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Parameter Estimation of the Inverted Kumaraswamy Distribution by Using L-Moments Method: An Application on Precipitation Data

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ABSTRACT

Modeling precipitation data plays a critical role in water resource and flood management. Statistical distributions are frequently used in describing hydrological variables. Different distributions and estimation methods have been presented in previous studies for modeling precipitation data. In this study, the inverted Kumaraswamy distribution is considered for its advantageous properties, and the L-moments and maximum likelihood methods are employed in estimating the parameters of the inverted Kumaraswamy distribution. In the application part, the annual maximum monthly precipitations recorded in the Rize, Türkiye are modeled with the inverted Kumaraswamy distribution. To the best of the author's knowledge, the L-moment method is considered for the first time to estimate the parameters of the inverted Kumaraswamy distribution. In addition, the efficiencies of the estimation methods are compared with a Monte-Carlo simulation study. For evaluating the performances of the estimation methods, the goodness of fit criteria including root mean square error, Kolmogorov Smirnov test, and coefficient of determination (R^2) are used in the application part of the study. The results show that for the data considered, the L-moments method yields more accurate results than the maximum likelihood method in estimating the parameters when the sample size is small. Accordingly, the corresponding distribution with L-moments estimations provides a better fit to precipitation data obtained from the Rize station.

Keywords: L-moments, Parameter estimation, Inverted kumaraswamy, Precipitation.

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Introduction

According to European Environment Agency [1] increased flooding is likely to be one of the most serious effects of climate change in Europe over the coming decades. In connection with flood management planning, modeling precipitation data has been one of the highlights in the field of hydrology. In recent years, there has been a growing interest in using probability distribution models to analyze precipitation data. The accurate prediction of precipitation is essential for effective flood management, water management, and hydrological modeling. Accordingly, the primary concerns of the studies using probability distribution models are to fit the most suitable probability density function (pdf) to data and estimate its parameters accurately.

In parameter estimation, several methods are available in the literature, such as maximum likelihood (ML), least squares, L-moment methods, and so on. The L-moment estimation is one of the methods for estimating the parameters of a probability distribution based on order statistics presented by Hosking [2]. The L-moment method is generally known for its applications in the hydrology field. It has some advantageous features that make it attractive for application to precipitation data where extreme rainfall events are possible. The use of L-moments has been shown to provide robust estimates of the parameters of distributions, especially when dealing with small sample sizes or skewed data [3]. As mentioned, another concern of using probability models is, fitting the most suitable distribution to data. Since every region has its own characteristics, the selection of the pdf mainly depends on the available precipitation data at a particular site [4]. Different pdf's are employed in modeling various natural phenomena including precipitation previously. The Kumaraswamy distribution is one of the distributions generally known for its applications in the hydrology field. A transformation of the Kumaraswamy distribution introduced by Abd Al-Fattah et al. [5] called the inverted Kumaraswamy (IKum) distribution presents some advantageous features such as having a longer right tail that increases the modeling ability of the distribution. The use of the IKum distribution in modeling ecological (precipitation and wind speeds) variables has been studied previously, see [6] and [7] respectively. Bagci et al. [7] have shown that the Ikum distribution can provide a better fit to given data compared to other well-known distributions, such as the Weibull, in some cases.

Consequently, the IKum distribution is employed in this study for modeling the precipitation data obtained from Rize province since it has some tractable properties. Rize is a province located in the eastern Black Sea region of Türkiye, which is a flood-prone area due to its geographical and climatic features. Unfortunately, Rize has been the subject of devastating flooding and landslides most recently. In addition, Rize is the rainiest province in Türkiye, having a total annual precipitation of over 2300 mm [8]. As a result, many studies are addressing the issue using different approaches. However, the studies in the region mostly concentrated on time series analysis. Such as Yozgatlıgil and Türkeş [9], modeled monthly maximum precipitation amounts for 60 year period using a pdf and time series analysis approach for several stations in the Black Sea region. They employed the generalized extreme value distribution in the analysis and the time series analysis. Cengiz et al. [10] analyzed historical precipitation changes at 16 stations during 1960–2015 in the Black Sea region of Türkiye. They emphasized the limitations in time series analysis such as normality, and independence of series, and used a combination of graphical and statistical methods to relax some of these restrictive assumptions. Aksu et al. [11] employed a comprehensive set of statistical methods to provide a detailed analysis of precipitation patterns in the Black Sea region mostly focused on trend analysis.

