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A Mathematical Model of Susceptible Diabetes Complication (SDC) Model in Discrete Time Fuzzy and Crisp Environment

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ABSTRACT

In this study, we examined the mathematical model of the discrete-time equation system with susceptible diabetes complication (SDC), which is known to be caused by environmental and genetic factors in a fuzzy environment. From the diabetes complication (DC) model, the susceptible diabetes complication (SDC) model is being developed. It was obtained using definitions of how the behavior of this model changes in a fuzzy environment. A nonlinear differential equation system transforms the sensitive diabetes complication (SDC) model into a discrete time equation system. Stability analysis of the model with jury criterion was examined. In addition, numerical solutions and graphics of the analysis of the discrete model in fuzzy environment are obtained by using the MATLAB package program.

Keywords: SDC model, Fuzzy difference equation, Generalized hukuhara.

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Introduction

Mathematical modeling is used to, examine the epidemiology of a disease and to analyze important questions arising from real-world problems. It is also used in fields such as bio-science, chemistry, engineering and also in some real world problems. Analysis of these disease models can also be made with discrete-time equation systems. With the development of science, mathematical modeling is used to study not only the spread of infectious diseases, but also non-communicable diseases. Diabetes model is a biological problem and diabetes is a chronical disease. Diabetes is an indicator of irregular metabolism, where a combination of inherited and environmental factors leads to abnormally high blood sugar levels. There are two types of diabetes: type 1 diabetes, where without insulin, body cells cannot absorb and commit glucose, resulting in increased blood sugar levels, and in type 2 diabetes, where the body is unable to produce enough insulin. Recently, many researchers offered research on diabetes modeling [1]. Sergre. arc. [2] Creates a mathematical model about Blood Sugar and diabetes. In the human body various symptoms due to chronic diabetes, such as hyperglycemia, are based on the acute symptom of excessive diabetes as extreme urine production, compensative thirst and increased liquid intake, blurry seeing, weight loss and drowsiness. Rosado [3] limits hormone activity and affects blood glucose levels, they also present a mathematical model that detects diabetes in patients based on a 5-hour glucose intolerance test and the magnification results recommended by Ackerman [4]. Insulin-dependent diabetes Mellitus (IDDM) is discriminated by the inactivity of the pancreas. Based on the actual data of an IDDM patient recently, a few substantial systems in the bio-system for mathematical modeling in diabetes mellitus have been obtained by Johansson and Stahl [5]. In 2004, Boutoyeb [6] et al. identified the diabetes complication model (DC) to find non-complicated diabetics (D) and complicated diabetics (C). One of the factors affecting lifestyle is sociable interplay. This interplay is an important factor influencing the lifestyle of a healthy sensitive individual to increase the potential in diabetes [6]. Hill et al. claims in [7] that it leads to interaction between patients with unhealthy lifestyles and healthy individuals, and a new group of individuals called (S), the number of susceptible individuals, is obtained to determine the number of likely interplays. The (DC) model is modified to the susceptible diabetes complication model (SDC), depending on the group of susceptible individuals[8].

$$\frac{dS}{dt} = \vartheta S + \vartheta(1-\rho)(D+C) - \frac{\beta SD}{N} - \mu S$$
$$\frac{dD}{dt} = \frac{\beta SD}{N} + \vartheta\rho(D+C) - (\lambda+\mu)D + \gamma C$$
$$\frac{dC}{dt} = \lambda D - (\gamma+\delta+\mu)C$$

it is being transformed into a discrete-time system of equations. Where $S(0) > 0, D(0) > 0, C\,(0) > 0$, and $h = 0.01$. The parameters $\alpha, \beta, \gamma, \delta, \lambda, \mu, \rho > 0$ and $0 \leq \rho \leq 1$, respectively, are birth rate, interaction rate, recovery rate of complications, complication-related mortality rate, occurrence rate of complications, and rate of genetic disorder at birth [7-9].

In this study, we examined the mathematical model of the discrete-time equation system with sensitive diabetes complication (SDC). It has been applied to the SDC model using the necessary definitions given in Chapter 2. In Chapter 3, the formulation of the model is shown with its analysis and symbols. In Chapter 4, the stability of the model and its numerical analyzes are obtained using the jury criterion. In Chapter 5, the analysis and stability of the model in fuzzy environment are examined.

