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Eradication Suggestions For Infectious Diseases Based on the Fractional Guinea-Worm Disease Model

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ABSTRACT

Guine-worm disease (GWD) is considered one of the most fascinating infectious diseases that almost no one is aware of. On the other hand, unfortunately, there is no medicine or vaccine to treat this tropical disease transmitted through drinking water. However, GWD is about to be miraculously eradicated. This feature makes it the first parasitic disease to be eradicated without biomedical interventions. Accordingly, this situation brings the question: How can a disease be eradicated without medicine, vaccine or immunity? In light of this question, the current study offers recommendations on how to stop the spread of infectious diseases. One of the best ways to eliminate existing diseases is to benefit from the strategies followed for diseases that have been eradicated. Our results obtained by utilizing the fractional Caputo derivative show that behavior change programs aimed at reducing or stopping the spread of infectious diseases are effective tools in eradicating the disease.

Keywords: Caputo fractional operator, Fractional modeling, Guinea-worm disease, Non-local derivative, Mathematical biology.

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Introduction

In the mathematical biology literature, many disease models have been theoretically and numerically investigated using various definitions of non-local fractional operators. As a result of the analyzes and simulations, it is mentioned whether the fractional operators used are advantageous for the model under consideration [1,2]. For example, in [3] the disease model of dengue fever was examined with different types of fractional operators. Also, the behavior of the disease was evaluated by performing comparative analysis, and it was stated that the fractional-order model produced better results than the classical model. In this and many other studies as can be seen in [4-7], diseases have been analyzed in the classical sense or with fractional derivative operators, and discussions have been made on simulations in the light of the parameters affecting the course of the disease. However, in this study, in addition to investigating a disease that has never been investigated before using fractional derivatives, a novel approach to eradicating today's dangerous diseases is given by utilizing a disease that is on the verge of extinction.

Guinea-worm disease (GWD) is one of the most remarkable diseases that nearly no one has heard of [4]. For the treatment of this neglected disease, which is spread through drinking water, there is no drug or vaccination. Regardless, GWD is about to disappear miraculously. Hence, it is the first parasitic disease to be eradicated, as well as the first parasitic disease to be eradicated without the need for biomedical intervention. Also, GWD is a disease that has been going on since ancient times. GWD afflicted 50 million people in most of Africa, Asia, and the Middle East in the 1950s. However, it is now on the verge of extinction, with only three African countries reporting less than 25 human cases in 2016. Therefore, this scourge has almost disappeared, and considering the factors that caused its extinction is a guide for how to eradicate diseases transmitted through contaminated water.

In [8,9] which contains some lessons that can be taken from the eradication process of smallpox for malaria eradication studies, the importance of determining the ways and methods to be followed to achieve this goal is mentioned by making use of smallpox eradication. Similarly, another study [10] has presented a malaria eradication strategy that incorporates lessons learned from the Global Polio Eradication Initiative (GPEI). For this reason, this present study proposes solutions using some popular mathematical tools such as Caputo fractional derivative, fractional numerical method for the eradication of contagious diseases we are still struggling with, using GWD, which is on the verge of extinction and is little-known. The eradication of diseases requires internationally coordinated approaches. Ensuring continuous participation from communities, politicians and funders, efficient organizations and well-managed programs have a very important place in eradication efforts. State and global disease communities should be informed about the experience of other disease eradication programs, including the smallpox eradication program, as they seek to achieve these goals. That is, it is important to benefit from disease eradication and elimination programs that have been successfully concluded. Problems such as encouraging international support for eradication studies, coordinating the programs organized, and providing the necessary financing should be taken into account. In addition to these, it may be considered a useful approach to review the literature of diseases that have disappeared or are about to disappear to evaluate how the difficulties that are likely to be encountered are overcome. On the other hand, various diseases have been examined using the fractional operators in the literature [11-27].

