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Description of 2_3^+, 0_3^+ Intruder States in [130]Xe Nucleus by Mixing of Transitional Hamiltonian and O(6) Casimir Operator

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ABSTRACT

In this paper, we tried to describe the 2_3^+ , 0_3^+ intruder stats in [130]Xe nucleus in a configuration mixing framework. To this aim, we have used a transitional Hamiltonian based in the affine SU(1,1) lie algebra in the framework of interacting boson model. Also, we perturbed this Hamiltonian in the version 2 with adding a new term, the O(6) Casimir operator, due to the nature of these intruder states. The results confirm the accuracy of this mixing configuration in the description of all considered energy levels and especially, the intruder states. Also, these results suggest same approach with adding other Casimir operators of different symmetry chains to extend the ability of this transitional Hamiltonian.

Keywords: Affine algebra, Casimir operator, Intruder state, Transitional Hamiltonian.

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Introduction

The existence of states belong to excited enrgy bands between the levels of ground band of energy refered to the mixing of symmetries. As have been shown in various spectroscopic selective experiments, *e.g.* transfer reactions in particular, very near to closed shells (the In and Sb nuclei at Z=50 but also in other mass regions, *e.g.* the Tl and Bi nuclei at Z=82) some low-lying extra states, so-called intruder states have been observed with a conspicuous energy dependence on the number of free valence neutrons, hinting for 2*p*-2*h* excitations as their origin [1-9]. If these excitations are proton excitations combined with the neutron degree of freedom appearing on both sides of the Z=50 closed shell, such as condition which are available for [130]Xe isotope, it is a natural step to suggest that low-lying extra 0^+ excitations will also show up in the even-even nuclei in between. Because the[130] Xe isotope with a large number of valence neutrons are situated near to the -stability line, they could be studied and must describe by such formalism which perturbed by some excitation [1-2]. The interacting boson model (IBM) Hamiltonian is regards as the most successful formalism in description of such concepts. IBM was written from the beginning in second quantization form in terms of the generators of the U(6) unitary lie algebra. The model assumes that low-lying collective excitations of the nucleus can be described in terms of the number *N* bosons. The bosons correspond to pairs of nucleons in valance shell, coupled to angular momentum j=0 and j=2 which correspond respectively to s and d boson and N is constant for a given nucleus and equal to half its number of valance nucleons [3-15]. The phenomenological IBM in terms of U(5), SU(3) and O(6) dynamical symmetries has been employed in describing the collective properties of several medium and heavy mass nuclei. These dynamical symmetries correspond to harmonic vibrator, axial rotor and $\gamma-$unstable rotor as the geometrical analogues, respectively. These symmetries are fairly successful in the investigation of low-lying nuclear states of nuclei located in three dynamical limits of IBM. On the other hand, the analytic description of structure at the critical point of phase transition is considered as issue, recently great analyses has been performed to describe them. Iachello in Refs [16-22] have established a new set of dynamical symmetries, i.e. *E*(5) and *X*(5), for nuclei which are located at the critical point of transitional regions. The *E*(5) symmetry describes a second order phase transition which corresponds to the transitional states in the region from *U*(5) to *O*(6) symmetries in the IBM [22-24].

Different studies on the[130] *Xe* nucleus report the excitation of as intruder states. The low-lying 2_3^+ and 0_3^+ levels arises either from the intruder 2p-2h or from the normal 0p-0h configuration ,low-lying of these isotope are formed from the mixing of two intrinsic states of different deformations These levels are classified as two phonon states where must describe by 2p-2h excitation [26-37] A method to describe the intruder excitations in the IBM framework proposed to associate the different shell-model spaces of 0P-0h, 2p-2h,etc excitations. In this approach, two or four protons in the *z*=50-82 major shell can be excited to next major shell. These excitations make the corresponding boson spaces including *N,N+2,…* bosons which N= $N_\pi + N_\nu$,The Hamiltonians of formalisms in description of such levels have the $\hat{H} = \hat{H}_{reg} + \hat{H}_{2p-2h}$ form where mixes the regular (0*p*-0*h*) and (2p-2h) configurations. The correlation between valence protons and neutrons is enhanced [8-22] due to this cross-shell excitation of protons resulting in the lowering of excited 2_3^+ and 0_3^+ states and therefore, classified them as intruder states.

In this paper we used i) the *U*(5)-*SO*(6) transitional Hamiltonian which is defined in the affine *SU*(1,1) algebra and ii) Due to the nature of the considered intruder levels, we added the *O*(6) Casimir operator as the perturbation to the both regular Hamiltonians. The efficiency of these both approaches are analyzed with comparison of their predictions for different energy levels and the experimental counterparts.

