Cumhuriyet Sci. J., 44(4) (2023) 741-752
DOI: https://doi.org/10.17776/csj.1221388

Cumhuriyet Science Journal

| csj.cumhuriyet.edu.tr | Founded: 2002 ISSN: 2587-2680 e-ISSN: 2587-246X Publisher: Sivas Cumhuriyet University

Application of Formable Transform for Solving Growth and Decay Problems, Logistic Growth Model and Prey-Predator Model

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Research Article

History
Received: 19/12/2022
Accepted: 15/10/2023

ABSTRACT

Integral transforms have become the focus of investigations, because they allow the solution of significant problems in the domains of science and engineering to be accomplished with a minimal number of straightforward calculations. In this study, growth and decay problems, which are crucial in fields such as biology, zoology, physics, chemistry, and economics, are solved utilizing the Formable transform. The Formable transform method is applied to the logistic growth model in population and prey-predator models. The effectiveness and simplicity of the use of the Formable transform in obtaining the solution to these problems are examples.

Keywords: Integral transforms, Formable transform, Growth and decay problem, Logistic growth model, Lotka-Volterra systems.

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Introduction

One of the earliest mathematical models for the dynamic change of populations was provided by Thomas Malthus. The Malthusian model posits that the rate of population growth in a country is directly proportional to its total population, denoted as $\mathcal{P}(t)$ at any given time t. Based on this theory, the growth of the population at a given time is directly proportional to the projected increase in population in the future. From a mathematical perspective, this assumption can be expressed such that κ is a constant of proportionality. The proportionality can be expressed using the following differential equation:

$$\frac{dP(t)}{dt} = \kappa P(t). \tag{1}$$

The mathematical representation of population growth is described as a first-order ordinary linear differential equation,

$$\frac{dP}{dt} = \kappa P$$

with initial condition

$$P(t_0) = P_0$$

where, P represents the population at time t, P_0 indicates the starting population at time t_0 and κ is a real number greater than zero. Mathematically, the decay problem of substances is written as a first-order ordinary linear differential equation

$$\frac{dP}{dt} = -\kappa P \tag{2}$$

with initial condition

$$P(t_0) = P_0$$

where, P represents the population at time t, P_0 represents the initial population at time t_0 and κ is a real number greater than zero. Based on these equations, it can be deduced that the population graph demonstrates exponential growth [1].

Due to limited resources such as food, space, and other factors, competition arises, resulting in a deviation from exponential population growth. As a result, the logistic model serves as a replacement for the Malthus model. The nonlinear biological models encompass a logistic growth model within a population, represented by the equation

$$\frac{dP(t)}{dt} = rP(t)\left(1 - \frac{P(t)}{\kappa}\right) \tag{3}$$

where r is a positive constant and κ is the carrying capacity. The function $P(t)$ denotes the population of the species at time t, while the expression $rP(t)\left(1 - \frac{P(t)}{\kappa}\right)$ represents the per capita growth rate. The non-dimensionalization of equation (3) is achieved by

$$v(\tau) = \frac{P(t)}{\kappa}, \qquad \tau = rt$$

which yields

$$\frac{dv}{d\tau} = v(1-v). \quad (4)$$

If the initial condition is given as $P(0) = P_0$, then $v(0) = \frac{P_0}{\kappa}$. Therefore, the analytical solution of equation (4) is obtained as

$$v(\tau) = \frac{1}{1+\left(\frac{\kappa}{P_0}-1\right)e^{-\tau}}. \quad (5)$$

A predator-prey relationship describes the dynamic between two species and how they affect one another. In this case, one species is in fact consuming the other species for food. A predator is an organism that consumes or hunts other organisms for food, while a prey is an organism that is slain by another organism for food. Examples of predators with their prey are the fox and the rabbit, the lion, and the zebra. The concept of predator-prey dynamics extends beyond animals and encompasses plants as well. The relationship between the grasshopper and the leaf serves as an illustrative example in this context. Consider the predator–prey models: Lotka–Volterra systems as an interacting species model to serve as a model for interacting species that are governed by

$$\frac{dN}{dt} = N(a-bP) \quad (6)$$

$$\frac{dP}{dt} = P(cN-d) \quad (7)$$

where a, b, c and d are constants [2]. Here $N = N(t)$ represents the prey population, and $P = P(t)$ represents the population of predators at the time t. The non-dimensionalization of the system (6)-(7) is achieved by

$$w(\tau) = \frac{cN(t)}{d}, \quad v(\tau) = \frac{bP(t)}{a}, \quad \tau = at, \quad \mu = d/a$$

and it turns into

$$\frac{dw}{d\tau} = w(1-v) \quad (8)$$

$$\frac{dv}{d\tau} = \mu[g(w,v)-v]. \quad (9)$$

Integral transforms are a valuable mathematical tool for solving a wide range of processes and phenomena in the fields of science, engineering, and real-life applications. These transforms allow us to express various complex problems in a mathematical framework, enabling their solution through rigorous mathematical techniques. The Laplace transform is widely recognized as the most commonly used among these various transforms. Many novel integral transformations, such as Laplace-Carson, Sumudu, Aboodh, Elzaki, Mohand, Sawi, Shehu, Sadik, Anuj, Rishi, Kamal, Kharrat-Toma, and Kashuri-Fundo have been developed in recent years in an effort to help scientists and engineers tackle increasingly complex issues in a variety of fields. Many researchers have analyzed the duality between various integral transforms, particularly the Laplace transform [3-10]. Rao [11] used the ZZ transform to handle natural growth and decay problems. Aggarwal et al. [12-18] applied Laplace, Elzaki, Kamal, Aboodh, Mahgoub, Mohand, Shehu transforms to solve growth and decay problems. Aggarwal and Bhatnagar [19] demonstrated how to use the Sadik transform to resolve growth and decay problems. Singh and Aggarwal [20] investigated population growth and decay problems with the Sawi transform. Verma et al. [21] scrutinized the applications of the Dines Verma Transform for handling population growth and decay problems. Bansal et al. [22] provided examples of applications for the Anuj transform. Pamuk and Soylu [23] utilized the Laplace transform method for logistic population growth and predator models. Additional works on integral transforms can be found in references [23-30].