Fuzzy Set and Fuzzy Difference Equations

The subject of Fuzzy difference equations has been developing rapidly recently. Applying Fuzzy difference equations is a natural way to model dynamical systems under probability uncertainty [10]. Fuzzy is a tool used for problems with uncertainty. Fuzzy derivative was first introduced by Lotfi A. Zadeh in 1965. The following studies were given by Prade and Dubois [11], Ralescu and Puri [12] and Voxman and Goetschel [13]. To solve the difference equation, there is an array that meets the equation, and this sequence is called a solution series of the equation. The Fuzzy difference equation is a difference equation in which the parameters and initial values are fuzzy numbers, and their solution is a sequence of fuzzy numbers. Fuzzy difference equations are developing rapidly.

Fuzzy difference equations were first introduced by Kandel and Byatt [14-15]. It was also rigorously studied by Kaleva for the initial value problem of these equations [16]. Zhang, Yang and Liao [17] investigated the positive solution and limitation of the fuzzy difference equation. These equations are suitable for financial problems and Chrysafis, Papadopoulos, Papaschinopoulos [18] have done studies on this subject. The Fuzzy difference equation is expressed as all derivative Hukuhara or generalized derivatives. Since the Hukahara difference is not always present, Bede et al. defined the generalized Hukuhara difference by generalizing the H-difference [19]. In recent years, there has been a great interest in research with fuzzy difference equations and stability of fuzzy difference equations. [20-32].

Definitions and Theorems

Definition 1 (Atanasov 1986). The intuitionistic fuzzy set defined on a non- empty set X as objects having the form $A=\{\langle x,\alpha_A(x),\beta_A(x)\rangle : x\in X\}$, where the functions $\alpha_A(x): X\to[0,1]$ and $\beta_A(x): X\to[0,1]$, denote the degree of membership and the degree of non-membership of each element $x\in X$ to the set A respectively, and $0\le\alpha_A(x)+\beta_A(x)\le 1$ for all $x\in X$. Clearly, when $\beta_A(x)=1-\alpha_A(x)$ for every $x\in X$, the set A becomes a fuzzy set [33].

Definition 2. Let the set $A\in F(\mathbb{R})$ be given. In this case

The set A (0) is limited.

It is a A grade convex set.

The B grade set is normal. So, for $\exists x_0\in\mathbb{R}, \mu_A(x_0)=1$.

The $\mu_A(x)$ membership function is top-half continuous. So, the set $\{x\in\mathbb{R}: \mu_A(x)>\alpha\}$ is closed for $\forall\alpha\in[0,1]$.

The set A that provides the conditions is called a graded number. Here the family of graded numbers on $\mathbb{R}$ is denoted by $\mathcal{F}_N(\mathbb{R})$[34].

Theorem 1 (Parametric form of the fuzzy number). Any fuzzy number can be stated by a couple functions such as $[q_i(\alpha),q_l(\alpha)], 0\le\alpha\le 1$ that satisfy the circumstance given as follows [35]:

$q_i(\alpha)$, is a bounded, non-decreasing, left-continuous function when $\alpha\in(0,1]$ and right-continuous function when $\alpha=0$.

$q_i(\alpha)$ is a bounded, non-increasing, left-continuous function when $\alpha\in(0,1]$ and right-continuous function when $\alpha=0$.

$q_i(\alpha)\le q_l(\alpha)$ for any $\alpha\in(0,1]$.

Definition 3. The membership function of the number of degrees $A\in\mathcal{F}_N(\mathbb{R})$ is $A_1\le A_2\le A_3$ so that the real numbers are,

$$\mu_A(x)=\begin{cases}0 & , \quad x<A_1\\ \dfrac{x-A_1}{A_2-A_1} & , \quad A_1\le x\le A_2\\ \dfrac{A_3-x}{A_3-A_2} & , \quad A_2\le x\le A_3\\ 0 & , \quad A_3<x\end{cases}$$

In this form, the number A is called the triangular grade number and is indicated by $A=(A_1,A_2,A_3)$[19].

Definition 4 (α-cut of the fuzzy set). Let α-cut be given by $A=(A_1,A_2,A_3)$.

$$A_\alpha=[A_1+\alpha(A_2-A_1), A_3-\alpha(A_3-A_2)], \forall\alpha\in[0,1].$$

Definition 5 (Hukuhara difference). Let $s,t\in\mathcal{F}_N(\mathbb{R})$. If $\exists v\in\mathcal{F}_N(\mathbb{R}^n)$ is present with $s=t+v$; Cluster v is called the Hukuhara difference (H-difference) of s and t clusters, denoted by $s\ominus_H t$. This is expressed as

$$s\ominus_H t=v\Leftrightarrow s=t+v$$

Theorem 2. Let s and $t\in\mathcal{F}_N(\mathbb{R})$. Single if $s\ominus_H t$ difference exists.

