
R^2	0.9274	0.8155	0.8771	0.9742	0.9602	0.8899	0.9912	0.9187	0.8938

Table 13 displayed the ARFIMA, ARFIMA FIGARCH and ARIMA GARCH selected models for Nigeria macro-economic variables forecast accuracy measures respectively. From the Results comparison of ARFIMA, ARFIMA FIGARCH and ARIMA GARCH modeling from Table 13, low estimates of the validation statistics such as RMSE, MAE and MAPE from ARFIMA FIGARCH and ARIMA GARCH models revealed the adequacy of the models in modeling the Inflation Rate and the exchange rate while estimates of validation statistics from ARFIMA model shows more adequacy in modeling the RGDP. This result revealed evidence of high volatilities in Nigeria Inflation and the exchange rate of Naira to US dollar. The low values of an unbiased statistic MAPE of ARFIMA FIGARCH models in Table 13 revealed the adequacy of the selected ARFIMA FIGARCH models. Moreover, the general error measures showed evidence of better forecast performance with ARFIMA FIGARCH models in forecasting the inflation and exchange values of Naira to US dollar.

Conclusion

The study investigated the relevance of two iterative methods of long memory dependency in modeling and forecasting the rate and volatilities of selected major macro-economic variables of Nigeria, these variables were chosen based on the crucial roles they played in influencing the overall economic performance of Nigeria economy; the selected predictors are some of the powerful instruments for upgrading developing nations from their current economic status. Hybrid of ARFIMA-FIGARCH and ARFIMA model were employed in modeling the series. FIGARCH techniques of modeling were applied to model the residuals sequence from the fitted ARFIMA model. ARIMA GARCH methods of modeling were also employed in analyzing the volatilities of Nigeria selected macroeconomic variables to enrich the study. Results comparison of ARFIMA, ARFIMA-FIGARCH and ARIMA

GARCH modeling revealed that ARFIMA-FIGARCH and ARIMA GARCH models are more adequate in modeling the Inflation Rate and the exchange rate while ARFIMA present more adequacies in modeling the RGDP. The selection of the best model is based on the minimum selection criteria. The result from the research revealed evidence of high volatilities in Nigeria inflation and the exchange rate of Naira to US dollar. Certainty of the fitted models was established by the evidence of minimal values of MAPE. Both conditional mean and conditional variance parameters of the long memory were statistically significant, this revealed the prevalence of the dual long memory property in the series and volatility.

Conflict of Interest

Author declares no conflict of interest.

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