Numerous studies have been presented to compare different parameter estimation methods in the literature additionally. Upon analysis of these studies, it becomes apparent that the most effective probability distribution and estimation method may differ based on the unique precipitation characteristics of the region being studied. For example, Lee et al. [12] investigated the estimation of rainfall data using L-moments. The authors tested their method using daily rainfall data from several stations in Korea and conducted a simulation study to test the effectiveness of the method for different sample sizes. Shabri et al. [13] proposed a new method called LQ-moment for estimating the parameters of the kappa distribution. This method relies on the concept of L-moment and is an extension of the L-moment method for estimating parameters of other distributions. The LQ-moment method is compared with the L-moment estimation. The results showed that the LQ-moment and L-moment methods performed similarly. Wan Zin et al. [14] modeled the maximum daily rainfall data obtained from several rain gauge stations in Malaysia. Ngongondo et al. [15] conducted a regional frequency analysis of rainfall extremes in Southern Malawi using the index rainfall and L-moment approaches. They found that the generalized extreme value distribution is the most suitable model for precipitation data analysis. Galorie et al. [16] conducted a study using rainfall data recorded for 66 years in Austria and estimated parameters of several well-known distributions using methods including the L-moment and ML. Rahman et al. [17] conducted a study to investigate appropriate probability distribution and associated parameter estimation procedures in at-site flood frequency analysis. They employed the ML, L-moment, and method of moments estimation for fitting annual maximum floods from Australia. According to their study different estimation methods and pdfs performed better for each station. Similarly, [18] used the L-moments approach to analyze daily rainfall extremes in Iran. They found that the generalized extreme value and three-parameter log-normal distributions are the best fit for their data. According to their study, the best-fitting distribution depends on the method of estimation and the data mostly follow the generalized logistic distribution, and the L-moment method performed better than the LQ-moment. Li et al. [19] conducted a study on the frequency analysis of extreme precipitation in the Heihe River basin in China, using daily precipitation observations between 1960-2010 years. They showed that theoretical return levels with the ML estimation show better approximations to the empirical ones for the precipitation data of the Heihe River. Zhou et al. [20] utilized Pearson type 3 and generalized extreme value distribution to forecast the annual maximum precipitation at the Changxi Station, located in the Taihu Basin. They employed different parameter estimation techniques, including the ML, L-moment, and conventional moment, and concluded that the generalized extreme value distribution with parameters estimated through the L-moment method yielded the best fit for the observed data at the Changxi Station. Khan et al. [21] investigated the effectiveness of different estimation methods including the L-moment, ML, and maximum product of spacing methods for modeling extreme precipitations. Their study found that the L-moment method performed the best in terms of accuracy and efficiency in estimating extreme values of precipitation data. It is also emphasized that the L-moment method can be preferred in case of small sample sizes.

In Türkiye, several studies have been carried out on modeling precipitation data. Some of them have already been discussed in the previous section. More studies using L-moment estimation are reviewed additionally. Although these studies employ the L-moment method in estimation, they generally focused more on fitting the most suitable pdf to data, a study [22] conducted a regional frequency analysis of rainfall data recorded in Trabzon province using L-moment estimation. The authors aimed to determine the most suitable pdf for the data and estimate the return periods of extreme rainfall events. Their study provides insights into extreme rainfall patterns and can contribute to the development of effective flood management strategies in the region. Seçkin et al. [23] conducted a flood frequency analysis employing the L-moment method. They analyzed flood events in six regions of Türkiye and found that the 3-parameter log-normal and Pearson type III distributions were the most suitable models for the precipitation data. [24] conducted a regional flood frequency analysis of the Çoruh Basin considering the L-moment method. Topcu et al. [25] analyzed the drought conditions of the Seyhan Basin by utilizing the L-moment method. Their findings highlighted the potential of these tools for drought analysis and management in the region. Also, Ghaei et al. [26] used the L-moment approach to conduct a regional intensity–duration–frequency analysis in the Eastern Black Sea region in Türkiye. They found that the generalized extreme value distribution provided the best fit for precipitation data.

