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Surface Energy Coefficient Determination in Global Mass Formula from Fission Barrier Energy

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Abstract: Semi-empirical mass formulae of the atomic nucleus describe binding energies of the nuclei. In the simple configuration and pattern of this formula, there are five terms related to the properties of the nuclear structure. The coefficients in each terms can be determined by various approach such as fitting on experimental binding energy values. In this paper, the surface energy coefficient in the formula which is a correction on total binding energy has been investigated by a method that is not previously described in the literature. The experimental fission barrier energies of nuclei have been used for this task. According to the results, surface energy coefficient in one of the most conventional formula has been improved by a factor of 3.4.

Keywords: Semi-empirical formula, fission barrier, surface term, Coulomb term

Fisyon Bariyer Enerjisi ile Kütle Formülündeki Yüzey Enerji Teriminin Tayini

Özet: Atom çekirdeğinin yarı ampirik kütle formülleri çekirdeklerin bağlanma enerjilerini tanımlar. Bu formüle ait basit yapılandırma ve modelde, nükleer yapı özellikleriyle ilgili beş terim vardır. Her bir terimdeki katsayılar, deneysel bağlanma enerji değerlerine uyma gibi çeşitli yaklaşımlarla belirlenebilir. Bu çalışmada, toplam bağlanma enerjisi üzerinde bir düzeltme etkisi olan yüzey enerji katsayısı, literatürde daha önce tanımlanmamış bir yöntemle araştırılmıştır. Bu amaçla çekirdeğin deneysel fisyon bariyer enerjileri kullanılmıştır. Elde edilen sonuçlara göre en geleneksel formüllerden birinde yüzey enerji katsayısı 3.4 kat arttırılmıştır.

Anahtar Kelimeler: Yarı ampirik formül, fisyon bariyeri, yüzey terimi, Coulomb terimi

1. INTRODUCTION

The nuclear mass formula is very important for describing nuclear properties and exploring the exotic structure of the nuclei such as halo structure, super-heavy nuclei structures and decays [1]. Liquid drop model plays a very crucial role for understanding about many nuclear phenomena which are unachievable by using the shell model of the nucleus. The semi-empirical mass formula based on this model of the nucleus was first proposed in 1935 by Bethe and von Weizsacker [2, 3]. According to the formula, the nuclear binding energy is expressed in terms of A and Z numbers of the nuclei. The conventional formula has simply five terms named as volume, surface, Coulomb, asymmetry and pairing energy terms. The surface term is a correction to the total binding energy due to deficit of binding energy for nucleons in the surface area. The magnitude of the nuclear surface energy is intimately related to the diffuseness of the nuclear surface and should provide a measure of the thickness of the nuclear surface. Since the surface energy is related to the lack of binding of the particles in the surface, it is clear that any attempt at a quantitative account of the nuclear

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surface energy will come up against difficulties due to our insufficient understanding of the nature of the effects responsible for nuclear cohesion [4]. Besides, the well-known force inside the nucleus is related to the Coulomb energy term, which can be regarded as a repulsive term among the protons. The coefficient in the Coulomb term can easily be calculated by using the formula $a_c = 3e^2/5r_0$. Recently, semi empirical mass formula has been extended by adding extra terms or has been modified slightly or completely [5, 6, 7, 8, 9, 10, 11, 12, 13]. These pioneering attempts provides better understanding how to investigate the binding energies of the nuclei on more precise scale. In the study of Kim and Cha [14], the coefficients and even the power of the A-number have

been determined in order to reach experimental values as close as possible. Also in that work, the nuclei are divided into different groups on the basis of their half-lives and investigated different coefficients for each group. The coefficients in each term can be determined by fitting the formula to the experimental binding energies on the atomic nuclei.

After the discovery of the fission, this phenomenon was started studying by considering nuclear drop model. If the Coulomb energy does not exceed a critical value, a charged drop is stable against fission. The surface energy in the drop model wants to keep the nucleus spherical, whereas Coulomb energy wants to deform it. Whether there will be a fission phenomenon or not, depends on the balance of these two effects. One can determine fissility parameter x, that is characterized by the ratio of surface and Coulomb energies. If x exceeds the value of 1, fission occurs immediately [15]. Throughout the years, the constant in the semi-empirical mass formula has been determined many times by using various procedures or on different data sets. Every determined coefficient is different from each other. In this study, we have applied a different approach to obtain a constant in the basic five term formula. We have used experimental fission barrier energies to determine the surface energy coefficient in semi-empirical mass formula. We have taken the measured fission barriers from Myers approach [16]. We have considered x to perform this task. Our aim was to obtain surface energy coefficient from experimental fission barrier energies and hence to reduce the mean square error value between theoretically determined binding energies of the nuclei and experimental ones.

