Cumhuriyet Science Journal

Cumhuriyet Sci. J., 45(3) (2024) 614-617
DOI: https://doi.org/10.17776/csj.1212015

| csj.cumhuriyet.edu.tr | Founded: 2002 ISSN: 2587-2680 e-ISSN: 2587-246X Publisher: Sivas Cumhuriyet University

Modeling of Growth in Turkeys by Nonlinear Regression Models

Esin Köksal Babacan [1,a,*], Umut Demir [2,b]

[1] Statistics Department, Science Faculty, Ankara University, Ankara, Türkiye.
[2] Tokat-Erzincan Avenue, Kemal Paşa Street, Suşehri District Police Department, Suşehri, Sivas, Türkiye.
*Corresponding author

Research Article

History
Received: 02/12/2022
Accepted: 20/08/2024

ABSTRACT

Nonlinear regression models are commonly used in many areas, such as physics, chemistry, biology, and engineering. In these models, the solution of normal equations is more difficult than normal equations of linear regression models. Iterative algorithms are used to solve these equations. It is highly important to choose an appropriate initial value while using these algorithms. The values obtained during the study or from previous studies can be used as an initial value. With an inappropriately chosen initial value, the number of iterations will increase, and convergence may also not occur. In this study, the nonlinear Gompertz, Richards, and Weibull models of turkey growth were considered by taking the data on female turkey weight, and it was investigated which of these models was the most appropriate model for turkey growth using the R program.

Keywords: Nonlinear regression, Growth models, Convergence.

[a] ekoksal@science.ankara.edu.tr https://orcid.org/0000-0002-9649-5276
[b] umut.1991.demir@gmail.com https://orcid.org/0000-0002-6196-0713

Introduction

Nonlinear regression models are commonly used in many areas, such as physics, chemistry, biology, and engineering. The model's nonlinearity originates from its geometry or the parameters in the model. Linearity refers to the linearity of the variables or parameters in the model. The linearity mentioned here is whether the parameters in the model are linear or not. In the model, if the value obtained when the first-degree derivative is taken according to the parameters is independent of the parameter, in other words, if it is a constant value, the model is called linear. Estimates in linear models are unbiased least squares estimates with minimum variance. However, estimates in nonlinear models are usually biased and may not have minimum variance. When a large sample is taken, the parameter estimation results will best fit the linear case approximation.

In nonlinear regression models, the solution of normal equations is more difficult than normal equations of linear regression models. Algorithms, such as Gauss-Newton, Newton-Raphson, and steepest descent, are used to solve these equations. It is highly important to choose an appropriate initial value while using these algorithms. The values obtained during the study or from previous studies can be used as an initial value. With an inappropriately chosen initial value, the number of iterations will increase, and convergence may also not occur.

In the literature, there are also some heuristic methods that are not affected much by the starting point and do not require derivatives. Nelder-Mead (1964) developed the Nelder-Mead simplex method, which allows finding the local minimum of a multivariate function [1]. In 1964 and 1973, Huber conducted studies on robust estimators and developed M-estimators [2,3].

In their study, Bates and Watts (1988) included nonlinear regression models and applications [4]. In the study by Çağlar (1995), the likelihood function was maximized by the Gauss-Newton method in nonlinear regression models, and the model parameters were estimated [5]. In the study by Aksoy (1996), after the nonlinear regression model was established, autocorrelations between model residuals were also included in the model, and nonlinear parameters were estimated by the linearization method [6].

Şengül and Kiraz (2003) used nonlinear regression models to obtain growth models for male and female turkeys [7]. In their study, they considered the Gompertz, Logistic, Morgan-Mercer-Flodin (MMF), and Richards nonlinear models, and at the end of the study, the researchers concluded that the Gompertz, Logistic, and Richards models were mathematically appropriate.

In their study, Çelik et al. (2022) expressed growth for male and female turkeys with a multiphasic model [8].

In the second section of the study, the Gompertz, Richards, and Weibull nonlinear models, which are the most used growth models in the literature, were given, and it was explained how the initial values to be used for parameter estimation in these models could be obtained by transforming the models. In the third section, namely the application section, the data on female turkey weight in the study by Çelik et al. (2022) were taken into account, and the suitability of these data to the Gompertz, Richards, and Weibull models was examined [9]. The conclusion and discussion were included in the final section.

Materials and Methods

The models most commonly used as growth models in living things in the literature are the Gompertz model, the Richards model, and the Weibull model. The formats of the models are given in Table 1, where x represents the time and y represents the body weight.