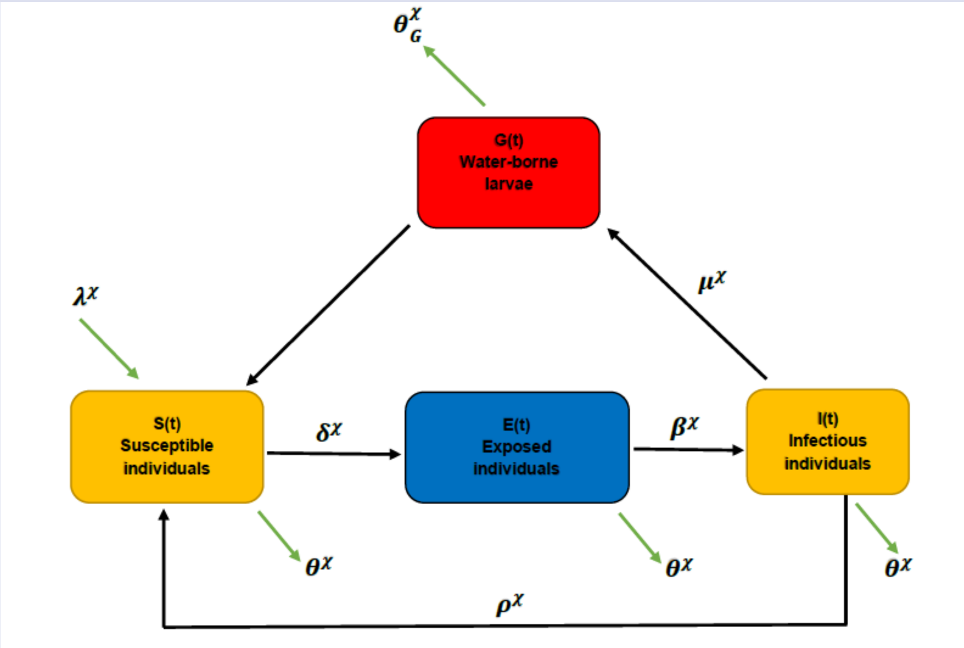

Figure 1. The fractional Guinea-worm disease model diagram.

The remainder of this study is presented as follows. In Section 2, we introduce the formulation of the classical GWD model and its fractional version under non-local Caputo operator including singular kernel. Then, in Section 3, a detailed mathematical analysis of the fractional GWD model is introduced in order to comprehend the dynamical behavior of the system handled. Finally, we present the numerical simulation and discussions section for the GWD model.

Mathematical Formulation of the Guinea-Worm Disease

In this section, we present two subsections wherein we describe the classical (integer-order) and fractional (non-integer order) versions of the GWD model. Thereby, it will be possible to compare the classical and fractional-type models by presenting them separately. Also, fundamental information about the GWD model is given to comprehend the results of the current study. In addition, the advantages of the fractional-order model are emphasized.

Classical GWD Model

We need to follow what comes in and what goes out in order to develop a mathematical model. The human population for GWD is divided into three groups. The first subgroup is susceptible individuals, and there are three possibilities for this group: birth, death, or infection. Individuals who are infectious recover or die, while those who are infected become infectious or die. There is, on the other hand, a worm population. When infected people put their feet in drinking water, the parasite is born as freshwater provides relief, and they die soon after. For model formulation, let S(t) represent susceptible individuals, E(t) represent exposed individuals, I(t) represent infected individuals, and G(t) represent the number of larvae in the water. Also, λ is the human birth rate, δ is the infection rate, β is the worm emergence rate, ρ is the recovery rate, and θ is the death rate. On the other hand, while infected individuals produce new larvae at the μ rate, the larvae are naturally cleared from the water at the θ_G ratio [4].

Interventions to change the course of the disease are carried out in 3 ways: filtration of the water supply, education given to the individuals, and chlorination. Here, the term "education" refers to teaching

individuals not to put infected limbs in the water. As a result, it is clear that the increase in education has a direct decreasing effect on the parasite birth rate, namely μ. Moreover, filtering is a technique for decreasing the parasite's capacity to infect humans-host and thus it has a reducing effect on δ. Also, chlorination has the effect of increasing the parasite death rate θ_G [4].

Now, let us give the mathematical model in the light of the information given above [4]:

$$\frac{ds}{dt}=\lambda-\delta S(t)G(t)-\theta S(t)+\rho I(t),$$
$$\frac{dE}{dt}=\delta S(t)G(t)-\beta E(t)-\theta E(t), \quad (1)$$
$$\frac{dI}{dt}=\beta E(t)-\rho I(t)-\theta I(t),$$
$$\frac{dG}{dt}=\mu I(t)-\theta_G G(t).$$

It should be noted here that state variables S, E, I, and G are not negative. Furthermore, because the quantities given are averages, it is not correct to assume that each person is infected with only one worm at a time. Because there have been instances where people have had seven worms at once. Mass-action transmission is employed because the interaction between aquatic parasites and people is present when humans drink parasite-containing water. Thus, each person's exposure to the parasite is approximately equal, as everyone in the village usually drinks from a single source. Also, we shall emphasize that in some areas, regular disease control can be difficult due to limited resources and infrastructure. In particular, chlorinating water regularly can be difficult or even impossible. The description of all parameters and the values used for this study can be seen in Table 1.