Theoretical Framework

The nature of $0_3^+, 2_3^+$ in ^{130}Xe is reported in Ref [9] which correspond with *SO* (6) symmetry of IBM. In the first part of this study we used a transitional Hamiltonian which describe both *U*(5) and *O*(6) limits as regular part to cover the spherical properties of the considered nucleus. The analytic description of nuclear structure at the critical point of phase transitions has attracted extensive interest in the recent decades. One has to employ some complicated numerical methods to diagonalize the transitional Hamiltonian in these situations but Pan et al. [23-24] have proposed a new solution based on algebraic technique and explores the properties of nuclei classified in the U(5)↔SO(6) transitional region of IBM. Hamiltonian with two control parameters was used in which the results for the control parameter of transitional Hamiltonian offer a combination of spherical and deformed shapes in different isotopic Also, this Hamiltonian extended by adding a corresponding *O*(6) operator to increase the effect of *O*(6) symmetry and increase the accuracy of this formalism in description of intruder levels as:

$$\hat{H}_{extended} = \hat{H}_{SU(1,1)} + \eta \hat{C}_{2(SO(6))}, \tag{1}$$

η is a constant which extract in comparison with experimental data. By employing the generators of SU(1,1) algebra, a Hamiltonian construct which is suitable for the investigation of such nuclei which located between U(5) and SO(6) limits The SU(1,1) algebra has been described in Refs[23-24]. Here, we briefly outline the basic ansatz and summarize the results. The lie algebra corresponds to the SU(1,1)lie algebra is generated by S^{ν}, $\nu = 0$ and $\pm$, which satisfies the following commutation relations:

$$[S^0, S^{\pm}] = \pm S^{\pm} \ , \quad [S^+, S^-] = -2S^0 \tag{2}$$

The Casimir operator of *SU*(1,1) group can be written as:

$$\hat{C}_2 = S^0(S^0 - 1) - S^+S^- \equiv \hat{C}_{2(SO(6))} \tag{3}$$

We would use this operator to describe some intruder states which have *SO*(6) nature which detailed are present in the following. Representations of *SU*(1,1) are determined by a single number κ, thus the representation of Hilbert space is spanned by orthonormal basis $|\kappa\mu\rangle$ where κ can be any positive number and $\mu = \kappa, \kappa+1, \ldots$. Therefore,

$$\hat{C}_2(SU(1,1))|\kappa\mu\rangle = \kappa(\kappa-1)|\kappa\mu\rangle \ , \ S^0|\kappa\mu\rangle = \mu|\kappa\mu\rangle \tag{4}$$

In IBM, the generators of s and $d-$bosons pairing algebra is created by:

$$S^+(d) = \frac{1}{2}(d^\dagger . d^\dagger) \ , \ S^-(d) = \frac{1}{2}(\tilde{d} . \tilde{d}) \ , \ S^0(d) = \frac{1}{4}\sum_{\nu}(d_\nu^\dagger d_\nu + d_\nu d_\nu^\dagger) \tag{5}$$

$$S^+(s) = \frac{1}{2}s^{\dagger 2} \quad , \quad S^-(s) = \frac{1}{2}s^2 \quad , \quad S^0(s) = \frac{1}{4}(s^\dagger s + s s^\dagger) \tag{6}$$

On the other hand, the infinite dimensional *SU*(1,1) algebra is generated by using of [23-24]:

$$S_n^{\pm} = \sum_t c_{s;\,t}^{2n+1} S^{\pm}(s;t) + c_{d;\,t}^{2n+1} S^{\pm}(d;t) \quad , \quad S_n^0 = \sum_t c_{s;\,t}^{2n} S^0(s;t) + c_{d;\,t}^{2n} S^0(d;t) \tag{7}$$

And the sum is over proton, π, and neutron, ν, indices Where c_s and c_d are real parameters and *n* can be $0, \pm 1, \pm 2, \ldots$. These generators satisfy the commutation relations,

$$[S_m^0, S_n^{\pm}] = \pm S_{m+n}^{\pm} \quad , \quad [S_m^+, S_n^-] = -2S_{m+n+1}^0 \tag{8}$$