Considering all of the reasons mentioned above, this paper investigates the efficiency of L-moment estimation with a particular focus on the application of the Inverted Kumaraswamy (IKum) distribution in modeling precipitation data from Rize, Türkiye. It should be noted that to the best of the author's knowledge L-moment estimation for parameters of the IKum distribution are obtained for the first time in this study. Moreover, with the L-moment estimation, the maximum likelihood (ML) method is considered in estimating parameters for comparison. In addition, a Monte-Carlo simulation study is conducted to examine these methods' efficiencies. Although, the L-moment method has the advantages of being robust and computationally simpler as emphasized in the study of [2] as is known the ML method is the most efficient estimation method under regularity conditions. It also stated that the L-moment estimates can even be more accurate than the ML estimates in small sample sizes. The comparison is done by using several well-known criteria including the root mean square error (RMSE), coefficient of determination (R^2), Kolmogorov Smirnov (KS) test and KS p-value.

Contributions of the study to related literature are given as follows. First, the L-moment estimates of unknown parameters of the IKum distribution are obtained and compared with ML estimates; second, a critical region in terms of flood risk is selected for analysis and the precipitation data of Rize is modeled for the first time using the IKum distribution.

The study is structured as follows: In Section 2 the methods used in the analysis along with L-moments of the IKum distribution described. Afterward, the presentation of precipitation data and modeling results are given. Finally, Section 4 discusses the findings of the study, which concludes with some final remarks.

Materials and Methods

In this section, primarily the data set is described. Then, the pdf of the IKum distribution is given and the L-moment estimations of its parameters are provided. Afterward, the results of the Monte-Carlo simulation are presented.

Data Set

In this study, the precipitation records obtained from the Rize gauge station are considered in the application. The data comprises monthly maximum precipitations between the dates of January 2021 and March 2023. The data are obtained from the Turkish State Meteorological Service. The data may be shared by the author upon request. As mentioned previously, Rize province experienced a devastating flood last year. For this reason, the data of this region is considered in this study and it is believed that it is important to obtain accurate predictions of the precipitations in the region.

The geographic location of Rize is given in Table 1.

Table 1. Geographical information of the Rize province

Latitude	Longitude
41.026089	40.518929

Descriptive statistics for precipitation data of Rize are provided in Table 2.

Table 2. Descriptive statistics

Descriptive Statistics	Values
Mean	45.6630
Skewness	1.723
Kurtosis	2.100
Variance	1196.713

The IKum Distribution

The IKum distribution is presented by Abd AL-Fattah et al. [5]. Due to its advantageous features, the IKum distribution has attracted attention. Consequently, many applications and extensions of it are presented in the literature, see [6] and [27].

Let X be a random variable following the IKum distribution with shape parameters α and θ, then its pdf is

$$f(x;\alpha,\theta)=\alpha\theta(1+x)^{-(\alpha-1)}(1-(1+x)^{-\alpha})^{\theta-1};$$
$$x>0,\alpha,\theta>0.$$

The Theoretical Background of the L-Moment Method

The L-moment theory proposed by Hosking [2] is based on order statistics. The method is generally adopted by the hydrology field. Hosking [2] showed that L-moments can be expressed by using linear combinations of probability-weighted moments PWMs (β_r) as well.

The first four theoretical L-moments are defined as

$$\lambda_1=\mathrm{E}[\mathrm{X}_{1:1}]=\int_0^1 \mathrm{x}(\mathrm{F})\mathrm{dF},$$
$$\lambda_2=\frac{1}{2}\mathrm{E}[\mathrm{X}_{2:2}-\mathrm{X}_{1:2}]=\int_0^1 \mathrm{x}(\mathrm{F})(2\mathrm{F}-1)\mathrm{dF},$$
$$\lambda_3=\frac{1}{3}\mathrm{E}[\mathrm{X}_{3:3}-2\mathrm{X}_{2:3}+\mathrm{X}_{1:3}]=\int_0^1 \mathrm{x}(\mathrm{F})(6\mathrm{F}^2-6\mathrm{F}+1)\mathrm{dF},$$

and

$$\lambda_3=\frac{1}{4}\mathrm{E}[\mathrm{X}_{4:4}-3\mathrm{X}_{3:4}+3\mathrm{X}_{2:4}-\mathrm{X}_{1:4}]=\int_0^1 \mathrm{x}(\mathrm{F})(20\mathrm{F}^3-30\mathrm{F}^2+12\mathrm{F}-1)\mathrm{dF}.$$

Here, X a real-valued random variable with cumulative distribution function $F(x)$ and quantile function $x(F)$ and $\mathrm{X}_{1:1}\le\mathrm{X}_{2:\mathrm{n}}\ldots\le\mathrm{X}_{\mathrm{n:n}}$ are the order statistics of a random sample of size n drawn from the distribution of X. Let n be the sample size, the r-th sample L-moment is

$$l_r=\Sigma\Sigma\ldots\Sigma r^{-1}\sum_{k=0}^{r-1}-1^k\binom{r-1}{k}x_{i_{r-k:n}}.$$
$$1\le i_1<i_2\ldots<i_r\le n,$$

L-moments can be obtained using the PWMs [28] given as follows,

$$\beta_r = \int_0^1 x(F)F^r dF$$

Here, F is non-exceedance probability.