A novel integral transform, the Formable transform, was described by Saadeh and Ghazal [31]. They proved some properties of this transform for handling both ordinary and partial differential equations. Additionally, they investigated the duality between the new transform and some existing transforms. Güngör [32] utilized Formable transform to solve linear Volterra integral equations of the convolution type. Ghazal et al. [33] introduced the concept of the double Formable transform, demonstrated its characteristics, and used it to solve partial integro-differential equations. Saadeh et al. [34] gave illustrative applications of heat equations via the use of numerical examples. For the purpose of resolving time-fractional partial differential equations, Saadeh et al. [35] used the Formable transform decomposition method. Prajapati and Meher [36] investigated a time-fractional Rosenau-Hyman model based on a KdV-like equation with compacton solutions using a robust homotopy analysis method with a formable transform. The Formable integral transform of the Hilfer-Prabhakar and its regularized variant of the Hilfer-Prabhakar fractional derivative were developed by Khalid and Alha [37].

The aim of this study is to present the concept of Formable transform as a technique that facilitates the solution of linear differential equations through growth and decay models as well as nonlinear differential equations through biological models, specifically those comprising a logistic growth model to study population dynamics and the prey-predator model to analyze ecological interactions.

Materials and Methods

This section will present the definition of the Formable transform, along with an exploration of its properties and its relationship to other widely recognized transforms.

Definition 1. [31] If there exists a positive number M that satisfying

$$|v(t)| \leq Me^{\alpha t}, M > 0, \alpha > 0, \forall t \geq 0$$

then the function $v(t)$ is said to have exponential order on every finite interval in $[0, +\infty)$.

Definition 2. [31] Over the set of functions

$$\mathcal{W} = \{v(t): \exists \gamma, \rho_1, \rho_2 > 0, |v(t)| < \gamma e^{\frac{t}{\rho_i}}, \text{if } t \in [0, \infty)\Big\},$$

the Formable integral transform of an exponential order function $v(t)$ is described as

$$\Re[v(t)] = \mathcal{V}(s,u) = s\int_0^\infty e^{-st} v(ut)dt. \quad (10)$$

This is equivalent to

$$\Re[v(t)] = \frac{s}{u}\int_0^\infty e^{\frac{-st}{u}} v(t)dt \quad (11)$$

where s and u are the variables of Formable transform.

The expression

$$\Re^{-1}[\mathcal{V}(s,u)] = v(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{1}{s} e^{\frac{st}{u}} \mathcal{V}(s,u)ds.$$

denotes the inverse Formable transform of a function $v(t)$.

Theorem 1. *(Linearity property)* [31] If $v_1(t)$ and $v_2(t)$ are two functions in $\mathcal{W}$, then $c_1v_1(t) + c_2v_2(t) \in \mathcal{W}$ where c_1 and c_2 are arbitrary constants, and

$$\Re[c_1v_1(t) + c_2v_2(t)] = c_1\Re[v_1(t)] + c_2\Re[v_2(t)].$$

Theorem 2. *(Formable transform of the derivative)* [31] Let's take the function $v^{(k)}(t)$ is the k-th derivative of the function $v(t)$. If $v^{(k)}(t) \in \mathcal{W}$ for $k = 0,1,2,\ldots$, then

$$\Re\left[v^{(k)}(t)\right] = \frac{s^k}{u^k}\mathcal{V}(s,u) - \sum_{m=0}^{k-1}\left(\frac{s}{u}\right)^{k-m} v^{(m)}(0).$$

The Formal transform and inverse of certain functions are presented below [31]:

$v(t)$	$\Re[v(t)] = \mathcal{V}(s,u)$
1	1
t	$\frac{u}{s}$
$\frac{t^n}{n!}$	$\frac{u^n}{s^n}$
$e^{\beta t}$	$\frac{s}{s-\beta u}$
$\frac{t^n}{n!}e^{\beta t}$	$\frac{su^n}{(s-\beta u)^{n+1}}$
$\sin(\beta t)$	$\frac{\beta su}{s^2+\beta^2u^2}$
$\cos(\beta t)$	$\frac{s^2}{s^2+\beta^2u^2}$
$\sinh(\beta t)$	$\frac{\beta su}{s^2-\beta^2u^2}$
$\cosh(\beta t)$	$\frac{s^2}{s^2-\beta^2u^2}$

$\mathcal{V}(s,u)$	$v(t) = \Re^{-1}[\mathcal{V}(s,u)]$
1	1
$\frac{u}{s}$	t
$\frac{u^n}{s^n}$	$\frac{t^n}{n!}$
$\frac{s}{s-\beta u}$	$e^{\beta t}$
$\frac{su^n}{(s-\beta u)^{n+1}}$	$\frac{t^n}{n!}e^{\beta t}$
$\frac{\beta su}{s^2+\beta^2u^2}$	$\sin(\beta t)$
$\frac{s^2}{s^2+\beta^2u^2}$	$\cos(\beta t)$
$\frac{\beta su}{s^2-\beta^2u^2}$	$\sinh(\beta t)$
$\frac{s^2}{s^2-\beta^2u^2}$	$\cosh(\beta t)$

The definitions of many integral transforms that can be found in [3-5, 7-10, 12-15, 17, 18, 20, 31] are given in a tabular format below:

Table 1. Definitions of some integral transforms

Integral Transform	***Definition***
Laplace transform	$\mathcal{L}[v(t)] = \int_0^\infty v(t)e^{-st}dt = F(s)$
Sumudu transform	$\mathcal{S}[v(t)] = \int_0^\infty v(st)e^{-t}dt = G(s)$
Elzaki transform	$E[v(t)] = s\int_0^\infty v(t)e^{\frac{-t}{s}}dt = C(s)$
Natural transform	$\mathbb{N}^+[v(t)](s,u) = \frac{1}{u}\int_0^\infty e^{\frac{-st}{u}}v(t)dt = N(s,u)$
Shehu transform	$\mathbb{S}[v(t)] = \int_0^\infty e^{\frac{-st}{u}}v(t)dt = Q(s,u)$
Aboodh transform	$\mathcal{A}[v(t)] = \frac{1}{s}\int_0 v(t)e^{-st}dt = A(s)$
Kamal transform	$\mathcal{K}[v(t)] = \int_0^\infty v(t)e^{\frac{-t}{s}}dt = H(s)$
Mohand transform	$\mathcal{M}[v(t)] = s^2\int_0^\infty v(t)e^{-st}dt = I(s)$
Sawi transform	$S[v(t)] = \frac{1}{s^2}\int_0^\infty v(t)e^{\frac{-t}{s}}dt = J(s)$

Let $\mathcal{V}(s,u)$ be the Formable transform of the function $v(t)$. Hence, the relationships between the Formable transform and other certain integral transforms are illustrated in the following manner:

- *Formable–Laplace duality* [31]: If $F(s)$ is the Laplace transform of the function $v(t)$, then

$$\mathcal{V}(s,1) = sF(s).$$

Indeed, it is clear that $\mathcal{V}(s,1) = s\int_0^\infty e^{-st}v(t)dt = sF(s)$ from (10).

- *Formable–Sumudu duality* [31]: If $G(u)$ represents the Sumudu transform of the function $v(t)$, then

$$\mathcal{V}(1,u) = G(u).$$

In fact, it is evident that $\mathcal{V}(1,u) = \int_0^\infty e^{-t}v(ut)dt = G(u)$ from (10).

- *Formable–Elzaki duality* [31]: If $C(u)$ is the Elzaki transform of the function $v(t)$, then

$$\mathcal{V}(1,u) = \frac{1}{u^2}C(u).$$

By using the equation (11), one gets $\mathcal{V}(1,u) = \frac{1}{u}\int_0^\infty e^{\frac{-t}{u}}v(t)dt = \frac{1}{u^2}\left(u\int_0^\infty v(t)e^{\frac{-t}{u}}dt\right) = \frac{1}{u^2}C(u)$.

- *Formable–Natural duality* [31]: If $N(s,u)$ is the Natural transform of the function $v(t)$, then

$$\mathcal{V}(s,u) = sN(s,u).$$

In fact, one obtains $\mathcal{V}(s,u) = \frac{s}{u}\int_0^\infty e^{\frac{-st}{u}}v(t)dt = s\frac{1}{u}\int_0^\infty e^{\frac{-st}{u}}v(t)dt = sN(s,u)$ from (11).

- *Formable–Shehu duality:* If $Q(s,u)$ is the Shehu transform of the function $v(t)$, then

$$\mathcal{V}(s,u) = \frac{s}{u}Q(s,u).$$

It is readily apparent that $\mathcal{V}(s,u) = \frac{s}{u}\int_0^\infty e^{\frac{-st}{u}}v(t)dt = \frac{s}{u}Q(s,u)$ from (11).

- *Formable–Aboodh duality:* If $A(u)$ is the Aboodh transform of the function $v(t)$, then

$$\mathcal{V}(1,u) = \frac{1}{u^2}A\left(\frac{1}{u}\right).$$

Indeed, one gets $\mathcal{V}(1,u) = \frac{1}{u}\int_0^\infty e^{\frac{-t}{u}}v(t)dt = \frac{1}{u^2}u\int_0^\infty e^{\frac{-t}{u}}v(t)dt = \frac{1}{u^2}A\left(\frac{1}{u}\right)$ by using the equation (11).

- *Formable–Kamal duality:* If $H(u)$ is the Kamal transform of the function $v(t)$, then

$$\mathcal{V}(1,u) = \frac{1}{u}H(u).$$

It is clear that $\mathcal{V}(1,u) = \frac{1}{u}\int_0^\infty e^{\frac{-t}{u}}v(t)dt = \frac{1}{u}H(u)$ from (11).

Applying of Formable Transform for Exponential Growth and Decay Model

In this part, the Formable transform is applied to find solutions to the problems of population growth and decay, which are mathematically described by equations by (1) and (2). Taking Formable transform on either side of the equation (1), we find

$$\Re\left[\frac{dP(t)}{dt}\right] = k\Re[P(t)].$$

By implementing the Formable transform of the derivative of the function, we get

$$\frac{s}{u}\Re[P(t)] - \frac{s}{u}P(0) = k\Re[P(t)].$$

Hence, we have

$$\left(\frac{s}{u} - k\right)\Re[P(t)] = \frac{s}{u}P_0$$
$$\Re[P(t)] = \frac{s}{s-ku}P_0. \qquad (12)$$

After applying the inverse Formable transform to either side of (12), we can write the result as

$$P(t) = \Re^{-1}\left[\frac{s}{s-uk}P_0\right]$$
$$P(t) = P_0\Re^{-1}\left[\frac{s}{s-ku}\right]$$
$$P(t) = P_0e^{kt}$$

which represents the desired population at the time t.