Table 1. Parameter values of GWD model [4].

Parameter	Definition	Sample Value	Units
S	Susceptible individuals	S(0)=λ/θ	people
E	Exposed individuals	E(0)=0	people
I	Infectious individuals	I(0)=0	people
G	Water-borne larvae	G(0)=200	larvae
λ	Birth rate	37	people/years
δ	Transmissibility	0.0255	1/larvae.years
θ	Death rate	0.0142	1/years
ρ	Recovery rate	8.760	1/years
β	Rate of worm emergence	1	1/years
μ	Parasite birth rate	100.000	larvae/people.years
θG	Parasite death rate	26	1/years

θ's average transmissibility can be calculated using (7 drink water per day) ×(365 days)/(100,000 larvae)=0.0255. This calculation shows the ratio of the total annual water ingested to the number of parasites. Also, the average life span of individuals, 1/θ, is assumed to be 70 years while the average infectious time 1/ρ is taken as 1 hour such that ρ=24× 365=8760 $years^{-1}$. The birth rates per 1000 population in the four endemic countries Mali, Ethiopia, Sudan, and Ghana are 46.09, 43.34, 33.25 and 28.09 with an average of 37, respectively [4].

Fractional GWD Model with Caputo Operator

This subsection provides the fractional version of the GWD system (1) under Caputo operator. Systems examined through fractional operators often give more reliable results than systems defined by classical derivatives. Fractional derivatives are used to examine the course of the disease in more detail and to obtain more sensitive results. We define the model as follows, using the Caputo derivative, which is known to be very advantageous in application.

$$ {}_{C}D_{0,t}^{\chi}S(t)=\frac{1}{1-\chi}\int_0^t\frac{S'(t)}{(t-\tau)^{\chi}}=\lambda^{\chi}-\delta^{\chi}S(t)G(t) -\theta^{\chi}S(t)+\rho^{\chi}I(t),$$
$$ {}_{C}D_{0,t}^{\chi}E(t)=\frac{1}{1-\chi}\int_0^t\frac{E'(t)}{(t-\tau)^{\chi}}=\delta^{\chi}S(t)G(t)-\beta^{\chi}E(t)-\theta^{\chi}E(t), \quad (2)$$
$$ {}_{C}D_{0,t}^{\chi}I(t)=\frac{1}{1-\chi}\int_0^t\frac{I'(t)}{(t-\tau)^{\chi}}=\beta^{\chi}E(t)-\rho^{\chi}I(t)-\theta^{\chi}I(t),$$
$$ {}_{C}D_{0,t}^{\chi}I(t)=\frac{1}{1-\chi}\int_0^t\frac{I'(t)}{(t-\tau)^{\chi}}=\mu^{\chi}I(t)-\theta_G^{\chi}G(t).$$

Now, we present the basic mathematical analysis of the GWD system by using the fractional Caputo derivative and make comments on the process of the disease under consideration.

Fractional Mathematical Analysis of Fractional GWD Model with Caputo Operator

In this section, we introduce some basic information on the proposed fractional system. A positive set of the presented fractional model is given. Also, the disease-free equilibrium (DFE) point is determined for computing the reproduction number (RN). Moreover, we analyze the stability of the system. On the other hand, the fractional model under examination is investigated for understanding the system behavior in detail by means of the Caputo operator. To comment on the eradication or persistence of the GWD, we determine the RN. For this purpose, some basic features of the fractional-order GWD model is presented. For obtaining invariant region of the fractional GWD model, let us consider the following theorem:

Theorem 1. The closed set $\Omega = \{(S,E,I,G) \in R_+^4 : 0 \leq S+E+I+G \leq K\}$ is a positive set of the fractional-order GWD model.