$\lambda_1 = \beta_0$,
$\lambda_2 = 2\beta_1 - \beta_0$,
$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0$,
$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0$ and sample L-moments are $l_1 = b_0, l_2 = 2b_1 - b_0, l_3 = 6b_2 - 6b_1 + b_0$ and $l_4 = 20b_3 - 30b_2 + 12b_1 - b_0$ when b_0, b_1, b_2 and b_3 are

$$b_0 = \frac{1}{n}\sum_{i=1}^{n} x_{i:n},\ b_1 = \frac{1}{n}\sum_{i=1}^{n}\frac{(i-1)}{(n-1)}x_{i:n},$$
$$b_2 = \frac{1}{n}\sum_{i=1}^{n}\frac{(i-1)(i-2)}{(n-1)(n-2)}x_{i:n},$$
$$b_3 = \frac{1}{n}\sum_{i=1}^{n}\frac{(i-1)(i-2)(i-3)}{(n-1)(n-2)(n-3)}x_{i:n}.$$

L-moment Estimation

The first four theoretical L-moments of the IKum distribution are

$$\lambda_1 = \theta\,\beta\left(\theta, 1-\frac{1}{a}\right) - 1,$$
$$\lambda_2 = 2\,\theta\beta\left(1-\frac{1}{a}, 2\,\theta\right) - \theta\,\beta\left(\theta, 1-\frac{1}{a}\right),$$
$$\lambda_3 = \theta\,\beta\left(\theta, 1-\frac{1}{a}\right) - 6\,\theta\,\beta\left(1-\frac{1}{a}, 2\,\theta\right) + 6\,\theta\,\beta\left(1-\frac{1}{a}, 3\,\theta\right)$$

and

$$\lambda_4 = 12\,\theta\,\beta\left(1-\frac{1}{a}, 2\,\theta\right) - \theta\,\beta\left(\theta, 1-\frac{1}{a}\right) - 30\,\theta\,\beta\left(1-\frac{1}{a}, 3\,\theta\right) + 20\theta\,\beta\left(1-\frac{1}{a}, 4\,\theta\right)$$

where $\alpha > 1$ and $\theta > 0$.

The L-moments estimations of the parameters of the IKum distribution are obtained by equating the first two theoretical L-moments to the corresponding sample moments, i.e., l_1 and l_2, respectively. Since these equations can not be solved explicitly, iterative techniques are employed in obtaining L-moment estimates of the parameters of the IKum distribution.

ML Estimation

The ML methodology aims to obtain the parameters maximizing the likelihood function. The ML estimators of the IKum distribution are not provided here for brevity since they were obtained previously in [5]. See [5] for more information on the ML estimators of the IKum distribution.

Simulation Study

As mentioned previously a Monte-Carlo simulation is conducted for comparing efficiencies of the ML and L-moment estimation methods. For the simulation study $n = 25, 50, 100, 500$, and 1000 sample sizes are utilized. Performances of these methods are compared using Mean Squared Error (MSE) values for the cases considered. The simulations are implemented using the optimization toolbox in MatlabR2021 software and results are provided in Tables 3, 4 and 5. The formulas for the MSE criterion are,

$$MSE(\hat{a}) = E(\hat{a} - a)^2 \text{ and } MSE(\hat{\theta}) = E(\hat{\theta} - \theta)^2.$$

Table 3. Simulation results for a=3 and θ=1

	a=3			θ=1		
	Mean	Var	MSE	Mean	Var	MSE
			n=25			
ML	3.3658	0.8573	0.9911	1.1266	0.1089	0.1249
L-Moment	3.3192	0.8465	0.9484	1.1319	0.1027	0.1201
			n=50			
ML	3.1469	0.3863	0.4079	1.061	0.043	0.0467
L-Moment	3.1904	0.4346	0.4709	1.0719	0.0756	0.0808
			n=100			
ML	3.0826	0.1689	0.1757	1.029	0.0197	0.0205
L-Moment	3.1338	0.2585	0.2764	1.0495	0.0424	0.0449
			n=500			
ML	3.0156	0.0345	0.0348	1.005	0.0036	0.0036
L-Moment	3.0205	0.0634	0.0639	1.0068	0.0091	0.0092
			n=1000			
ML	3.0131	0.0168	0.017	1.0045	0.0018	0.0018
L-Moment	3.0165	0.0332	0.0334	1.0059	0.0049	0.0049