2. THEORETICAL FORMALISM

The most conventional simple semi-empirical mass formula considered in this work has been presented in Eq. (1). This formula simply composed of five terms, named as volume, surface, Coulomb, asymmetry and pairing terms. The coefficients in each term are calculated mostly by fitting to experimentally measured masses of nuclei. They usually vary depending on the fitting methodology.

$$B(Mev) = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_a \frac{Z(A-2Z)^2}{A} + a_p \frac{k}{A^{\frac{3}{4}}}. \quad (1)$$

In the above mentioned formula, k takes the values of +1, 0 or -1 for even-even, even-odd or odd-odd nuclei, respectively. In fission process in which nuclear shape deviates from spherical shape, the surface energy of the nuclei increases and the Coulomb energy decreases because charge density is reduced. The other terms contributing to the total binding energy of the nuclei are not appreciable changed when the nuclei split into two fragments. The total potential energy is determined by the sum of these two terms given in Eq. (2) and Eq. (3)

$$E_c^0 = a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} \quad (2)$$

$$E_s^0 = a_s A^{\frac{1}{3}}. \quad (3)$$

The ratio of these terms are depicted in Eq. (4), known as the fissility parameter x. Stable, unstable, and metastable states are defined by using the fissility parameter, the released energy, and the fission barrier [16].

$$x = \frac{E_c^0}{2E_s^0} = \frac{a_c}{2a_s} \frac{Z(Z-1)}{A} \quad (4)$$

Here, and are the Coulomb and surface energies of the spherical nucleus, respectively. If the changes in the Coulomb and surface energies are equal to each other according to their spherical states, the nucleus becomes unstable against fission. This parameter is reached to 1 for *Z(Z-1)/A≈50*. Hence, according to the drop model of the nucleus, nuclei with *Z(Z-1)/A>50* are unstable against fission [17].

The liquid drop model of the nucleus permits calculation of the change in potential energy of the nucleus when it deviates from spherical shape [18].

In this case, the potential energy of the nucleus increases. The contributions to this change comes from surface and Coulomb energy terms. The Coulomb energy repulsion wants to deform spherical shape while the surface energy wants to keeps them in spherical configuration. The total change in potential energy are related to the total deformation energy and are mentioned as, in Eq. (5).

$$\Delta E = (E_s + E_c) + (E_s^0 - E_c^0) = E_s^0[\frac{2}{5}(1-x)a_2^2 - \frac{4}{105}(1+2x)a_3^2 + ...] \quad (5)$$

where E_s and E_c are the surface and Coulomb energy of the deformed nucleus, respectively .

We can calculate the maximum of Eq. (5) as

$$\frac{d\Delta E}{da_2} = E_s^0[\frac{4}{5}(1-x)a_2 - \frac{4}{35}(1+2x)a_2^2 \quad (6)$$

The first root ($a_2 = 0$) corresponds to minimum of the spherical nucleus and the second ($a_2 = 7.(1 - x/1 + 2x)$) is fission barrier maximum. If we substitute the second root to Eq. (5) we can obtain fission barrier maximum in MeV. The fission barrier maximum is determined as difference between the saddle-point and ground state masses. This can be calculated theoretically by using Eq. (7)

$$E_b = \frac{98(1-x)^3}{15(1+2x)^2}.E_s^0. \quad (7)$$

where and E_b are the surface and barrier energies of the spherical nucleus, respectively. If experimental barrier energies of the fissionability nuclei used in this formula and with the inclusion of the are once investigated, then surface energies of the nuclei are calculated more easily. After obtaining this energy values for different nuclei, it is easy to have surface energy coefficient a_s.

3. RESULTS AND DISCUSSIONS

We have used the barrier maximum formula Eq. (8) in order to obtain surface term coefficient in the semi-empirical mass formula.

$$E_b = \frac{98(1-\frac{E_c^0}{2E_s^0})^3}{15(1+\frac{E_c^0}{2E_s^0})^2}.E_s^0 \quad (8)$$

where E_b, and are maximum energy of fission barrier, Coulomb energy and surface energy for spherical nuclei, respectively. By solving this cubic equation, we have obtained surface energy (E_S) of the nuclei. We have considered Eq. (2) for Coulomb energy and taken the coefficient a_c=*0.72*, as given in the coefficient from Krane [19]. We have thought that if one can take any experimental values to derive a formula, this procedure can be one of the best way for this aim. Therefore, we have used experimental fission barrier height in MeV [20]. This data _le includes total 36 isotopes of the nuclei from Lu (Z=71) to Cf (Z=98). After determination of by Eq. (8), we have used Eq. (3) to get surface term coefficient. As can be seen in Table 1 that the surface term coefficients have been calculated for different isotopes which have experimental barrier data. From all 36 isotopes, we have calculated the average value of the coefficient. According to the results, the coefficient has been redefined as 16.481.