Proof. In order to prove the desired result, we follow that

$$
\begin{aligned}
{}_cD_{0,t}^{\chi}S(t)|_{S(t)=0} &= \rho^{\chi}I(t) \geq 0,\\
{}_cD_{0,t}^{\chi}E(t)|_{E(t)=0} &= \delta^{\chi}S(t)G(t) \geq 0,\\
{}_cD_{0,t}^{\chi}I(t)|_{I(t)=0} &= \beta^{\chi}E(t) \geq 0,\\
{}_cD_{0,t}^{\chi}G(t)|_{G(t)=0} &= \mu^{\chi}I(t) \geq 0,
\end{aligned}
$$

and this means that the solutions of the suggested model are non-negative. Furthermore, from the sum of the equations of the GWD model, we obtain

$$
{}_cD_{0,t}^{\chi}N(t) \leq \lambda^{\chi} - \theta^{\chi}N(t),
$$

where N(t) is the total population size. Utilizing the property of fractional operator, one can have

$$
N(t) \leq \left(N(0) - \frac{\lambda^{\chi}}{\theta^{\chi}}\right)\boldsymbol{E}_{\chi}(-\theta^{\chi}t^{\chi}) + \frac{\lambda^{\chi}}{\theta^{\chi}}, \tag{3}
$$

where $\boldsymbol{E}_{\chi}(.)$ is Mittag-Leffler (M-L) function. Also, if we use the properties of the ML function, the expression (3) can be written as

$$
N(t) \leq \frac{\lambda^{\chi}}{\theta^{\chi}} \cong K,
$$

and so we get $N(t) \leq K$. Finally, it can be said that Ω is the positive invariant region of the fractional GWD model including the Caputo differential operator.

On the other hand, we shall note that

$$
{}_cD_{0,t}^{\chi}(S+E+I) = \lambda^{\chi} - \theta^{\chi}(S+E+I),
$$

and thus

$$
S(t)+E(t)+I(t) \leq \frac{\lambda^{\chi}}{\theta^{\chi}}. \tag{4}
$$

Using (4), we have

$$
{}_cD_{0,t}^{\chi}I \leq \frac{\beta^{\chi}\lambda^{\chi}}{\theta^{\chi}} - (\rho^{\chi}+\theta^{\chi})I,
$$

and from the properties of the fractional derivatives, it can be readily obtained the following result

$$
I(t) \leq \left(I(0) - \frac{\beta^{\chi}\lambda^{\chi}}{\theta^{\chi}(\rho^{\chi}+\theta^{\chi})}\right)\boldsymbol{E}_{\chi}(-(\rho^{\chi}+\theta^{\chi})t^{\chi}) + \frac{\beta^{\chi}\lambda^{\chi}}{\theta^{\chi}(\rho^{\chi}\theta^{\chi})}.
$$

Because of the fact that ρ is large, the M-L function term is small. So, we reach

$$
I(t) \leq \frac{\beta^{\chi}\lambda^{\chi}}{\theta^{\chi}(\rho^{\chi}\theta^{\chi})}.
$$

Hence

$$
{}_cD_{0,t}^{\chi}G \leq \frac{\mu^{\chi}\beta^{\chi}\lambda^{\chi}}{\theta^{\chi}(\rho^{\chi}+\theta^{\chi})} - \theta_G^{\chi}G,
$$

and so we get

$$
G \leq \frac{\mu^{\chi}\beta^{\chi}\lambda^{\chi}}{\theta^{\chi}\theta_G^{\chi}(\rho^{\chi}+\theta^{\chi})}.
$$

The above inequalities may cause the number of parasite in the water to be overestimated. However, they help us to estimate them without solving the fractional system of GWD.

Now, we discuss the dynamical properties of the fractional-order GWD model. DFE point is obtained in order to investigate the RN. Moreover, we analyze the stability of the fractional model under examination. Let $E_0(S^*,E^*,I^*,G^*)$ be the equilibrium points of the proposed model. By setting the right-hand side of all differential equations of the system equal to zero, we get a steady state of the GWD model as follows:

$$
\begin{aligned}
&\lambda^{\chi} - \delta^{\chi}S(t)G(t) - \theta^{\chi}S(t) + \rho^{\chi}I(t) = 0,\\
&\delta^{\chi}S(t)G(t) - \beta^{\chi}E(t) - \theta^{\chi}E(t) = 0,\\
&\beta^{\chi}E(t) - \rho^{\chi}I(t) - \theta^{\chi}I(t) = 0,\\
&\mu^{\chi}I(t) - \theta_G^{\chi}G(t) = 0.
\end{aligned}
$$

For obtaining the DFE points, we consider the system in the absence of the GWD. Hence, the suggested model reduces to

$$
\lambda^{\chi} - \theta^{\chi}S = 0,
$$

and if we solve, we have $S^* = \frac{\lambda^{\chi}}{\theta^{\chi}}$ Thus, the DFE is

$$E_0(S^*, E^*, I^*, G^*) = \left(\frac{\lambda^\chi}{\theta^\chi}, 0,0,0\right).$$

