Contribution of Neutralino and Chargino to Diagonal Form Factor of Majorana Neutrino in the Minimal Supersymmetric Standard Model

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ABSTRACT

In this study, we have calculated the diagonal form factor and charge radius of Majorana neutrinos in the Minimal Supersymmetry Standard Model (MSSM) using Feynman-'t Hooft gauge and dimensional regularization. From the obtained result of calculations, we have seen that the main contribution come from chargino particles in MSSM and its contribution is very small than SM contribution. It is found that \( q^2 = 1.66 \cdot 10^{-32} \text{ cm}^2 \) for the charge radius of electron neutrinos which is in good agreement with results obtained from the scattering experiments.

Keywords: Standard model, Minimal supersymmetric model, Lepton, Form factors, Majorana neutrino.

Introduction

The Standard Model (SM) is a theory concerning the electromagnetic, weak and strong interactions. The SM of elementary particles is a gauge quantum field theory. The internal symmetry that defines the SM is the local \( SU(3) \times SU(2) \times SU(1) \) gauge symmetry and the most general Lagrangian that describes the dynamics of the fields [1-3]. Although the SM is a beautiful model with predictive power, it includes almost 30 parameters which we are not able to know where they came from as well as some other minor problems. For example, one of them is to determine the absolute scale of the neutrino mass or the nature of the neutrino, whether it is a Dirac or a Majorana particle. We know that neutrinos are massive because neutrino oscillations have been observed in many experiments. The SM must be extended to account for the neutrino masses. Physics beyond the SM has attracted the attention of physicists for a long time. One of the most appealing theories to describe physics at the TeV scale is the minimal supersymmetric extensions of the SM (MSSM) [4-6]. Besides providing a solution to the hierarchy problem, it also provides us with a good candidate for cold dark matter (CDM), namely, the lightest neutralino which is a Majorana particle.

It is known that in quantum field theory (QFT), the interaction vertex between a single photon and a fermion can be characterized in terms of four electromagnetic (EM) form factors. In the limit of vanishing momentum transfer between the photon and fermion, the form factors encode the static EM properties of the particle [7-22].
conserves helicity, whereas the interactions generated by the magnetic and electric dipole moment form factors that flips helicity.

Let us now consider the helicity conserving (charge and anapole) parts of neutrino electromagnetic vertex function for diagonal case in Eq.(1), as

$$A_μ^{QA}(q) = (γ_μ q^2 - q_μ \slashed{q})\left[f_0(q^2) + f_A(q^2) q^2 γ_5\right]$$ (4)

In the SM the neutrino electromagnetic form factor for small values of $q^2$ is given by

$$f(q^2) \equiv \left(\frac{r_2^2}{6} - a\right) q^2$$ (5)

where $(r_2^2)$ is the neutrino charge radius and $a$ is the neutrino anapole moment. For this massless case, Dirac or Majorana neutrinos are equivalent. Thus, in the SM the form factor $f(q^2)$ can be interpreted as a neutrino charge radius or as an anapole moment (or as a combination of both).

A Majorana neutrino is characterized by just one flavor diagonal electromagnetic form factor for diagonal case: the anapole moment, that in the static limit corresponds to the axial vector charge radius $(r_2^2)$.

It is easy to understand the physical significance of the form factors in the nonrelativistic limit. In this limit, the interaction energy with an external electromagnetic field takes the following form

$$H_{\text{int}} = -μ(\vec{σ} \cdot \vec{B}) - d(\vec{σ} \cdot \vec{E}) - a(\vec{σ} \cdot \vec{j})$$ (6)

where $\vec{B}$ and $\vec{E}$ are the magnetic and electric fields, $\vec{σ}$ is the Pauli spin matrix, and $\vec{j} = \left(\vec{V} \times \vec{B} - \frac{B}{c^2} \vec{E}\right)$ is the electric current density at the point where the particle is situated. The form factors of the neutrinos (uncharged leptons) have been calculated in gauge field theories. The questions of whether neutrinos have masses and if so, and whether the neutrinos are Dirac or Majorana particles have been two of the most important issues in both particle physics and astrophysics. It is known that the neutrinos are massless and that it is Weyl neutrinos in the SM. There are many possible extensions of SM which reveal massive neutrino[23].