Table 1. Growth Models

Gompertz model	$y = \beta_1 e^{-\beta_2 e^{-\beta_3 x}} + \varepsilon$
Richards model	$y = \frac{\beta_1}{(1+\beta_2 e^{-\beta_3 x})^{\frac{1}{\beta_4}}} + \varepsilon$
Weibull model	$y = \beta_1 - \beta_2 e^{-\beta_3 x^{\beta_4}} + \varepsilon$

where $x = 1,2,\ldots,n$, β_1 represents the maximum value that y will take in case of $n \to \infty$, β_2 represents the scale parameter, β_3 refers to the location parameter, β_4 denotes the inflection parameter that determines the shape of the function, and ε represents the Normally distributed random error term with a mean of 0 and a variance of σ^2.

As is seen, these models are nonlinear models according to their parameters. Accordingly, the model's parameters can be estimated using parameter estimation methods in nonlinear regression models. The most important thing for the parameter estimation of the models is to determine the appropriate initial values. When the appropriate initial value is not given, the parameter estimation results cannot be achieved, and an error occurs. Therefore, it is crucial to determine the initial values of the parameters appropriately. In this study, transformations to the models were used to determine the initial values of the models' parameters [10]. How the initial values for each model are obtained is explained in detail in the following section.

Let us consider the Gompertz model first. A logarithmic transformation can be applied to the model to determine the initial values where the model is

$$y = \beta_1 e^{-\beta_2 e^{-\beta_3 x}} + \varepsilon_x \quad (1)$$

when the natural logarithm of both sides is taken, where

$$ln(y) = ln(\beta_1) - \beta_2 e^{\beta_3 x} + ln(\varepsilon_x)$$

and, if necessary arrangements are made, it is obtained as follows:

$$ln\left(ln\left(\frac{\beta_1}{y}\right)\right) = ln(\beta_2) - \beta_3 x$$

In this equation obtained, if

$$y^* = ln\left(ln\left(\frac{\beta_1}{y}\right)\right), \beta_2^* = ln(\beta_2), \beta_3^* = -\beta_3$$

is taken,

$$y^* = \beta_2^* + \beta_3^* x \quad (2)$$

linear regression model is obtained. The initial value can be taken as $\hat{\beta}_1^{(0)} > max(y)$ for β_1. Accordingly, the parameters of the linear regression model (2) obtained by applying the mentioned transformations are estimated to be able to determine the initial values for other parameters. Thus, by substituting the estimates for β_2^* and β_3^*, the initial values for the parameters are obtained with the inverse transformation:

$$\hat{\beta}_2^{(0)} = e^{\hat{\beta}_2^*}, \hat{\beta}_3^{(0)} = -\hat{\beta}_3^*.$$

Second, let us consider the Richards model. A logarithmic transformation can be applied to the model to determine the initial values where the model is

$$y = \frac{\beta_1}{(1+\beta_2 e^{-\beta_3 x})^{1/\beta_4}} + \varepsilon_x \quad (3)$$

when the natural logarithm of both sides is taken, where

$$ln(y) + \frac{1}{\beta_4} ln\left(1 + \beta_2 e^{-\beta_3 x}\right) = \ln(\beta_1)$$

and, if necessary arrangements are made, it is obtained as follows:

$$\left(\frac{\beta_1}{y}\right)^{\beta_4} - 1 = \beta_2 e^{-\beta_3 x}$$

$$ln\left(\left(\frac{\beta_1}{y}\right)^{\beta_4} - 1\right) = ln\beta_2 - \beta_3 x$$

where, if

$$y^* = ln\left(\left(\frac{\beta_1}{y}\right)^{\beta_4} - 1\right), \beta_2^* = ln\beta_2 \text{ and } \beta_3^* = -\beta_3 \quad (4)$$

is taken,

$$y^* = \beta_2^* + \beta_3^* x$$

linear regression model is obtained. The initial value can be taken as $\hat{\beta}_1^{(0)} > max(y)$ for β_1, and the initial value can be taken as $\hat{\beta}_4^{(0)} = 1$ for β_4. Accordingly, the parameters of the linear regression model (4) obtained by applying the mentioned transformations are estimated to be able to determine the initial values for other parameters. Then, by substituting these parameter values, the initial values for the parameters are obtained with the inverse transformation:

$$\hat{\beta}_2^{(0)} = e^{\hat{\beta}_2^*}, \hat{\beta}_3^{(0)} = -\hat{\beta}_3^*.$$