Now, we calculate the reproductive ratio R_0 of the fractional GWD model by means of the next-generation method (NGM). Here, $R_0 = \rho(FV^{-1})$ such that ρ is the spectral radius of FV^{-1} called next-generation matrix. The matrix **F** includes new infections terms and the matrix **V** including the remaining terms of the fractional GWD model are

$$\boldsymbol{F} = \begin{bmatrix} \delta^\chi SG \\ 0 \\ 0 \end{bmatrix}, \qquad \boldsymbol{V} = \begin{bmatrix} \beta^\chi E + \theta^\chi E \\ \beta^\chi E + \rho^\chi I + \theta^\chi I \\ -\mu^\chi I + \theta_G^\chi G \end{bmatrix},$$

and so we have

$$F = \begin{bmatrix} 0 & 0 & \delta^\chi S \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} \beta^\chi + \theta^\chi & 0 & 0 \\ \beta^\chi & \rho^\chi + \theta^\chi & 0 \\ 0 & -\mu^\chi & \theta_G^\chi \end{bmatrix}$$

Thereby, we can readily obtain that

$$FV^{-1} = \begin{bmatrix} 0 & 0 & \delta^\chi S \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\beta^\chi + \theta^\chi} & 0 & 0 \\ \frac{\beta^\chi}{(\beta^\chi + \theta^\chi)(\rho^\chi + \theta^\chi)} & \frac{1}{\rho^\chi + \theta^\chi} & 0 \\ \frac{\beta^\chi \mu^\chi}{\theta_G^\chi (\beta^\chi + \theta^\chi)(\rho^\chi + \theta^\chi)} & \frac{\mu^\chi}{\theta_G^\chi (\rho^\chi + \theta^\chi)} & \frac{1}{\theta_G^\chi} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\delta^\chi \beta^\chi \mu^\chi S^*}{\theta_G^\chi (\beta^\chi + \theta^\chi)(\rho^\chi + \theta^\chi)} & \frac{\delta^\chi \mu^\chi S^*}{\theta_G^\chi (\rho^\chi + \theta^\chi)} & \frac{\delta^\chi S^*}{\theta_G^\chi} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Owing to the property of upper triangular matrix, the eigenvalues of the above matrix are on the diagonal. Thus, if we use the properties of the next-generation method, the largest eigenvalue is obtained as

$$R_0 = \frac{\lambda^\chi \beta^\chi \mu^\chi \delta^\chi}{\theta^\chi (\beta^\chi + \theta^\chi)(\rho^\chi + \theta^\chi)\theta_G^\chi}. \tag{5}$$

Now, let us interpret the resulting R_0 value. If $R_0 < 1$, then it can be said that DFE is stable and is the only equilibrium. If $R_0 > 1$, then DFE is unstable and endemic equilibrium exists. On the other hand, it is useful to note that the value of R_0 increases with $\lambda^\chi, \beta^\chi, \mu^\chi$, and δ^χ, and decreases with $\theta^\chi, \theta_G^\chi$, and Furthermore, the education given to individuals is aimed at preventing people from placing their limbs in drinking water. So, this decreases the value of μ^χ. Additionally, filtering drinking water reduces δ^χ while θ_G^χ increases with continuous chlorination of water. Hence, all these interventions cause a decrease in R_0 [4].

It is worth mentioning that continuous chlorination in the water is neither possible nor desirable, so chlorination is assumed to occur at different t times. During these times, the number of larvae in the water decreases and this gives rise to an impulsive differential equation. Also, in some regions, regular disease control can be quite difficult due to limited resources of infrastructure. In particular, it may be difficult or impossible to chlorinate water at certain times.

To examine the three important control parameters for the GWD model in more detail, fix all the other parameters to the sample values and take $R_0 = 1$. The equation (5) can be solved for the parameter δ^χ (which is trivial) and then μ^χ and θ_G^χ can be taken as independent variables. This situation allows us to obtain a 3D surface plotted in Figure 2. Combinations of parameters below the surface can be said to induce eradication. Furthermore, the disease persists due to a combination of parameters above the surface. Changes in μ^χ have a significant impact on the outcomes. On the other hand, we can not eliminate the disease even if we increase θ_G^χ 100 times, but due to the log scale, we have to drop δ^χ to very low levels. In Figure 2, it can be observed that eradication occurs if the infection rate is greatly reduced by filtration of drinking water and the parasite death rate is increased by more than 100 times by chlorination or the parasite birth rate is reduced to about 1 percent of its current value through education. Moreover, Figure 2 shows the eradication thresholds for the three separate parameters that most affect R_0. Accordingly, eradication will occur if the infection rate is reduced to well below its current value by filtering drinking water, and if the parasite mortality rate is increased more than a hundredfold by chlorination, or if the parasite birth rate is reduced to 1 percent through education.