Applying the Formable transform on either side of equation (2), we find

$$\Re\left[\frac{dP(t)}{dt}\right] = -k\Re[P(t)].$$

Now, by implementing the Formable transform of the derivative of the function, we obtain

$$\frac{s}{u}\Re[P(t)] - \frac{s}{u}P(0) = -k\Re[P(t)].$$

Therefore, we have

$$\left(\frac{s}{u} + k\right)\Re[P(t)] = \frac{s}{u}P_0$$
$$\Re[P(t)] = \frac{s}{s+ku}P_0. \qquad (13)$$

After applying the inverse Formable transform on either side of (13), we get

$$P(t) = \Re^{-1}\left[\frac{s}{s+ku}P_0\right]$$
$$P(t) = P_0\Re^{-1}\left[\frac{s}{s+ku}\right]$$
$$P(t) = P_0e^{-kt}$$

which represents the desired population at time t.

Applying of Formable Transform for Logistic Growth Model

Take into account the model equation in the form of

$$\frac{dv}{dt} = v - h(v), \; v(0) = v_0 \qquad (14)$$

where h represents a nonlinear function of v. Therefore, we suppose that the solution, v of (14) has a representation in the form of an infinite series

$$v = v(t) = \sum_{n=0}^{\infty} c_n t^n \qquad (15)$$

and it fulfills the necessary requirements for the existence of the Formable transform. Once Formable transform is applied to either side of the Equation (14), it is obtained as

$$\Re\left[\frac{dv}{dt}\right] = \Re[v] - \Re[h(v)]$$

$$\frac{s}{u}\mathcal{V}(s,u) - \frac{s}{u}v(0) = \mathcal{V}(s,u) - \mathcal{H}(s,u)$$

where $\Re[v] = \mathcal{V}(s,u)$ and $\Re[h(v)] = \mathcal{H}(s,u)$ are Formable transforms of the functions $v(t)$ and $h(v)$, respectively. By rearranging the terms in the equation, the expression $\mathcal{V}(s,u)$ can be determined as

$$\mathcal{V}(s,u) = v_0\frac{s}{s-u} - \frac{u\mathcal{H}(s,u)}{s-u}. \qquad (16)$$

Therefore, under the assumption that the inverse Formable transform $\Re^{-1}$ exists and applying it to the expression (16), the equation can be expressed as follows:

$$v(t) = v_0e^t - \Re^{-1}\left[\frac{u\mathcal{H}(s,u)}{s-u}\right].$$

Applying of Formable Transform for Prey-Predator Model

Let us consider the system of non-linear differential equations that determines the predator-prey model.

$$\frac{dw}{dt} = w - h(w,v) \qquad (17)$$
$$\frac{dv}{dt} = \mu[g(w,v) - v] \qquad (18)$$

with initial conditions

$$w(0) = w_0, v(0) = v_0 \qquad (19)$$

where h and g are nonlinear functions of w and v and also μ be a positive constant. It is assumed that the solutions w and v of the system (17)-(18) possess infinite series expansions in the following form:

$$w(t)=\sum_{n=o}^{\infty} a_n t^n, v(t)=\sum_{n=0}^{\infty} c_n t^n. \quad (20)$$

Furthermore, the necessary criteria for the existence of their Formable transforms are satisfied by them. By utilizing the Formable transform for the equations (17)-(18) and utilizing equation (19), we obtain

$$\frac{s}{u}\mathcal{W}(s,u)-\frac{s}{u}w_0=\mathcal{W}(s,u)-\mathcal{H}(s,u) \quad (21)$$

$$\frac{s}{u}\mathcal{V}(s,u)-\frac{s}{u}v_0=\mu[G(s,u)-\mathcal{V}(s,u)] \quad (22)$$

where
$\Re[w(t)]=\mathcal{W}(s,u)$, $\Re[h(w(t),v(t))]=\mathcal{H}(s,u)$, $\Re[v(t)]=\mathcal{V}(s,u)$, $\Re[g(w(t),v(t))]=G(s,u)$ are the Formable transforms of the functions $w(t), h(w(t),v(t)), v(t)$ and $g(w(t),v(t))$, respectively. By solving the equations (21)-(22) for $\mathcal{W}(s,u)$ and $\mathcal{V}(s,u)$, one gets

$$\mathcal{W}(s,u)=\frac{s}{s-u}w_0-\frac{u}{s-u}\mathcal{H}(s,u) \quad (23)$$

$$\mathcal{V}(s,u)=\frac{s}{s+\mu u}v_0+\frac{u\mu}{s+\mu u}G(s,u). \quad (24)$$

Assuming inverse Formable transforms exist and utilizing them to the system, we obtain

$$w(t)=w_0e^t-\Re^{-1}\left[\frac{u\mathcal{H}(s,u)}{s-u}\right]$$

$$v(t)=v_0e^{-\mu t}+\mu\Re^{-1}\left[\frac{u}{s+\mu u}G(s,u)\right]$$

desired solutions to the initial value problem (17)-(19).

Applications

This section presents numerical examples to illustrate the efficacy of the Formable transform in problem-solving.