Then, $\{S_m^{\mu}, \mu=0,+,-;\pm1,\pm2,...\}$ generates an affine lie algebra *SU*(1,1) without central extension. By employing the generators of *SU*(1,1) algebra, the following Hamiltonian is constructed for the transitional region between $U(5)\leftrightarrow SO(6)$ limits [23-24]:

$$\hat{H} = g\, S_0^+ S_0^- + \varepsilon\, S_1^0 + \gamma_1\, \hat{C}_2(SO_\pi(5)) + \gamma_2\, \hat{C}_2(SO_\nu(5)) + \delta_1\, \hat{C}_2(SO_\pi(3)) + \delta_2\, \hat{C}_2(SO_\nu(3)) + \delta\, \hat{C}_2(SO(3)) \tag{9}$$

g, ε, γ and δ are real parameters and $\hat{C}_2(SO(3))$ and $\hat{C}_2(SO(5))$ denote the Casimir operators of these groups. It can be seen that Hamiltonian (9) would be equivalent with $SO(6)$ Hamiltonian if $c_s = c_d$ and with $U(5)$ Hamiltonian when $c_s = 0$ & $c_d \neq 0$. Therefore, the $c_s \neq c_d \neq 0$ requirement just corresponds to the $U(5)\leftrightarrow SO(6)$ transitional region. To control this transitional Hamiltonian via single parameter and also due to the *O*(5) sub algebras which joint symmetry for both *U*(5) and *O*(6) dynamical limits, we take c_d (=1) constant value and c_s vary between 0 and c_d.

Eigenstates of Hamiltonian(9) can obtain with using the Fourier-Laurent expansion of eigenstates and *SU*(1,1) generators in terms of unknown $c-$number parameters x_i with $i = 1, 2, ..., k$. It means, one can consider the eigenstates as [23-24]:

$$\left|k;\beta;\nu_s^\pi,\nu_s^\nu,\nu^\pi,\nu^\nu;n_\Delta^\pi L_\pi,n_\Delta^\nu L_\nu;LM\right\rangle = \sum_{n_i\in Z} a_{n_1}a_{n_2}...a_{n_k}\, x_1^{n_1}x_2^{n_2}...x_k^{n_k}\, S_{n_1}^+ S_{n_2}^+ ... S_{n_k}^+ \left|lw\right\rangle \tag{10}$$

Due to the analytical behavior of wave functions, it suffices to consider x_i near zero. With using the commutation relations between the generators of SU(1,1) algebra, i.e. Eq(10), wave functions can be considered as:

$$\left|k;\beta;\nu_s^\pi,\nu_s^\nu,\nu^\pi,\nu^\nu;n_\Delta^\pi L_\pi,n_\Delta^\nu L_\nu;LM\right\rangle = N S_{x_1}^+ S_{x_2}^+ ... S_{x_k}^+ \left|lw\right\rangle \tag{11}$$

N is the normalization factor and

$$S_{x_i}^+ = \sum_t \frac{c_{s;\,t}}{1-c_{s;\,t}^2 x_i} S^+(s;t) + \frac{c_{d;\,t}}{1-c_{d;\,t}^2 x_i} S^+(d;t) \tag{12}$$

The c-numbers x_i are determined through the following set of equations as:

$$\frac{\epsilon}{x_i} = \sum_t g\left(\frac{c_{s;\,t}{}^2(\nu_s^t + \frac{1}{2})}{1-c_{s;\,t}{}^2 x_i} + \frac{c_{d;\,t}^2(\nu^t + \frac{5}{2})}{1-c_{d;\,t}^2 x_i}\right) - \sum_{i\neq j}\frac{2}{x_i - x_j} \qquad \text{for i=1,2,...,k} \tag{13}$$

Eigenvalues of Hamiltonian (9), i.e. $E^{(k)}$, can be expressed as

$$E^{(k)} = \sum_{i=1}^{k}\frac{\varepsilon}{x_i} + \gamma_1\nu^\pi(\nu^\pi+3) + \gamma_2\nu^\nu(\nu^\nu+3) + \delta_1 L_\pi(L_\pi+1) + \delta_2 L_\nu(L_\nu+1) + \delta L(L+1) + \varepsilon\Lambda_1^0$$
$$\Lambda_1^0 = \sum_t \frac{1}{2}[c_{s;\,t}^2(\nu_s^t + \frac{1}{2}) + c_{d;\,t}^2(\nu^t + \frac{5}{2})] \tag{14}$$