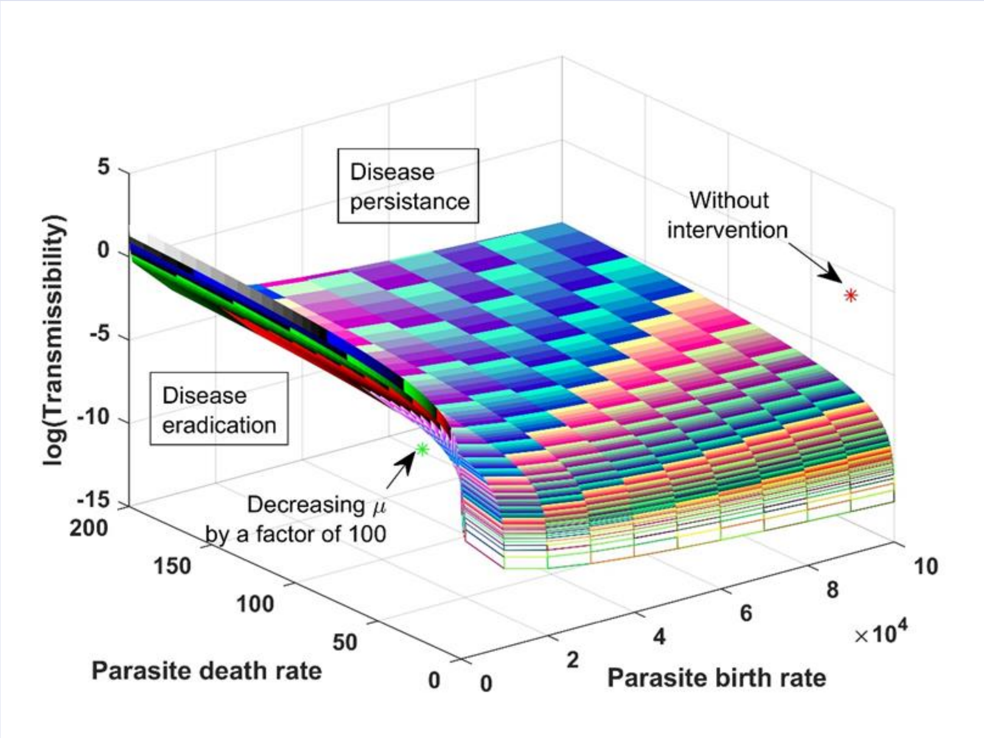

Figure 1. X-ray diffraction patterns of the $(Bi_{2-x}Ga_x)Sr_2CaCu_2O_2$ superconducting samples. (+) Bi-2212 phase, and (∗) Ga_2O_3 secondary phases.

On the other hand, the endemic equilibrium (EE) is presented for predicting the long-term outcome of the GWD. If the disease continues, we can say that there is an endemic equilibrium, which is shown as $\bar{E} = (\bar{S}, \bar{E}, \bar{I}, \bar{G})$. Also, the DFE is stable when EE does not exist, and hence, we do not have any infection. However, the GWD persists if the EE exists. The EE of the fractional GWD model is as follows:

$$\bar{S} = \frac{\theta_G^\chi}{\delta^\chi \mu^\chi}\left(\rho^\chi + \theta^\chi + \frac{\rho^\chi \theta^\chi}{\beta^\chi} + \frac{\theta^{2\chi}}{\beta^\chi}\right),$$
$$\bar{E} = \frac{\rho^\chi + \theta^\chi}{\beta^\chi}\bar{I},$$
$$\bar{G} = \frac{\mu^\chi}{\theta_G^\chi}\bar{I},$$
$$\bar{I} = \frac{\lambda^\chi \delta^\chi \mu^\chi \beta^\chi - \theta^\chi \theta_G^\chi(\rho^\chi \beta^\chi + \theta^\chi \beta^\chi + \rho^\chi \theta^\chi + \rho^\chi \theta^{2\chi})}{(\beta^\chi + \rho^\chi + \theta^\chi)\delta^\chi \mu^\chi \theta^\chi}$$