Finally, let us consider the Weibull model. If two logarithmic transformations are applied to the model to determine the initial values where the model is

$$y = \beta_1 - \beta_2 e^{-\beta_3 e^{\beta_3 x^{\beta_4}}} + \varepsilon_x \qquad (5)$$

and necessary arrangements are made,

$$\beta_1 - y = \beta_2 e^{-\beta_3 e^{\beta_3 x^{\beta_4}}}$$

$$ln\left(-ln\left(\frac{\beta_1 - y}{\beta_2}\right)\right) = \ln(\beta_3) + \beta_4 . ln(x)$$

is obtained. In this equation obtained,

$$y^* = ln\left(-ln\left(\frac{\beta_1 - y}{\beta_2}\right)\right), \beta_3^* = \ln(\beta_3), \beta_4^* = \beta_4 \text{ and } x^* = \ln(x)$$

where, if

$$y^* = \beta_3^* + \beta_4^* x^* \qquad (6)$$

linear regression model is obtained. Since the initial value is $\hat{\beta}_1^{(0)} > max(y)$ for β_1 and it is $y_0 = \beta_1 - \beta_2$, it is $\beta_1 = \beta_2$ and the initial value is taken as $\hat{\beta}_2^{(0)} = \hat{\beta}_1^{(0)} > max(y)$ for β_2. Accordingly, the parameters of the linear regression model (6) obtained by applying the mentioned transformations are estimated to be able to determine the initial values for other parameters. Then, by substituting the obtained parameter estimates, the initial values for the parameters are obtained with the inverse transformation in the following way:

$$\hat{\beta}_3^{(0)} = e^{\hat{\beta}_3^*}, \hat{\beta}_4^{(0)} = \hat{\beta}_4^* .$$

Application Study

In this study, weekly body living weight values of female turkeys between 4-60 weeks, which were obtained from Kahramanmaraş Sütçü İmam University, Faculty of Agriculture, Department of Zootechnics, Livestock Research and Application Center (Haymer) and used in the master's thesis entitled "Multiphasic Growth Functions and Some Applications," were taken into account, and it was attempted to obtain the most appropriate growth model for the growth of female turkeys. The data are presented in Table 2.

It was aimed to determine the appropriate growth model using weekly weight values of female turkeys. Therefore, three widely used nonlinear growth models given in Table 1 were considered, and it was attempted to determine which model was the most appropriate for the data by making parameter estimations regarding the models. In these three models taken for comparison, the weeks and female turkey weights are expressed by x and y, respectively.

Table2. Weights of female turkeys

week	weight	week	weight	week	weight
4	439	23	5250	42	11250
5	614.5	24	5432	43	11680
6	835	25	5578	44	11680
7	1183	26	5838.5	45	11685
8	1483	27	5807	46	11908
9	1845	28	6075.5	47	12110
10	2165	29	6283.5	48	12345
11	2520	30	6445.5	49	12460
12	2652	31	6432	50	12578
13	2935.5	32	6783	51	12614
14	3207	33	7245	52	12723
15	3505	34	7936	53	12715
16	3784	35	8746	54	12812
17	3994.5	36	9102	55	12894
18	4049.5	37	9457	56	12889
19	4342.5	38	9364	57	12892
20	4577	39	9809	58	13053
21	4823.5	40	10182	59	13046
22	5007.5	41	10895	60	13055

In the linear regression models obtained by applying the transformations described, parameter estimations were made using the R program, and the initial values of the parameters were obtained for each model with the necessary transformations. The initial values of the parameters for each model are presented in Table 3.

Table 3. Parameter initial values of the Gompertz, Richards, and Weibull models

Model	β_1	β_2	β_3	β_4
Gompertz	13056	12.2509	0.1215	-
Richaards	13056	44.7727	0.14	1
Weibull	13056	13056	0.0020	1.844

With these initial values obtained, parameter estimates of the three growth models were obtained using the "nls" command, which performs analysis with the nonlinear least squares method in the R program. This command calculates the parameter estimates using the Gauss-Newton algorithm [11]. According to the parameter estimates obtained, the graphs of the curves fitted with the data are shown in Figure 1.

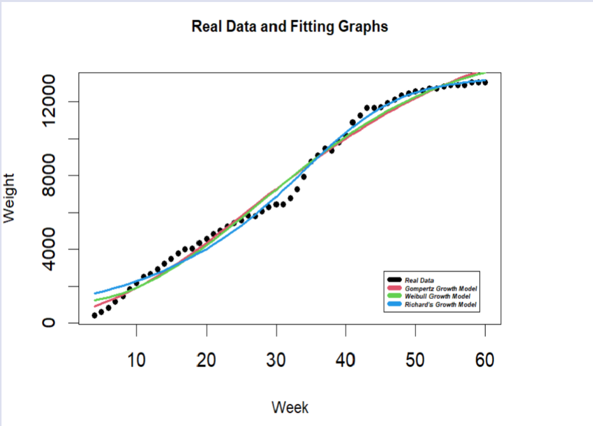

Figure 1. Graph of the estimate and actual values of the real data application

It was seen that the nonlinear regression models obtained were quite compatible with the data. When we are fitting several models to certain sample data and the aim is to select the preferable model among these models, we use *F*-test such that, where p is the number of parameters and n is the number of observations,