Table 1. Measured fission barrier [20], Coulomb and surface energies and surface term coefficient for 36 isotopes.

Z	N	A	E_b	E_c^0	E_s^0	a_c
71	102	173	28.00	642.2048	498.428	16.054
73	106	179	26.10	671.4858	513.417	16.165
75	110	185	24.00	701.2963	527.681	16.253
76	110	186	23.40	718.9572	537.645	16.500
76	111	187	22.70	717.6733	534.824	16.355
76	112	188	24.20	716.3986	538.180	16.399
77	112	189	22.60	734.2031	545.352	16.599
77	114	191	23.70	731.6314	546.807	16.487
80	118	198	20.40	780.7186	568.856	16.754
81	120	201	22.30	796.4811	585.002	17.049
83	124	207	21.90	828.3890	604.401	17.272
83	126	209	23.30	825.7381	607.044	17.237
84	126	210	20.95	844.5333	611.723	17.314
84	128	212	19.50	841.8691	605.130	17.020
85	128	213	17.00	860.8038	608.137	17.051
88	140	228	8.10	902.3108	591.541	15.850
90	138	228	6.50	944.0320	605.706	16.230
90	140	230	7.00	941.2877	607.629	16.187
90	142	232	6.30	938.5751	601.014	15.918
90	144	234	6.65	935.8934	601.968	15.852
92	140	232	5.40	980.9926	618.870	16.391
92	142	234	5.80	978.1897	620.498	16.340
92	144	236	5.75	975.4186	618.472	16.195
92	146	238	5.90	972.6787	618.055	16.093
92	148	240	5.80	969.9693	615.672	15.942
94	144	238	5.30	1015.6662	638.268	16.620
94	146	240	5.50	1012.8370	638.326	16.529
94	148	242	5.50	1010.0391	636.691	16.395
94	150	244	5.30	1007.2718	633.371	16.221
94	152	246	5.30	1004.5347	631.773	16.092
96	146	242	5.00	1053.7127	657.730	16.937
96	148	244	5.00	1050.8258	656.054	16.801
96	150	246	4.70	1047.9703	651.594	16.597
96	152	248	5.00	1045.1455	652.755	16.537
96	154	250	4.40	1042.3510	645.431	16.264
98	154	252	4.80	1083.5862	673.167	16.873
					average(a_s)	16.481

We have tested semi-empirical mass formula Eq. (1) with Krane coefficient (a_v=15.5, a_s=16.8, a_c=0.72, a_a=23 and a_p=34). The mean square error (MSE) value between theoretical masses of the nuclei and the experimental masses has been obtained 100.9 for 3245 isotopes from A=20 to 295. If we use the new surface coefficient as as=16.481 in same formula, the MSE value has been achieved as 29.6 which gives 3.4 factor better result than Krane surface coefficient gives. In Figure 1a, the differences between experimental binding energies (BE_{exp}) and theoretical binding energies (BE_{theo}) calculated by Krane coefficients have been shown. The deviations from experimental values are lied between about -10 to 40 MeV. In Figure 1b, we have also shown these differences by redefined surface coefficient. The deviations are lied between about -10 to 20 MeV.

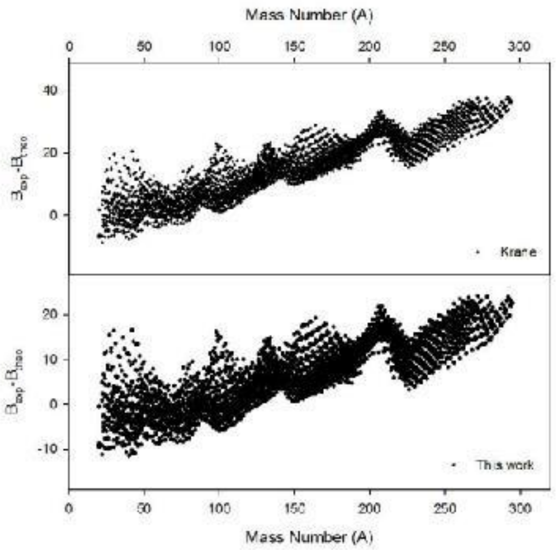

Figure 1. The difference between experimental and theoretical binding energies with Krane coefficient (a) and the coefficient obtained in this work (b).

3. CONCLUSIONS

In this work, the experimental fission barrier energy values exist for 36 nuclei have been used for determination of the surface energy coefficient in semi-empirical mass formula of liquid drop model of the nucleus. We have considered conventional mass formula with Krane coefficient. We have borrowed Coulomb energy coefficient as existing value and calculated surface energy terms and then their coefficients for each 36 nuclei. After obtaining the coefficients, we have calculated the average value. The redefined value of the surface coefficient is as=16.481. Other coefficients remain same, when we used this coefficient in semi-empirical formula, the result is 3.4 factor better the result of Krane surface coefficient.

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