Numerical Simulation and Discussions

Biological and technical feasibility, costs and benefits, societal and political considerations are listed as distinguishing criteria for eradicating a contagious disease. The model we have presented meets these criteria. It is very important to know which of the ways to be followed may be optimal for the eradication of the disease. Despite the prospects for the extinction of diseases such as malaria, yaws, and yellow fever in the 20th century, and the eradication programs currently underway, such as polio and leprosy, smallpox remains the only disease that has been eradicated. On the other hand, hepatitis A and B, measles, rubella diseases are seen as suitable candidates for eradication. In other words, if necessary efforts are made for these diseases, which are technically and biologically possible to disappear, they will disappear just like smallpox. This study aims to offer solutions by presenting in detail the ways followed for GWD, which has already come to the threshold of eradication. While suggesting these solution proposals, it is aimed to shorten and facilitate the process by making use of the Caputo fractional derivative.

It can be observed that the most effective way to eliminate GWD is to reduce the parasite birth rate. This is possible by training people not to put their infected limbs in water. Although changing people's behavior is generally difficult, GWD eradication programs have been successful in this regard. If 99 percent of people can be persuaded not to put their infected feet in water, GWD will disappear completely. Chlorination, on the other hand, can theoretically control the disease, but numerical simulations clearly show that education is much more effective. Therefore, this study points to the importance of education in the last move towards the eradication of a disease. Although the results show that education is the most important intervention method, a combination of education with chlorination and filtration will be necessary to reach the final steps in a long eradication journey. By bringing together scientific and cultural resources, it will be possible to eradicate one of the oldest diseases in history.

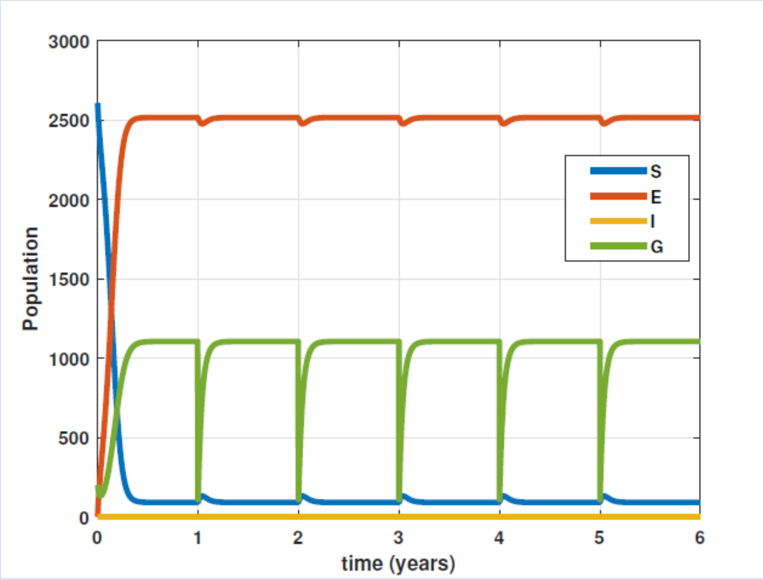

Figure 3. Persistence of the disease as a result of annual chlorination.

The effect of annual chlorination is shown in Figure 3 by using the values in Table 1. Although a significant reduction in larvae numbers is observed after chlorination, the population increases rapidly. While the number of susceptible individuals remains low, nearly all individuals continue to be infected. Additionally, it can be observed that chlorination is applied annually under the assumption that it is 90 percent successful. It is worth mentioning that infection levels are low, as individuals are contagious for a short time when their feet are submerged in water. On the other hand, the parasite birth rate is reduced by 99 percent, as seen in Figure 4. In this situation, the number of people who are exposed and infectious is almost zero, and no one is infected. All values except μ are the same as in Table 1.