In 1980, the anapole moment of the electron was calculated by Dombey and Kennedy [18] in the SM. In 1987, H.Czyz et al. [24-25] discussed the anapole moment of charged leptons in the context of the SM. In 2017, Whitcomb and Latimer[26] performed a scattering calculations that probes the anapole moment with a spinless charged particle. They showed that, in the nonrelativistic limit, the cross sections are compatible with the quantum mechanical computation of the cross section for a spinless current scattered by an infinitesimally thin toroidal solenoid. As a result, they have obtained the effect of the charge radius or anapole moment on the scattering. Lately, it has been observed a noticeable surge of interest in the study of charge radius or anapole moment from the astrophysical as well as the particle physics point of view[27-29]. These studies also serve as basis and provide contributions to explain the dark matter [30-32].

Neutrino oscillation results imply that the flavor neutrino fields $\nu_{Ll}(x)$ are the mixtures of the left-right handed components of the fields of the neutrinos, with masses defined as

$$\nu_{Ll}(x) = \frac{1}{2}(1 - γ_5)\nu_l(x) = \sum_{i=1}^{3} U_{i}^* \nu_{Li}(x), \ l = e^-, μ^-, τ^-$$ (7)

where $U$ is the unitary PMNS mixing matrix relating the neutrino mass eigenstates to the weak eigenstates and $\nu_l(x)$ is the field of neutrino (Majorana or Dirac) with mass $m_l(x)$ Flavor fields $\nu_L(x)$ enter into the SM charged current (CC)

$$L^{CC}_l(x) = -\frac{g}{2\sqrt{2}} \sum_{i=e, μ, \tau} \bar{ν}_{Li}(x) γ^\mu ν_{Ll}(x) W^\μ(x) + h.c$$ (8)

and neutral current (NC) interactions

$$L^{NC}_l(x) = -\frac{g}{2\sqrt{2}} \sum_{i=e, μ, \tau} \bar{ν}_{Li}(x) γ^\μ ν_{Ll}(x) Z^\μ(x)$$ (9)

is the neutrino NC. $W^\μ(x)$ and $Z^\μ(x)$ are the fields of $W$ and $Z$ vector bosons. $g$ is the electroweak interaction constant and $θ_\text{W}$ is the weak (Weinberg) angle [39]. The vector and axial-vector couplings are

$$g_{V} \equiv t_{3L}(l) - 2q_{l} sin^2θ_\text{W}, \ \ g_{A} \equiv t_{3L}(l)$$

where $t_{3L}(l) = -1/2$ is the weak isospin charged leptons $l$ and $q_{l}$ is the charge of $l$ in units of $e$, which is equal to $g sinθ_\text{W}$, is the positron electric charge. The interactions of the lepton and sneutrino with chargino (lepton-sneutino-chargino vertices), the charged lepton and the charged slepton with the neutralino (lepton-slepton-neutralino vertices) and the neutrino and slepton with chargino are correspondingly given by Lagrangian as [33-34]

$$L^{\text{SY}}_{1}(x) = \sum_{i} \left\{ \left[\left[(NL_{L} + N^R P_{R})χ^0\right] + [L_{L} P_{L} + R_{R} P_{L})χ^{\nu}\right] + \left[(C_{L} P_{L} + C_{R} P_{L})\chi^{\nu}\right] + \left[C^{\nu} P_{L} \nu_{L}\right] \right\} + h.c$$ (10)

where $P_{L,R} = \frac{1}{2}(1 ± γ_5)$,

$$N_{L} = -\frac{g}{\sqrt{2}} \sum_{A \times} \left\{ m_{1i} M_{\nu} cosβ N_{A}(R_{L1} + 2N_{A} tanθ_{R} R_{L1}) \right\}$$ (11)
\[ N^R = -\frac{g}{\sqrt{2}} \sum_{\alpha \chi} \{ -N_{\alpha 2} - N_{\alpha 1} \tan \theta_W \} R_{\chi i}^l + \frac{m_{\mu}}{M_w \cos \beta} N_{\alpha 3} R_{\chi i+3}^l \] 