Similarly, the eigenvalues of Hamiltonian (1), i.e. E extended (k), can be expressed as:

$$\hat{C}_2[SO(6)] = h^{(k)} \quad , \qquad h^{(k)} = \sum_{i=1}^{k}\frac{\varepsilon}{x_i} \tag{15}$$

$$E_{extended}(k) = E(k) + \sum_{i=1}^{k}\frac{\varepsilon}{x_i} \quad , \quad E^{(k)} = \sum_{i=1}^{k}\frac{\varepsilon}{x_i} + \gamma_1\nu^\pi(\nu^\pi+3) + \gamma_2\nu^\nu(\nu^\nu+3) + \delta_1 L_\pi(L_\pi+1) + \delta_2 L_\nu(L_\nu+1) + \delta L(L+1) + \varepsilon\Lambda_1^0 \tag{16}$$

The quantum number *k*, is related to total boson number *N*, by

$$2k = N_\pi + N_\nu - \nu_s^\pi - \nu_s^\nu - \nu^\pi - \nu^\nu$$

To obtain the numerical results for E(k), we have followed the prescriptions have introduced in Refs.[38-39], namely a set of non-linear Bethe-Ansatz equations (BAE) with $k-$ unknowns for $k-$ pair excitations must be solved. To this aim we have changed the variables as:

$$\grave{o}=\frac{\varepsilon}{g}(g=1\ \ keV)\quad ,\quad c\ =\frac{c_s}{c_d}\le 1\quad ,\quad y_i=c_d^2 x_i \tag{17}$$

so, the new form of Eq.(13) would be

$$\frac{\epsilon}{y_i}=\sum_t\left(\frac{c_t^{\ 2}(v_s^t+\frac{1}{2})}{1-c_t^{\ 2}y_i}+\frac{(v^t+\frac{5}{2})}{1-y_i}\right)-\sum_{i\neq j}\frac{2}{y_i-y_j}\qquad \text{for i=1,2,...,k} \tag{18}$$

We have solved Eq.(18) with definite values of c and ε for $i=1$ to determine the roots of Beth-Ansatz equations (BAE) with specified values of v_s and v, similar to procedure which have done in Refs [38-39]. Then, we have used “Find root” in the Maple17 to get all $y_j^{'}$ s. We carry out this procedure with different values of c and ε to provide energy spectra (after inserting γ and δ) with minimum variation as compared to the experimental counterparts:

$$\sigma=\left(\frac{1}{N_{tot}^2}\sum_{i=1}\left|E_{\exp}(i)-E_{cal}(i)\right|^2\right)^{1/2}$$

Which N_{tot} is the number of energy levels where are included in extraction processes. We have extracted the best set of Hamiltonian’s parameters, i.e. γ and δ, via the available experimental data [25-29] for excitation energies of selected states, $0_1^+,2_1^+,4_1^+,0_2^+,2_2^+,4_2^+$ and *etc*, e.g. 12 levels up to 2_4^+, or two neutron separation energies for nuclei which are considered in this study. In summary, we have extracted γ and δ externally from empirical evidences and other quantities of Hamiltonian, e.g. c and ε would determine through the minimization of σ.

Results

In this paper We try to increase the accuracy of heoretical prediction in the determination of energy levels in ^{130}Xe nucleus with using an extension of transitional Hamiltonian. As have explained in the previous parts, due to the nature of considered intruder levels, 2*p*-2*h* excitation, we added a *O*(6) Casimir to improve our methods. we calculated all the roots of Eq (18) and then extracted the constants of energy formula with least square method, then, we determined the energy of different states with Eq (14), namely without the mixing term. Also we carried out these calculations with using different values of c_s, δ, γ and ε to get the best corresponding between theoretical predictions and experimental counterparts. These results are presented in Table 1 and the best arrangement which yield by C_s=*0.7 and* ϭ =137 Also, for excited energy levels and especially, intruder levels, we need to add new terms to optimize our theoretical predictions. The new results which yield with using Eq (16) are presented in Table 2. Also, the best corresponding between theoretical predictions and experimental counterparts yield via C_s=*0.8 and* ϭ =183, are presented in Table 2.

Table 1. The parameters of IBM-2 Hamiltonian for ^{130} Xe isotope which are extracted by least square method from experimental data were taken from [25-29] ϭ is regarded as the quality for extraction processes (N=5 the Boson number)

Cs	γ_1	γ_2	δ_1	δ_2	Δ	ε	ϭ
0.2	76.41	43.75	80.33	-105.49	11.74	500	228
0.3	80.35	42.77	85.42	-110.08	11.09	500	302
0.4	88.26	42.32	94.64	-118.57	10.99	500	270
0.5	91.76	41.98	110.21	-133.77	9.89	500	230
0.6	101.43	40.64	150.55	-140.58	7.83	500	244
0.7	120.65	39.85	170.76	-146.04	3.16	500	137
0.8	118.87	44.45	180.90	-190.41	2.75	500	267
0.9	112.98	45.32	210.48	-220.88	1.74	500	280