Theorem 3. Let s and $t\in\mathcal{F}_N(\mathbb{R})$ and let $s(\alpha)=[s_1(\alpha),s_2(\alpha)]$ and $t=[t_1(\alpha),t_2(\alpha)]$ be the α-cut of the graded numbers s and t respectively. α-cut of difference $s\ominus_H t$ is

$$(s\ominus_H t)(\alpha)=[s_1(\alpha)-t_1(\alpha), s_2(\alpha)-t_2(\alpha)]$$

Definition 6 (Hukuhara differentiable).

Let $f:(a,b)\to\mathcal{F}_N(\mathbb{R})$ and $x\in(a,b)$.

If $f(x+h)\ominus_H f(x)$,$f(x)\ominus_H f(x-h)$ is present for $\forall h>0$,

$$\lim_{h\to 0^+}\frac{f(x+h)\ominus_H f(x)}{h}=\lim_{h\to 0^+}\frac{f(x)\ominus_H f(x-h)}{h}=f'_H(x)$$

if $f'_H(x) \in \mathcal{F}_N(\mathbb{R})$ then f is called Hukuhara differentiable at x. This limit value is called the Hukuhara derivative of f in x.

Definition 7 (Generalized Hukuhara difference). Let s and $s \in \mathcal{F}_N(\mathbb{R})$. If,

$$s \ominus_{gH} t = v = \begin{cases} (i) & s = t + v \\ (ii) & t = s + (-1)v \end{cases}$$

the number of $\exists v \in \mathcal{F}_N(\mathbb{R}^n)$ grades is available, the number v is called the generalized Hukuhara difference (gH-difference) of s, t grades and is indicated by $s \ominus_{gH} t$.

Theorem 4. If $s, t \in \mathcal{F}_N(\mathbb{R})$ and α-cut sets $s(\alpha) = [s_1(\alpha), s_2(\alpha)]$ and $t = [t_1(\alpha), t_2(\alpha)]$ $s \ominus_{gH} t$ are available, the α-cut set of the gH-difference is defined as [36]

$$\left[s \ominus_{gH} t\right](\alpha) = [\min\{s_1(\alpha) - t_1(\alpha), s_2(\alpha) - t_2(\alpha)\}, \max\{s_1(\alpha) - t_1(\alpha), s_2(\alpha) - t_2(\alpha)\}]$$

Definition 8 (Generalized Hukuhara differentiable). Let $f: (a,b) \to \mathcal{F}_N(\mathbb{R})$ and $x \in (a,b)$. If $f(x+h) \ominus_H f(x)$, $f(x) \ominus_H f(x-h)$ is present for $\forall h > 0$,

$$\lim_{h\to 0^+} \frac{f(x+h) \ominus_{gH} f(x)}{h} = \lim_{h\to 0^+} \frac{f(x) \ominus_{gH} f(x-h)}{h} = f'_{gH}(x)$$

if $f'_{gH}(x) \in \mathcal{F}_N(\mathbb{R})$ then f is called Generalized Hukuhara differentiable at x. This limit value is called the Generalized Hukuhara differentiable of f in x [36].

Theorem 5. Let the α-cut section of the function $f: (a,b) \to F_N(\mathbb{R}))$ be $f(x,\alpha) = [f_1(x,\alpha), f_2(x,\alpha)]$ for each $\alpha \in [0,1]$. In this case,

If f is differentiable in the sense of (i), the functions $f_1(x,\alpha)$ and $f_2(x,\alpha)$ are differentiable and $[f'(x)](\alpha) = [f_1'(x,\alpha), f_2'(x,\alpha)]$.

If f is differentiable in the sense of (ii), the functions $f_1(x,\alpha)$ and $f_2(x,\alpha)$ are differentiable and $[f'(x)](\alpha) = [f_1'(x,\alpha), f_2'(x,\alpha)]$.