$$F = \frac{MSR}{MSE} = \frac{SS_{Reg}/p}{SS_e/n-p-1} \quad (7)$$

when the sum of squared residuals (errors) is denoted by SS_e and the sum of squared regression is represented by SS_{Reg}, so that:

$$SS_{Reg} = \sum_{i=1}^{n}(y_i - \bar{y})^2 - \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (8)$$

$$SS_e = \sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (9)$$

It is better to use a model with a considerable and higher F value. The models can be compared using the information theory-based Akaike information criterion (AIC) in order to provide a more thorough assessment of their performance. The AIC value is calculated as follows,

$$AIC = nln\left({SS_e}/{n}\right) + 2p$$

A smaller value of AIC criteria indicate a preferable model [12].

In Table 4, the parameter estimate values, standard errors, number of iterations, and AIC values required for model comparison are given for each model.

Table 4. The models' results of turkey growth

Model	Parameter Estimate	AIC	Number of Iterations	SE
Gompertz	$\beta_1 = 16867$ $\beta_2 = 3.5083$ $\beta_3 = 0.0476$	874.0791	9	495.4
Richards	$\beta_1 = 13305$ $\beta_2 = 970.3538$ $\beta_3 = 0.1694$ $\beta_4 = 2.9424$	861.8387	42	441.3
Weibull	$\beta_1 = 14669$ $\beta_2 = 13573$ $\beta_3 = 0.00053$ $\beta_4 = 2.0627$	874.0259	13	491.1

According to the results in Table 4, the model with the smallest AIC value was the second model, the Richards model. Furthermore, the model with the smallest standard error was the Richards model. Therefore, it can be said that the most appropriate model for turkey growth based on the available data was the Richards model.

Conclusion and Discussion

In the study, the Gompertz model, Richards model, and Weibull model, which are nonlinear growth models, were fitted to the 60-week living weight growth data of turkeys, and the models' parameter estimations were made. One of the most important problems in parameter estimation in nonlinear models is to determine the initial values of parameters. In this study, linear regression models were obtained by applying transformations to the models in order to determine the initial values of the parameters, and the initial values of the original model parameters were determined by the inverse transformation method by making parameter estimations in these linear regression models. In conclusion, it was seen that all three models fitted the data quite well. The AIC values of the models were calculated to determine the best model, and it was observed that the model with the smallest AIC value was the Richards model. This result was consistent with the results of previous studies in the literature.

Conflicts of interest

There are no conflicts of interest in this work.

References

[1] Nelder J. A. , Mead R. A., Simplex Method for Function Minimization, *The Computer Journal*, 7(4) (1965) 308–313.

[2] Huber P.J., Robust Estimation of a Location Parameter, *Ann. Math. Statist*., 35(1) (1964) 73-101.

[3] Huber P.J., Robust Regression: Asymptotics, Conjectures and Monte Carlo, *Annals of Statistics*, 1 (1973) 799-821.

[4] Bates D.M., Watts D.G., Nonlinear Regression Analysis and Its Applications, Wiley, New York (1988).

[5] Çağlar A., Forming Confidence Regions for Parameter Estimation in Nonlinear Models and a Comparison, MSc Thesis, Gazi University, Ankara, Turkey (1995).

[6] Aksoy S., Otokorelasyonlu Hata Terimli Doğrusal Olmayan Regresyon Modellerinde Parametre Tahmini, PhD Thesis, Gazi University, Ankara, Turkey (1996).

[7] Şengül T. ,Kiraz S., Non-Linear Models for Growth Curves in Large White Turkeys, *Turk J. Vet. Anim. Sci*., 29 (2005) 331-337.

[8] Çelik A., Aslan Y., Efe E., Investigation of Growth in Turkeys Using a Multiphasic Model, *Black Sea Journal Of Engineering And Science*, 5(1) (2022) 18-25.

[9] Çelik A., Multiphasic Growth Functions and Some Applications, MSc Thesis, Kahramanmaras Sutcu Imam University, Kahramanmaras, Turkey (2013).

[10] Ghanim Al-Ani Ban., Statistical modeling of the novel COVID-19 epidemic in Iraq, *Epidemiol. Methods*, 10(1) (2021) 20200025.

[11] Ritz C., Streibig J. C., Nonlinear Regression with R, Use R!, Springer Verlag (2008).

[12] Teleken J. T., A. C. Galvão, Robazza W. S., ‘‘Comparing Non-linear Mathematical Models to Describe Growth of Different Animals.’’ Acta Scientiarum, *Animal Sciences,* 39(1) (2017) 73–81.