(12)

\[ C^\ell = g \sum_{\chi} \frac{m_{\mu}}{M_w \cos \beta} V_{\alpha 2} R_{\chi i}^\ell \] 

(13)

\[ C^{\prime \ell} = \sum_{\chi} (g V_{\alpha 1} R_{\chi i}^l m_{\mu} N_{\alpha 3} R_{\chi i+3}^l) \] 

(14)

and \( P_{L,R} \) are the project operators. \( N \) and \( U,V \) are the diagonalized matrices of neutralino and chargino mass matrix, respectively. \( R \) is the diagonalized matrix of slepton or sneutrino mass matrix. The indices \( A \) (1...4 for neutralinos; 1, 2 for charginos) and \( X \) (1...6 for sleptons; 1, 2, 3 for sneutrinos) run over the dimensions of the respective matrices, whereas \( i \) as usual runs over the generations, \( m_i \) is the mass of the \( i^{th} \) charged lepton and rest of the parameters carry the standard definitions.

This paper is structured as follows: In Section 2, we present the calculations of the Majorana neutrinos’ form factor in the framework of the MSSM. Finally, we present our discussion and conclusion in Section 3.

**Calculation**

The basic diagrams which contribute to the form factor of Majorana neutrino are shown in Fig. 1. No diagrams involving Higgs particle contribute to parity violation to this order. The simplest gauge to choose for calculating these diagrams is Feynman-'t Hooft gauge, and the calculations are performed using the dimensional regularization produce which preserves gauge in variance. This calculation is done, using above interactions Lagrangians, in framework of the minimal supersymmetric extension of the Standard Model (MSSM).

There are the first three diagrams (i, ii and iii) in the SM [20,35].

We choose the Breit frame in which the neutrino has momentum initially \((p - q/2)\) and \((p + q/2)\), then we find the matrix elements for the MSSM in Fig. 1(i – v), respectively.

\[ M_i = \frac{e g^2}{8} \int \frac{d^2 w_k}{(2\pi)^2} \left( \gamma^\mu (1 - \gamma_5) \left( k + p + \frac{q}{2} + m_\chi \right) \right) \frac{\gamma^\mu \left( -k + p + \frac{q}{2} + m_\chi \right)}{\left( -k + p + \frac{q}{2} - m_\chi^2 \right)} \] 

(15)

\[ M_{ii} = \frac{e g^2}{8} \int \frac{d^2 w_k}{(2\pi)^2} \left( \gamma^\mu (1 - \gamma_5) \left( k + m_\chi \right) \right) \frac{\gamma^\mu \left( -k + p + \frac{q}{2} + m_\chi \right)}{\left( -k + p + \frac{q}{2} - m_\chi^2 \right)} \] 

(16)

\[ M_{iii} = \frac{i g}{2 \sin 2\theta_w} \frac{1}{q^2 - M_\nu^2} \pi_{\mu \nu}^{2\gamma}(q^2) \] 

(17)

where \( \pi_{\mu \nu}^{2\gamma}(q^2) \) is the mixing tensor for the \( \gamma - Z \) mixing diagrams[28]. Dombey and Kennedy obtained numerically value as

\[ \pi_{\mu \nu}^{2\gamma}(q^2) = (3.83 \times 10^{-3}) e^2 q^2 g_{\mu \nu} + 0(q^4) \] 