Theorem 6 (Jury Theorem). For this criterion for $|\theta| < 1$ values

$$\theta^3 + a_3\theta^2 + a_2\theta + a_1 = 0$$

the roots of the cubic equation can be shown by the following conditions.

$$\begin{aligned} &1 + a_3 + a_2 + a_1 > 0, \\ &3 + a_3 - a_2 - 3a_1 > 0, \\ &1 - a_3 + a_2 - a_1 > 0, \\ &1 + a_3a_1 - a_2 - a_1^2 > 0, \end{aligned} \quad [37].$$

Mathematical Model of the Susceptible Diabetes Complication mModel (SDC)

The model is given below with the symbols to analyze and develop.

Notations

$\boldsymbol{S(n)}$: Number of susceptible individuals
$\boldsymbol{D(n)}$: Diabetes without complications (D)
$\boldsymbol{C(n)}$: Complications of diabetes (C)
$\boldsymbol{\gamma}$: Recovery rate of complications
$\boldsymbol{\vartheta}$: Birth rate
$\boldsymbol{\delta}$: Complication-related mortality
$\boldsymbol{\lambda}$: Rate of occurrence of complications
$\boldsymbol{\mu}$: Death rate
$\boldsymbol{\rho}$: Genetic disorder in childbirth
$\boldsymbol{\beta}$: Interaction rate

Formula of the model

The bio mathematical model examines the behavior of the system. This mathematical model for diabetes is used by Hill et al.[7]. It had been obtained before and is expressed in the following way by converting this model into the difference equation using the initial conditions previously accepted.

$$\begin{aligned} S(n+1) &= S(n) + \vartheta S(n) + \vartheta(1-\rho)\big(D(n) + C(n)\big) - \tfrac{\beta S(n)D(n)}{N} - \mu S(n) \\ D(n+1) &= D(n) + \tfrac{\beta S(n)D(n)}{N} + \vartheta\rho\big(D(n) + C(n)\big) - (\lambda+\mu)D(n) + \gamma C(n) \\ C(n+1) &= C(n) + \lambda D(n) - (\gamma+\delta+\mu)C(n) \end{aligned} \tag{1}$$

The equilibrium points are $S(n=0) = S_0, D(n=0) = D_0, C(n=0) = C_0$.

Analysis of the Model

This section uses the information given about the SDC model to find stability and analysis.

Stability Analysis of the Model

The nonlinear difference system given above can be inscribed as follows [8]:

$$\begin{aligned} S(n+1) &= S(n) + \vartheta S(n) + \vartheta(1-\rho)\big(D(n) + C(n)\big) - \tfrac{\beta S(n)D(n)}{N} - \mu S(n) \\ D(n+1) &= D(n) + \tfrac{\beta S(n)D(n)}{N} + \vartheta\rho\big(D(n) + C(n)\big) - (\lambda+\mu)D(n) + \gamma C(n) \\ C(n+1) &= C(n) + \lambda D(n) - (\gamma+\delta+\mu)C(n) + 0S(n) \end{aligned}$$

Model (1) Linearized matrix in $E = (0,0,0)$ equilibrium points,

$$A = \begin{pmatrix} 1+\vartheta-\mu & \vartheta(1-\rho) & \vartheta(1-\rho) \\ 0 & 1+\vartheta\rho-\lambda-\mu & \vartheta\rho+\gamma \\ 0 & \lambda & 1-\gamma-\delta-\mu \end{pmatrix}$$

Let us make the equation simpler here and say $X = \vartheta-\mu, Y = \vartheta(1-\rho), Z = \vartheta\rho-\lambda-\mu, T = \vartheta\rho+\gamma, W = \gamma+\delta+\mu$.
Characteristic equation

$$|A-\theta I| = 0$$

and the cubic equation

$$\theta^3 + a_3\theta^2 + a_2\theta + a_1 = 0$$

is obtained where the first patient given in Table 1, the parameters $\vartheta, \beta, \gamma, \delta, \lambda, \mu, \rho > 0$ and $0 \le \rho \le 1$ written

$$a_3 = X - Z + W - 3 < 0,$$
$$a_2 = 3 + 2\mathrm{Z} - 2\mathrm{W} - 2\mathrm{X} - \mathrm{XZ} + \mathrm{XW} - \mathrm{ZW} - \lambda\mathrm{T} > 0$$
$$a_1 = -1 + W - Z + ZW + \lambda T + X - XW + XZ - XZW - XT\lambda > 0$$

Therefore, according to theorem 6,

$$q_1 = 1 + a_3 + a_2 + a_1 > 0,$$
$$q_2 = 3 + a_3 - a_2 - 3a_1 > 0,$$
$$q_3 = 1 - a_3 + a_2 - a_1 > 0,$$
$$q_4 = 1 + a_3a_1 - a_2 - a_1^2 > 0$$ the system is stable.