Table 4. Simulation results for a=2 and θ=2

	a=2			θ=2		
	Mean	Var	MSE	Mean	Var	MSE
			n=25			
ML	2.165	0.2676	0.2948	2.3199	0.7615	0.8638
L-Moment	2.3101	0.341	0.4372	2.5948	1.4917	1.8455
			n=50			
ML	2.0922	0.1108	0.1193	2.1524	0.2503	0.2735
L-Moment	2.1858	0.1661	0.2006	2.3179	0.5267	0.6277
			n=100			
ML	2.0518	0.0523	0.055	2.0867	0.1105	0.118
L-Moment	2.1181	0.0976	0.1115	2.2123	0.3014	0.3465
			n=500			
ML	2.0084	0.0096	0.0097	2.0205	0.018	0.0184
L-Moment	2.0436	0.0265	0.0284	2.0836	0.0684	0.0754
			n=1000			
ML	2.0033	0.0052	0.0052	2.0081	0.0089	0.009
L-Moment	2.022	0.0172	0.0177	2.0414	0.04	0.0417

Table 5. Simulation results for a=5 and θ=0.5

	a=5			θ=0.5		
	Mean	Var	MSE	Mean	Var	MSE
			n=25			
ML	5.8202	4.1115	4.7842	0.552	0.0198	0.0225
L-Moment	5.7911	4.9961	5.622	0.5378	0.0424	0.0439
			n=50			
ML	5.3649	1.5977	1.7309	0.5212	0.0086	0.0091
L-Moment	5.3999	2.0187	2.1786	0.5239	0.0188	0.0194
			n=100			
ML	5.1719	0.6874	0.717	0.5084	0.0036	0.0036
L-Moment	5.1828	0.9579	0.9913	0.508	0.008	0.0081
			n=500			
ML	5.0287	0.1265	0.1273	0.503	0.0007	0.0007
L-Moment	5.0301	0.1843	0.1852	0.5027	0.0016	0.0016
			n=1000			
ML	5.0112	0.0619	0.062	0.5008	0.0003	0.0003
L-Moment	5.0069	0.0958	0.0958	0.5	0.0008	0.0008

According to Table 3, for both of the parameters, when sample size n= 25, the L-moment estimation performed better and for the other sample sizes the ML estimation provided more effective estimations. When both of the parameters are set to 2, for all sample sizes ML estimation performed better as presented in Table 4. Similarly, for the parameter settings, α=5 and θ=0.5 ML estimations are superior for all sample sizes. Overall, it can be said that although ML estimations are superior in general, for larger samples MSE values are close between the rivals. For the small samples, although ML estimations provided good performance in one scenario the L-moment method is better. Consequently, the L-moment estimation has shown a performance worth examining especially for small sample sizes.

Return Period

The return period is the average time between the occurrence of a specific event, such as a natural disaster or weather event, of a certain magnitude or greater. If an event of magnitude x_T occurs once in T years, where x represents the precipitations, the probability (P) of the variable exceeding or equaling x in any given year is expressed as follows

$$P(x \geq x_T) = 1/T$$

Application

In this section, the data are modeled using the IKum distribution. In estimating the parameters, the ML and L-moments methods are used and estimates of the parameters are given in Table 6, in addition, precipitations for the 10, 25, 50, and 100-year return periods are provided.

Table 6. Estimates of the parameters of the IKum distribution

Parameters	$\hat{\alpha}$	$\hat{\theta}$
L-Moment	2.22678	1793.4614
ML	1.6121	210.1573

Values of evaluating criteria for the L-moment and ML methods are provided in Table 7. As is known lower values of the KS and RMSE, higher values of KS (p-value) and R^2 criteria indicate a better fit.