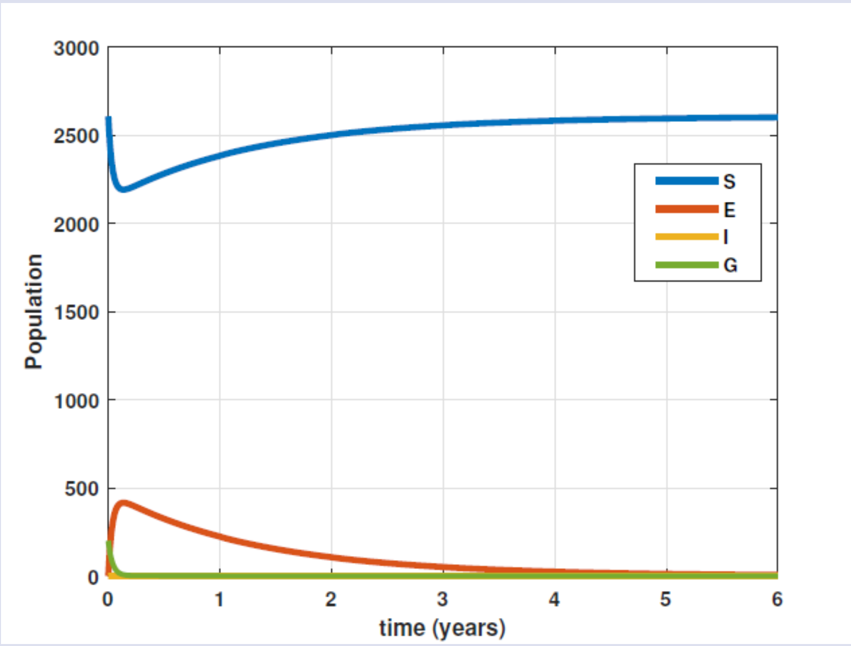

Figure 4. Eradication of the disease in case of reduced parasite birth rate when μ=1000..

Concluding Remarks

Some important results of this study are listed below:

- In the mathematical biology literature, theoretical and numerical results of some diseases have been analyzed using various fractional derivative definitions. As a result of the analyzes and simulations, it is mentioned whether the fractional derivative is advantageous for the model examined. In light of these studies, GWD, which has not been examined through a fractional derivative before, was analyzed with the Caputo derivative. It is thought that this analysis may be guiding in eradication studies of various diseases.

- The present study's methodology demonstrates that education about disease prevention is the most successful method of intervention. However, depending on the type of disease, a combination of education and certain additional interventions may be needed to reach the final stages of a prolonged eradication process.
- It is necessary to carry out internationally coordinated studies to eliminate the diseases that exist today. It has been emphasized that efficient organizations and well-managed programs are important in disease eradication studies. Therefore, it is crucial to review the literature on diseases that have been or are about to be eradicated to examine how the challenges likely to be faced by disease eradication efforts are addressed.
- Continuous chlorination to eliminate GWD is neither possible nor desirable. Therefore, if it is assumed that chlorination takes place at different t_k times, the following fractional impulsive differential equation system emerges, with the number of larvae in the water decreasing at the rate r:

$$
\begin{aligned}
{}_{C}D_{0,t}^{\chi}S(t) &= \frac{1}{1-\chi}\int_0^t \frac{S'(t)}{(t-\tau)^{\chi}} = \lambda^{\chi} - \delta^{\chi}S(t)G(t) - \theta^{\chi}S(t) + \rho^{\chi}I(t), \qquad t \neq t_k \\
{}_{C}D_{0,t}^{\chi}E(t) &= \frac{1}{1-\chi}\int_0^t \frac{E'(t)}{(t-\tau)^{\chi}} = \delta^{\chi}S(t)G(t) - \beta^{\chi}E(t) - \theta^{\chi}E(t), \qquad t \neq t_k \\
{}_{C}D_{0,t}^{\chi}I(t) &= \frac{1}{1-\chi}\int_0^t \frac{I'(t)}{(t-\tau)^{\chi}} = \beta^{\chi}E(t) - \rho^{\chi}I(t) - \theta^{\chi}I(t), \qquad t \neq t_k \\
{}_{C}D_{0,t}^{\chi}I(t) &= \frac{1}{1-\chi}\int_0^t \frac{I'(t)}{(t-\tau)^{\chi}} = \mu^{\chi}I(t) - \theta_G^{\chi}G(t), \qquad t \neq t_k \\
\Delta G &= -rG(t), \qquad t \neq t_k
\end{aligned}
$$

It should be noted that in some regions, regular disease control may be difficult due to limited resources and infrastructure. In particular, it may be difficult or impossible to chlorinate water at constant period. Also, numerical simulations clearly show that education is a much more effective factor in eradicating the disease under investigation.

- As a result, one of the most important conclusions to be drawn from this study is that the precautionary training provided to the public during the eradication studies of a disease can be considered the most important factor. On the other hand, using an effective fractional operator like Caputo derivative can speed up the process of disease eradication.

Conflicts of interest

There are no conflicts of interest in this work.

Ethical Approval Statement

The author of this article declares that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

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