Example 1. The population of a city experiences growth at a rate that is directly proportional to the current number of people living there. Estimate how many people lived in the city at the beginning if the population has doubled after three years and 30000 after four years.
The mathematical model of the mentioned problem can be expressed as

$$\frac{dP(t)}{dt}=kP(t). \quad (25)$$

where k is the constant of proportionality and P represents number of individuals that are currently living in the city at any given time t. Suppose that P_0 represents the beginning population of the city at $t=0$. By applying the Formable transform to both sides of equation (25), the following result is obtained:

$$\Re\left[\frac{dP(t)}{dt}\right]=k\Re[P(t)]$$

$$\frac{s}{u}\Re[P(t)]-\frac{s}{u}P(0)=k\Re[P(t)].$$

Substituting the condition $P(0)=P_0$ at $t=0$, we acquire

$$\left(\frac{s}{u}-k\right)\Re[P(t)]=\frac{s}{u}P_0$$

$$\Re[P(t)]=\frac{s}{s-ku}P_0. \quad (26)$$

Upon applying the inverse Formable transform on either side of (26), we find

$$P(t)=\Re^{-1}\left[\frac{s}{s-ku}P_0\right]$$

$$P(t)=P_0\Re^{-1}\left[\frac{s}{s-ku}\right]$$

$$P(t)=P_0e^{kt}.$$

Since $P=2P_0$ at $t=3$, we can write

$$P_0e^{3k}=2P_0$$

$$e^{3k}=2$$

$$k=\frac{1}{3}\ln 2\cong 0.23104. \quad (27)$$

Now, substituting the value of k found in (27) and using the condition $P=30000$ at $t=4$, we have

$$30000=P_0e^{4\frac{1}{3}\ln 2}$$

$$P_0\cong 11905$$

which is the required number of people who lived in the city at the beginning.

Example 2. Bacteria in a certain culture increase at a rate proportional to the number present. Estimate the number of bacteria at the beginning of a certain culture after six hours, if the number of bacteria grows from 500 to 1500 in three hours.
The mathematical model of the mentioned problem can be expressed as

$$\frac{dP(t)}{dt}=kP(t). \quad (28)$$

where P denotes the number of bacteria at any time t and k is the constant of proportionality. The result of using the Formable transform on either side of (28) is

$$\frac{s}{u}\Re[P(t)]-\frac{s}{u}P(0)=k\Re[P(t)].$$

Since the initial amount is 500 at $t = 0$, we have

$$\left(\frac{s}{u} - k\right)\Re[P(t)] = 500\frac{s}{u}$$

$$\Re[P(t)] = \frac{s}{s - ku}500 \quad (29)$$

Having applied inverse Formable transform on either side of (29), we find

$$P(t) = \Re^{-1}\left[\frac{s}{s - ku}500\right]$$

$$P(t) = 500\Re^{-1}\left[\frac{s}{s - ku}\right]$$

$$P(t) = 500e^{kt}.$$

By using the another given condition $P = 1500$ at $t = 3$, we find

$$500e^{3k} = 1500$$

$$e^{3k} = 3$$

$$k = \frac{1}{3}\ln 3 \cong 0.23104 \quad (30)$$

We are looking for $t = 6$, so we get the number of bacteria present in a certain culture as

$$P(6) = 500e^{6\frac{1}{3}\ln 3} \cong 4500$$

by substituting the value of k found in (30).

Example 3. The decay of a radioactive substance is recognized to occur at a rate that is directly proportional to the quantity of the substance present. Find the half-life of the radioactive substance, if 50 milligrams of the substance are originally present and the radioactive substance has lost 20 percent of its original mass after five hours.
The mathematical model of the mentioned problem can be expressed as

$$\frac{dP(t)}{dt} = -kP(t) \quad (31)$$

where P represents the quantity of radioactive substance at time t and k is the proportionality constant. Suppose that P_0 is the initial amount of the radioactive substance at time $t = 0$. When we apply the Formable transform on either side of (31), we get the following result:

$$\Re\left[\frac{dP(t)}{dt}\right] = -k\Re[P(t)]$$

$$\frac{s}{u}\Re[P(t)] - \frac{s}{u}P(0) = -k\Re[P(t)].$$

Substituting the condition $P(0) = P_0$ at $t = 0$, we acquire

$$\left(\frac{s}{u} + k\right)\Re[P(t)] = \frac{s}{u}P_0$$

$$\Re[P(t)] = \frac{s}{s + ku}50. \quad (32)$$

Having applied the inverse Formable transform on either side of (32), we obtain

$$P(t) = \Re^{-1}\left[\frac{s}{s + ku}50\right]$$

$$P(t) = 50\Re^{-1}\left[\frac{s}{s + ku}\right]$$

$$P(t) = 50e^{-kt}. \quad (33)$$

At time $t = 5$, the radioactive substance has lost 20 percent of its original mass of 50 milligrams. Hence $P = 50 - 10 = 40$. By using this in (33), we get

$$40 = 50e^{-5k}.$$

$$e^{-5k} = 0.8$$

$$k = -\frac{1}{5}\ln 0.8 \cong 0.04462 \ldots \quad (34)$$

We required t when $P = \frac{P_0}{2} = 25$. Therefore, we can write

$$25 = 50e^{-kt}$$

from (31). By substituting the value of k from (34), we obtain

$$25 = 50e^{-0.04462t}$$

$$e^{-0.04462t} = 0.5$$

$$t = -\frac{1}{0.04462}\ln 0.5$$

$t \cong$15.531 hours

which is the desired half-time of the radioactive substance.

In Examples 1-3, it is demonstrated that Formable transform successfully determines the solutions to population growth and decay problems solved by other integral transforms studied in [11-22]. Upon individually solving the instances provided in the references, it will be seen that the same results are obtained with the Formable transform.