(18)

\[ M_{iv} = e \int \frac{d^2 w_k}{(2\pi)^2} \left( \frac{C^{\ell \mu} p_\ell}{k^2 - m_\ell^2} \left( -k + p + \frac{q}{2} + m_\chi \right) \left( C'^{\ell \mu} p_k \right) \right) \] 

(19)

\[ M_v = 2e \int \frac{d^2 w_k}{(2\pi)^2} \left( \frac{C^{\ell \mu} p_\ell}{k^2 - m_\ell^2} \left( -k + p + \frac{q}{2} \right) \left( C'^{\ell \mu} p_k \right) \right) \] 

(20)
Since the anti-particle of Majorana neutrino is equal to its particle ($\nu = \nu^c$), we didn’t need to write the matrix elements of Fig.1 ($i^i - v^v$). For obtaining the total result for the Majorana neutrino, we will multiply the contribution of Fig.1 $i^i - v$ by a factor 2.

Projecting out the axial parts proportional to

$$\frac{i e}{16\pi^2} \gamma_5 r^\mu g^2 \frac{q}{m_{W}^2}$$

and thereafter long but simple calculations, neglecting the terms of order $(m_i/M_w)^2$, we obtain the Majorana neutrino form factor expressions as follows

\[ a_i^+ = \ln \left( \frac{m_W^2}{m_i^2} \right) - \frac{13}{12} \]
\[ a_{ii}^+ = -\frac{7}{24} \]
\[ a_{iv}^+ = \left( \frac{\cos \theta_w}{4} \right) 3.83 \times 10^{-3} \]
\[ a_{iv}^+ = \left( \frac{M_W^2}{m_i^2} \right) \left[ -\frac{7}{6} + 2 \ln \left( \frac{m_i}{m_H^2} \right) \left( \frac{1}{g^2} |C^{ij}|^2 \right) \right] \]
\[ a_v^+ = \left( \frac{m_v^2}{m_\chi^2} \right) \left[ -\frac{1}{6g^2} |C^{ij}|^2 \right] \]

(Differences to Fig. 1(i)-(v)). Then, the total form factor of the Majorana neutrinos is

\[ a_i^+ = 2(a_i^+ + a_{ii}^+ + a_{iv}^+ + a_{iu}^+ + a_v^+) \]

**Conclusion**

We calculated the contribution to Majorana neutrino form factor arising from the exchange of $Z(W)$ and the leptons in the loop, as well as from the exchange of charginos (neutralino) and sleptons in the framework of the MSSM. Even though this model would be perhaps an unrealistic model, it is one of the models extending the SM. Limitations that can be obtained for this interaction from the entire body of available experimental data are investigated. We think that the most compelling constraints arise from experiments with polarized electrons. It is well known that there are many scenarios in SUSY. The leading effect in Fig. 1 is approximately in the large $\tan \beta$ limit. In this study we have considered only the scenario for large $\tan \beta$ values and we obtained

\[ a_i^+ = \ln \left( \frac{m_W^2}{m_i^2} \right) - \frac{33}{12} + \frac{m_i^2}{8} \left\{ -\frac{3m_i^2}{2(m_H^2-m_\chi^2)} + \frac{m_\chi^2}{(m_H^2-m_\chi^2)^2} \right\} + \frac{9m_i^2}{(m_H^2-m_\chi^2)^2} ln \left( \frac{m_\chi^2}{m_H^2} \right) \]

(Differences to Fig. 1(i)-(v)). Then, the total form factor of the Majorana neutrinos is

\[ a_i^+ = 2(a_i^+ + a_{ii}^+ + a_{iv}^+ + a_{iu}^+ + a_v^+) \]

It can be seen that, since $m_i$ and $m_H$ are bigger than $m_i$, therefore $m_i/m_H$ and $m_i/m_\chi$ become much smaller than 1. As a result, it seems unlikely possible from SM that in the MSSM, even taking the limited values (see Table 1), the contribution of chargino and neutralino is extremely smaller than the contribution of SM’s particle. Therefore it is unnecessary to obtain results for the different values of the parameters. This means that the terms coming from SUSY are strongly effective due to the properties of neutralino and chargino.