Table 2. The parameters of extended IBM-2 Hamiltonian for ^{130} Xe isotope which are extracted by least square method from experimental data were taken from [25-29]. б is regarded as the quality for extraction processes.(N=5 the Boson number)

Cs	γ_1	γ_2	δ_1	δ_2	Δ	ε	б
0.2	101.32	80.65	110.95	-90.76	13.99	500	230
0.3	110.67	77.09	130.09	-95.54	11.74	500	210
0.4	118.21	71.39	145.88	-100.89	9.09	500	225
0.5	124.12	68.44	160.37	-110.55	7.78	500	218
0.6	133.45	60.56	177.05	-116.04	6.45	500	190
0.7	155.38	57.34	198.29	-126.47	5.63	500	198
0.8	160.44	44.45	188.94	-130.66	3.88	500	183
0.9	140.77	49.32	200.44	-149.68	2.95	500	205

Obviously, this extended formalism improve theoretical predictions and confirm our idea to add this term in description of intruder levels of ^{130}Xe nucleus. A detailed report on the results of these two formalisms about each levels are showed in Table 3 and Figure 1.

Table 3. Theoretical predictions of both normal and extended Hamiltonian for ^{130}Xe nucleus. All energies are in *keV*.

level	$E_{experimental}$	$E_{SU(1,1)}$	$E_{extended\ SU(1,1)}$
0_1^+	0	0	0
2_1^+	536	499	510
4_1^+	1204	1290	1180
6_1^+	1944	2015	1902
2_2^+	1122	1200	1100
3_1^+	1632	1550	1675
4_2^+	1808	1700	1878
0_2^+	1793	1500	1750
2_3^+	2150	2430	2220
0_3^+	2017	2221	2430

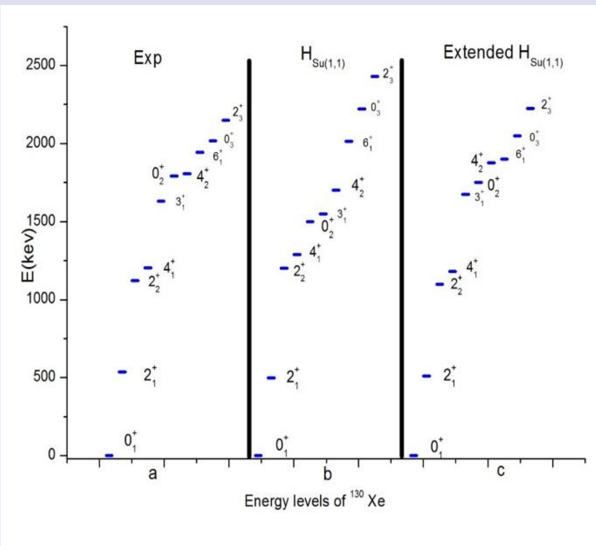

Figure 1. The basal latencies of rats before and after operation (paired student t-test, *p<0.05)

Figure1.energy spectra of ^{130}Xe nucleus, a) experimental spectra together b) theoretical predictions based on *SU*(1,1) transitional Hamiltonian and c) extended Hamiltonian.

Conclusions and Summary

These results show the advantages of the considered transitional Hamiltonian in the description of only the states of ground band. This confirm the spherical nature of this nucleus and therefore, explain the reason of using such transitional Hamiltonian for this nucleus. On the other hand, we got the biggest differences between theoretical predictions and experimental data in 2_3^+ and 0_3^+ states which as reported have *O*(6) nature. To optimize our theoretical predictions, we need to add new term as explained in Eq.(1) and due to the properties of these intruder states, the $\hat{C}_{2(SO(6))}$, e.g. Eq.(3), is our selection. By using same method which have used to get roots and predictions of the transitional Hamiltonian In transitional Hamiltonian, we changed the control parameter of this model which describe symmetries mixing, together the excitation term to describe the energy spectra with high accuracy. The results of normal and perturbed Hamiltonians show the advantages of perturbed one for all of the considered states and especially 2_3^+ and 0_3^+ intruder states. Also, the changes in the control parameters of this transitional Hamiltonian verify the superposition of two *U*(5) and *O*(6) nature of this nucleus. The better agreement which yield by the extended Hamiltonian suggest the partial symmetry-like structure in this nucleus which is type-II and only, these intruder states belong to this additional symmetry.

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Conflict of interests

Conflict-of-interest statement the authors have no conflicts of interest to declare. The authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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