Example 4. Let us consider the logistic growth model equation (14) where $P_0 = 2$ and $\kappa = 1$. Hence v_0 can be expressed as $v_0 = \frac{P_0}{\kappa} = 2$. We set $h(v) = v^2$ as in (4) so that one finds

$$h(v) = \left(\sum_{n=0}^{\infty} c_n t^n\right)^2 = (c_0 + c_1 t + c_2 t^2 + \cdots + c_n t^n + \cdots)^2$$
$$= c_0^2 + 2c_0c_1 t + (2c_0c_2 + c_1^2)t^2 + (2c_0c_3 + 2c_1c_2)t^3 + \cdots.$$

Implementing Formable transform to each side of the equation

$$\Re[h(v)] = \mathcal{H}(s,u) = c_0^2 + 2c_0c_1 \frac{u}{s} + (2c_0c_2 + c_1^2)\, 2!\, \frac{u^2}{s^2} + (2c_0c_3 + 2c_1c_2)\, 3!\, \frac{u^3}{s^3} + \cdots.$$

One obtains

$$\mathcal{V}(s,u) = v_0 \frac{s}{s-u} - \frac{u\mathcal{H}(s,u)}{s-u}$$
$$= \frac{2s}{s-u} - \left[\frac{uc_0^2}{s-u} + \frac{2c_0c_1u^2}{s(s-u)} + 2!\frac{(2c_0c_2 + c_1^2)u^3}{s^2(s-u)} + 3!\frac{(2c_0c_3 + 2c_1c_2)u^4}{s^3(s-u)} + \cdots\right]$$
$$= \frac{2s}{s-u} - \left[\left(\frac{s}{s-u} - 1\right)c_0^2 + \left(\frac{s}{s-u} - 1 - \frac{u}{s}\right)2c_0c_1 + 2!\left(\frac{s}{s-u} - 1 - \frac{u}{s} - \frac{u^2}{s^2}\right)(2c_0c_2 + c_1^2)\right.$$
$$\left. + 3!\left(\frac{s}{s-u} - 1 - \frac{u}{s} - \frac{u^2}{s^2} - \frac{u^3}{s^3}\right)(2c_0c_3 + 2c_1c_2) + \cdots\right]$$

by using (16). Upon application of the inverse Formable transform to this equation yields

$$c_0 + c_1 t + c_2 t^2 + c_3 t^3 + \cdots = 2\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right)$$
$$-(c_0^2 + 2c_0c_1 + 4c_0c_2 + 2c_1^2 + 12c_0^2 2c_0c_3 + 12c_1c_2 + \cdots)\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right)$$
$$+(c_0^2 + 2c_0c_1 + 4c_0c_2 + 2c_1^2 + 12c_0^2 2c_0c_3 + 12c_1c_2 + \cdots)$$
$$+(2c_0c_1 + 4c_0c_2 + 2c_1^2 + 12c_0^2 2c_0c_3 + 12c_1c_2 + \cdots)t$$
$$+(2c_0c_1 + c_1^2 + 6c_0^2 2c_0c_3 + 6c_1c_2 + \cdots)t^2 + (2c_0c_3 + 2c_1c_2 + \cdots)t^3 + \cdots$$
$$= 2\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right) - c_0^2 t - \left(\frac{c_0^2}{2} + c_0c_1\right)t^2 - \left(\frac{c_0^2}{6} + \frac{c_0c_1}{3} + \frac{2c_0c_2}{3} + \frac{c_1^2}{3}\right)t^3 - \cdots$$
$$= 2 + (2 - c_0^2)t + \left(1 - \frac{c_0^2}{2} - c_0c_1\right)t^2 + \left(\frac{1}{3} - \frac{c_0^2}{6} - \frac{c_0c_1}{3} - \frac{2c_0c_2}{3} - \frac{c_1^2}{3}\right)t^3 + \cdots$$

from (15). When the coefficients of power t are equated, the result is

$$c_0 = 2,$$
$$c_1 = 2 - c_0^2 \Rightarrow c_1 = -2,$$
$$c_2 = 1 - \frac{c_0^2}{2} - c_0c_1 \Rightarrow c_2 = 3,$$
$$c_3 = \frac{1}{3} - \frac{c_0^2}{6} - \frac{c_0c_1}{3} - \frac{2c_0c_2}{3} - \frac{c_1^2}{3} \Rightarrow c_3 = -\frac{13}{3},$$
$$\vdots$$

and so on. Consequently, the solution $v(t)$ is obtained from (15) as follows

$$v(t) = 2 - 2t + 3t^2 - \frac{13}{3}t^3 + \cdots$$

that is the closed form exact solution obtained in (5). This solution is identical to the one discovered in [23].

The nth partial sums of the series (15) is represented by $\varphi_n(t)$ which is equivalent to

$$\varphi_n(t) = \sum_{m=0}^{n} c_m t^m. \qquad (35)$$

Based on the observation of Figure 1, it is evident that a highly accurate approximation of the exact solution for the logistic growth model within the time interval [0,0.20] has been achieved by computing only four terms of the series in (35). This indicates that the rate at which the Formable transform method converges is highly rapid. Furthermore, it is possible to minimize the overall errors and obtain a reasonably accurate estimation of the exact solution for $t \geq 0.2$ by incorporating new terms into the series.

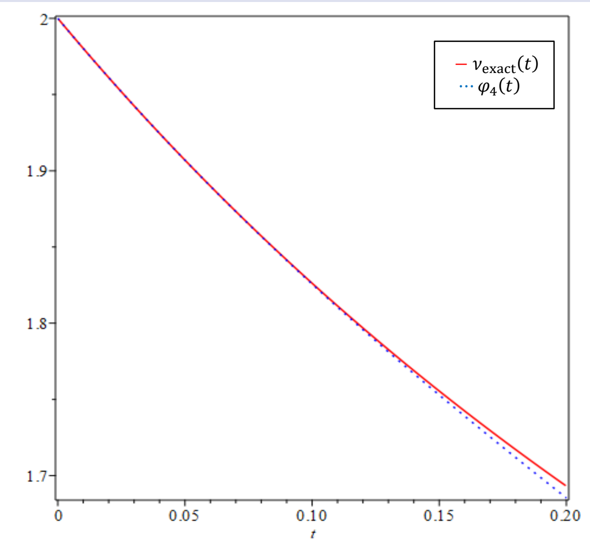

Figure 1. The resolution of the logistic growth model in population dynamics

Example 5. Consider the differential equation system that governs the predator and prey model