Here, we note that there is a long discussion on the possibility of obtaining for the neutrino charged radius which is a gauge-independent and finite quantity [36]. In the corresponding calculations, performed in the one-loop approximation including additional terms from the photon ($\gamma$) and neutral boson ($Z$) mixing and the box diagrams involving charge vector boson ($W$) and neutral vector boson ($Z$), the gauge invariant result for the neutrino charge radius (in fact, it is the charged radius squared) have been obtained [13,37-39].

The numerical values of the Majorana neutrinos’ charge radius for different neutrinos are obtained as

\[ \langle r^2 \rangle \approx \begin{cases} 
1.66 \cdot 10^{-32} \text{ cm}^2 \text{ for } \nu_\mu \\
0.96 \cdot 10^{-32} \text{ cm}^2 \text{ for } \nu_\mu \\
0.30 \cdot 10^{-32} \text{ cm}^2 \text{ for } \nu_e 
\end{cases} \]
using the corresponding leptons value of $m_l$ and mass values given in Table 1. The current constraints on the flavour neutrino charge radius $\langle r_{\nu_e,\mu,\tau}^2 \rangle \leq (10^{-32} - 10^{-31}) \text{ cm}^2$ was obtained from the scattering experiments [38-40]. The theoretical results of $\langle r_{\nu_e}^2 \rangle$ are smaller than the experimental results in an magnitude of only about one order.

Table 1: The particle masses values used in the calculations [40]

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass [GeV]</th>
<th>Particle</th>
<th>Mass [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^-$</td>
<td>$5,11 \times 10^{-8}$</td>
<td>$\chi^0_1$</td>
<td>$\geq 46$</td>
</tr>
<tr>
<td>$\mu^-$</td>
<td>$0.105$</td>
<td>$\chi^\pm_1$</td>
<td>$\geq 94$</td>
</tr>
<tr>
<td>$\tau^-$</td>
<td>$1,777$</td>
<td>$\tilde{e}$</td>
<td>$\geq 107$</td>
</tr>
<tr>
<td>$Z^0$</td>
<td>$91,18$</td>
<td>$\mu$</td>
<td>$\geq 94$</td>
</tr>
<tr>
<td>$W^\mp$</td>
<td>$80,39$</td>
<td>$\tilde{\tau}$</td>
<td>$\geq 81,9$</td>
</tr>
<tr>
<td>$H$</td>
<td>$125$</td>
<td>$\tilde{\nu}$</td>
<td>$\geq 41$</td>
</tr>
</tbody>
</table>

Some constant values

| $G_F$ | $1,16 \times 10^{-5}$ | $1/u$ | $137$ |
| $\sin^2 \theta_W$ | $0.229$ | $\tan \beta$ | $20$ |

Without performing calculations, we can’t say nothing about order of magnitude of considered contributions. A neutrino charge radius contributes to the neutrino scattering cross section on electrons and thus can be constrained by corresponding laboratory experiments.

The effect of charge radius can be included just as a shift of the vertex coupling constant

$C_V \rightarrow C_V + \left( \sqrt{2} \pi / 3 G_F \right) \langle r_{\nu_e}^2 \rangle$ in the weak contribution to the cross section. It has also some impact on astrophysical phenomena and cosmology. Performing at present time and planning in future very sensitive experiments on lepton magnetic, electric dipole, anapole moments and charge radius are one of main research direction in low energy experiments. For this reason presented in this work calculations have direct significance for experiment.

Figure 1. Basic diagram of the form factor of the Majorana neutrinos in the minimal supersymmetry standard model.

Conflicts of interest

There are no conflicts of interest in this work.

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