Table 7. Fitting performance for the IKum distribution using estimation methods

Criteria	R^2	RMSE	KS (*p*-value)
L-Moment	0.9758	0.0451	0.1016 (0.9169)
ML	0.9438	0.0594	0.1582 (0.4616)

It can be seen from Table 7 that the L-moment provided the smallest values for the KS and RMSE and the largest values for the R^2. Thus, the L-moments method performed better in all criteria and provided more accurate estimates than ML in modeling the precipitation records of Rize province. It should be noted that the sample size in this study can be regarded as small (n=27). Furthermore, fitted density plots for the ML and L-moment methods are provided in Figure 1.

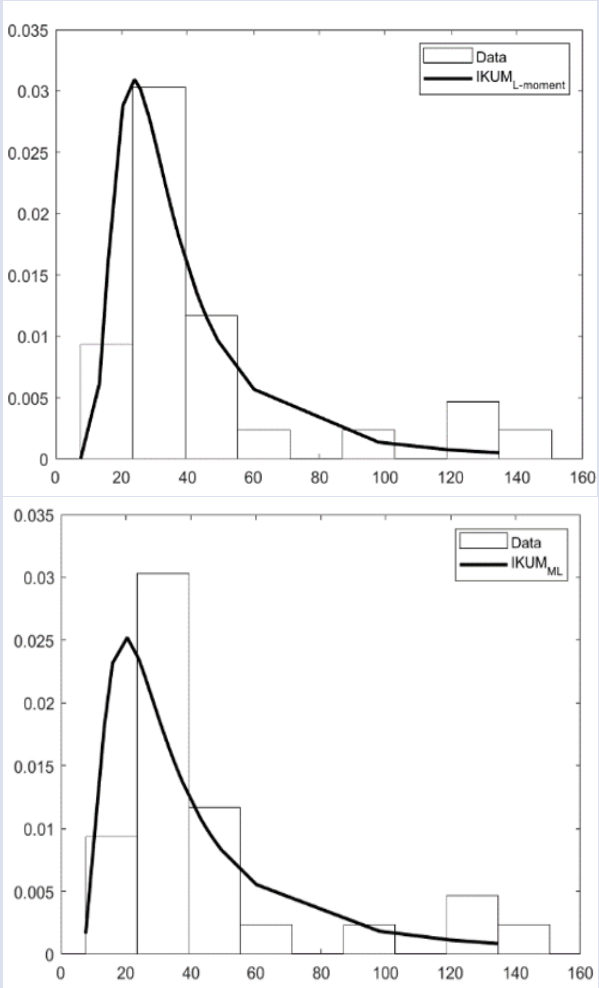

Figure 1. Fitted density plots of the IKum distribution using the L-moments and ML methods.

Figure 1 shows that the ML performed poorly than the L-moment at the peak of the distribution, although it fitted quite satisfactorily for the remaining part of the distribution. It can also be seen that the IKum distribution presents the advantages of having a longer right tail and the precipitation data in the right tail of the distribution are modeled quite well. Consequently, the results pointed out in Table 7 are also supported by the plots.

Return period information is a valuable tool for policymakers and planners, enabling them to make informed decisions within defined timeframes. The return periods are obtained for the precipitations of the Rize station for the ML and L-moment estimations. When the ML method is utilized, the precipitations equivalent to 10, 25, 50, and 100-year return periods are 110.4272, 199.6232, 309.3570, and 477.5745, respectively. However when the L-moment estimation is considered the precipitations are calculated as 78.4441, 120.6133, 165.7886, and 227.2125 for the given return periods. For instance, the precipitation value of 78.4441 can be considered as the average occurrence expected once every 10 years when the L-moment method is used. Thus, one can see how much the estimation method can differentiate the results. This situation may pose risks in terms of planning and emphasizes the importance of the accurate prediction of precipitation is essential for effective hydrological modeling.

Conclusion

Modeling precipitations using probability distributions generally paired with the L-moment method in parameter estimation. In this study, the estimations of the L-moment method and the ML are compared. In this context, the L-moment estimations of parameters for the IKum distribution are obtained for the first time. It has been noted that, although the simulation study favored the MLE method, the L-moments method was more effective in one scenario, when the sample size was small. In the application part, the annual maximum monthly precipitations recorded in Rize are modeled and results show that the L-moment method allows for estimating parameters more accurately. While simulation results often favor the MLE, real data applications may not always align with these results as in this case. Overall, the prediction of precipitation amounts is an important issue for the planning and design of flood protection systems. It is hoped that this study will contribute to the management of risks arising from floods.

Conflicts of interest

There are no conflicts of interest in this work.

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