$$\frac{dw}{dt} = w - wv \tag{36}$$

$$\frac{dv}{d\tau} = wv - v \tag{37}$$

with initial data $w(0) = 1.3, v(0) = 0.6$.
Assume that $w(t) = \sum_{n=o}^{\infty} a_n t^n, v(t) = \sum_{n=0}^{\infty} c_n t^n$ be solutions of the system of (36)-(37). Therefore, we have

$$h(w,v) = g(w,v) = wv = \left(\sum_{n=0}^{\infty} a_n t^n\right)\left(\sum_{n=0}^{\infty} c_n t^n\right)$$
$$= a_0c_0 + (a_0c_1 + a_1c_0)t + (a_0c_2 + a_1c_1 + a_2c_0)t^2 + (a_0c_3 + a_1c_2 + a_2c_1 + a_3c_0)t^3 + \cdots.$$

The corresponding Formable transform of these functions are

$$\Re[wv] = a_0c_0\Re[1] + (a_0c_1 + a_1c_0)\Re[t] + (a_0c_2 + a_1c_1 + a_2c_0)\Re[t^2] + (a_0c_3 + a_1c_2 + a_2c_1 + a_3c_0)\Re[t^3] + \cdots$$

$$= a_0c_0 + (a_0c_1 + a_1c_0)\frac{u}{s} + (2a_0c_2 + 2a_1c_1 + 2a_2c_0)\frac{u^2}{s^2} + (6a_0c_3 + 6a_1c_2 + 6a_2c_1 + 6a_3c_0)\frac{u^3}{s^3} + \cdots.$$

Here $\mathcal{H}(s,u) = G(s,u) = \Re[wv]$. From (23) and (24), it is found that

$$\begin{aligned}
\mathcal{W}(s,u) &= \frac{s}{s-u}1.3 - \frac{u}{s-u}\mathcal{H}(s,u) \\
&= \frac{s}{s-u}1.3 - \frac{u}{s-u}a_0c_0 - (a_0c_1 + a_1c_0)\frac{u^2}{s(s-u)} - (a_0c_2 + a_1c_1 + a_2c_0)\frac{2u^3}{s^2(s-u)} \\
&\quad -(a_0c_3 + a_1c_2 + a_2c_1 + a_3c_0)\frac{6u^4}{s^3(s-u)} + \cdots \\
&= \frac{s}{s-u}1.3 - a_0c_0\left(\frac{s}{s-u} - 1\right) - (a_0c_1 + a_1c_0)\left(\frac{s}{s-u} - 1 - \frac{u}{s}\right) \\
&\quad -2!\,(a_0c_2 + a_1c_1 + a_2c_0)\left(\frac{s}{s-u} - 1 - \frac{u}{s} - \frac{u^2}{s^2}\right) \\
&\quad -3!\,(a_0c_3 + a_1c_2 + a_2c_1 + a_3c_0)\left(\frac{s}{s-u} - 1 - \frac{u}{s} - \frac{u^2}{s^2} - \frac{u^3}{s^3}\right) + \cdots
\end{aligned}$$

$$\begin{aligned}
\mathcal{V}(s,u) &= \frac{s}{s+u}0.6 + \frac{u}{s+u}G(s,u) \\
&= \frac{s}{s+u}0.6 + \frac{u}{s+u}a_0c_0 + (a_0c_1 + a_1c_0)\frac{u^2}{s(s+u)} + (a_0c_2 + a_1c_1 + a_2c_0)\frac{2u^3}{s^2(s+u)} \\
&\quad +(a_0c_3 + a_1c_2 + a_2c_1 + a_3c_0)\frac{6u^4}{s^3(s+1)} + \cdots \\
&= \frac{s}{s+u}0.6 + a_0c_0\left(1 - \frac{s}{s+u}\right) + (a_0c_1 + a_1c_0)\left(-1 + \frac{u}{s} + \frac{s}{s+u}\right) \\
&\quad +2!\,(a_0c_2 + a_1c_1 + a_2c_0)\left(1 - \frac{u}{s} + \frac{u^2}{s^2} - \frac{s}{s+u}\right) \\
&\quad +3!\,(a_0c_3 + a_1c_2 + a_2c_1 + a_3c_0)\left(-1 + \frac{u}{s} - \frac{u^2}{s^2} + \frac{u^3}{s^3} + \frac{s}{s+u}\right) + \cdots .
\end{aligned}$$

When the inverse Formable transform is applied to these equations, one gets

$$a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots = 1.3\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right) - a_0 c_0 t - (a_0 c_0 + a_0 c_1 + a_1 c_0)\frac{t^2}{2!}$$
$$-(a_0 c_0 + a_0 c_1 + a_1 c_0 + 2a_0 c_2 + 2a_1 c_1 + 2a_2 c_0)\frac{t^3}{3!} - \cdots$$
$$= 1.3 + (1.3 - a_0 c_0)t + (1.3 - a_0 c_0 - a_0 c_1 - a_1 c_0)\frac{t^2}{2!}$$
$$+(1.3 - a_0 c_0 - a_0 c_1 - a_1 c_0 - 2a_0 c_2 - 2a_1 c_1 - 2a_2 c_0)\frac{t^3}{3!} + \cdots$$

and

$$c_0 + c_1 t + c_2 t^2 + c_3 t^3 + \cdots = 0.6\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \cdots\right) - a_0 c_0 t - (a_0 c_0 + a_0 c_1 + a_1 c_0)\frac{t^2}{2!}$$
$$-(a_0 c_0 + a_0 c_1 + a_1 c_0 + 2a_0 c_2 + 2a_1 c_1 + 2a_2 c_0)\frac{t^3}{3!} - \cdots$$
$$= 0.6 + (a_0 c_0 - 0.6)t + (0.6 - a_0 c_0 + a_0 c_1 + a_1 c_0)\frac{t^2}{2!}$$
$$+(-0.6 + a_0 c_0 - a_0 c_1 - a_1 c_0 + 2a_0 c_2 + 2a_1 c_1 + 2a_2 c_0)\frac{t^3}{3!} + \cdots \ .$$

If the coefficients are equalized to powers of t, it is found as

$$a_0 = 1.3$$

$$a_1 = 1.3 - a_0 c_0$$
$$a_1 = 0.52$$

$$a_2 = \frac{1}{2!}(1.3 - a_0 c_0 - a_0 c_1 - a_1 c_0)$$
$$a_2 = -0.013$$

$$a_3 = \frac{1}{3!}(1.3 - a_0 c_0 - a_0 c_1 - a_1 c_0 - 2a_0 c_2 - 2a_1 c_1 - 2a_2 c_0)$$
$$a_3 = -0.1122$$
$$\vdots$$

$$c_0 = 0.6$$

$$c_1 = a_0 c_0 - 0.6$$
$$c_1 = 0.18$$

$$c_2 = \frac{1}{2!}(0.6 + a_0 c_1 + a_1 c_0 - a_0 c_0)$$
$$c_2 = 0.183$$

$$c_3 = \frac{1}{3!}(-0.6 + a_0 c_0 - a_0 c_1 - a_1 c_0 + 2a_0 c_2 + 2a_1 c_1 + 2a_2 c_0)$$
$$c_3 = 0.0469$$
$$\vdots$$

The subsequent terms of the series can be obtained using this method. By substituting these terms into equation (20), we obtain the approximate solutions for the problem described by equations (36)-(37):

$$w(t) = 1.3 + 0.52t - 0.013t^2 - 0.1122t^3 - \cdots$$
$$v(t) = 0.6 + 0.18t + 0.183t^2 + 0.0469t^3 + \cdots \ .$$

The outcomes we have presently achieved exhibit congruence with the findings derived from the study conducted in reference [23].

Figure 2 shows the approximate solutions to system (36)-(37) which are obtained by Formable transform using only four terms of the series (20). This system's numerical solutions are shown in Figure 3. The system's numerical solutions are obtained using Ode45, which is a built-in ordinary differential equation solver in MATLAB.

The comparison of the two figures reveals a significant level of closeness between the two solutions for w (prey population) and v (predator population) within the time interval of $[0,2]$. Adding more terms to the series provides an even closer approximation to the numerical answer for $t \geq 1{,}6$, as previously mentioned in the context of the logistic growth model.

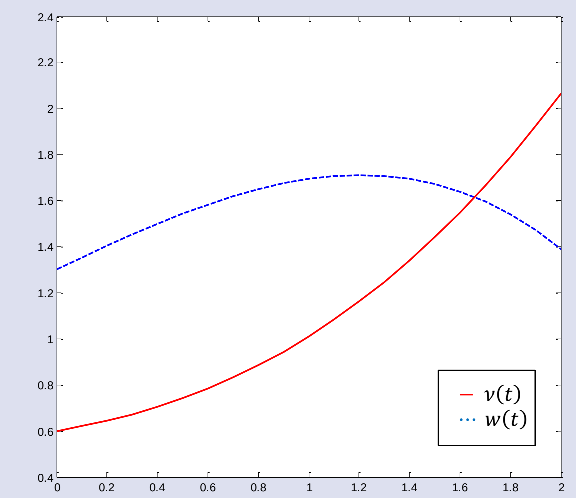

Figure 2. Approximate solutions to the system (36)-(37) by Formable transform method

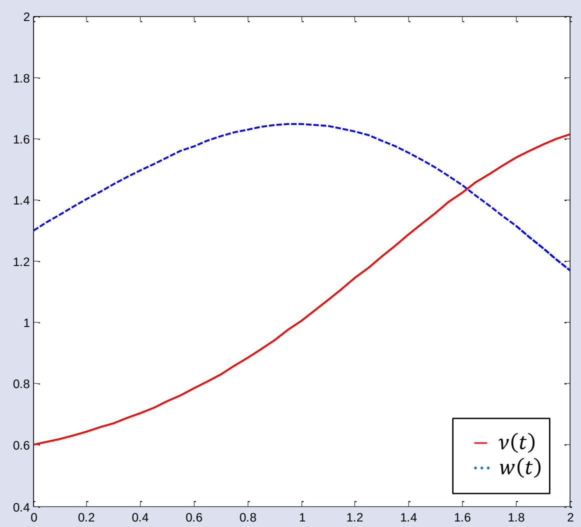

Figure 3. Numerical solutions to the system (36)-(37)

Conclusions

Differential equations are of paramount importance in the development of mathematical models to describe physical phenomena. In this study, we successfully applied the Formable transform to growth and decay problems. We further strengthened its application to growth and decay problems with several numerical examples, showing that the Formable transform is a highly handy approach for solving differential equations. These applications demonstrate how the Formable transform may be used to resolve growth and decay problems without complex calculations. The Formable transform method provides very accurate approximate solutions to nonlinear problems in mathematics, biology, physics, etc. Furthermore, it does not necessitate the process of linearization or rely on biologically implausible assumptions. As a result of its efficiency and convenience, this transform is a powerful tool for numerically solving a wide variety of interesting mathematical models involving differential equations.

Conflict of interests

The authors state that did not have conflict of interests.

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