Numerical Analysis on the Model

For the equation (1), the table (see Table 1) shown below shows the analysis of stability on three patients for different parameter values [24].

Table 1. Stability analysis of the SDC model

Parameters	First Patient	Second Patient	Third Patient
γ	0.37141	0	0
ϑ	0.01623	0.017	0.01642
δ	0.0068	0.0078	0.0588
λ	0.67758	0.77758	0.67765
μ	0.00764	1.91774	1.91864
ρ	0.077	0.077	0.077
β	0.16263	0.16263	0.16263
Value Of Variable	$a_1 = 2.9905 \times 10^{-5} > 0$ $a_2 = 0.0535 > 0$ $a_3 = 3.6788 > 0$ $a_4 = 0.0285 > 0$	$a_1 = 10.0107 > 0$ $a_2 = -1.8086 < 0$ $a_3 = -0.0042 < 0$ $a_4 = 0.0500 > 0$	$a_1 = 9.9082 > 0$ $a_2 = -1.7880 < 0$ $a_3 = -0.0016 < 0$ $a_4 = 0.0285 > 0$
Stability Status	**Stable**	**Unstable**	**Unstable**

It was observed that the jury criterion was stable or unstable when evaluating parameter values taken for different patients.

Analysis of the Model in Fuzzy Environment

Let us assume that the initial conditions of the model $\tilde{S}_0, \tilde{D}_0, \tilde{C}_0$ are Fuzzy numbers. The model defined in the third section becomes

$$\begin{aligned} \tilde{S}(n+1) &= \tilde{S}(n) + \vartheta\tilde{S}(n) + \vartheta(1-\rho)\left(\tilde{D}(n) + \tilde{C}(n)\right) - \frac{\beta\tilde{S}(n)\tilde{D}(n)}{N} - \mu\tilde{S}(n) \\ \tilde{D}(n+1) &= \tilde{D}(n) + \frac{\beta\tilde{S}(n)\tilde{D}(n)}{N} + \vartheta\rho\left(\tilde{D}(n) + \tilde{C}(n)\right) - (\lambda+\mu)\tilde{D}(n) + \gamma\tilde{C}(n) \\ \tilde{C}(n+1) &= \tilde{C}(n) + \lambda\tilde{D}(n) - (\gamma+\delta+\mu)\tilde{C}(n) \end{aligned} \tag{2}$$

with $\tilde{S}(n=0) = \tilde{S}_0, \tilde{D}(n=0) = \tilde{D}_0, \tilde{C}(n=0) = \tilde{C}_0$.

Models are being converted to the fuzzy difference equation due to the presence of the fuzzy variable. Here we can use the fuzzy difference equation concept for the solution of the model and stability analysis.
Six different situations arise:

i. $\tilde{S}(n), \widetilde{D}(n)$ and $\tilde{C}(n)$, $(i) - gH$ differentiable
ii. $\tilde{S}(n)$, $(i) - gH$, $\widetilde{D}(n)$, $(i) - gH$ and $\tilde{C}(n)$,$(ii) - gH$ differentiable
iii. $\tilde{S}(n)$, $(i) - gH$, $\widetilde{D}(n)$, $(ii) - gH$ and $\tilde{C}(n)$,$(i) - gH$ differentiable
iv. $\tilde{S}(n)$, $(ii) - gH$, $\widetilde{D}(n)$, $(i) - gH$ and $\tilde{C}(n)$,$(ii) - gH$ differentiable
v. $\tilde{S}(n)$, $(ii) - gH$, $\widetilde{D}(n)$, $(ii) - gH$ and $\tilde{C}(n)$,$(i) - gH$ differentiable
vi. $\tilde{S}(n), \widetilde{D}(n)$ and $\tilde{C}(n)$ $(ii) - gH$ differentiable

It is not substantial to find all situations if we can understand the method for any of the situations, then we can find the result of other states.
In this study, we will consider the initial situation, i.e., $\tilde{S}(n), \widetilde{D}(n)$ and $\tilde{C}(n)$, $(i) - gH$ may be differentiable. The above difference equation $\tilde{S}(n), \widetilde{D}(n)$and $\tilde{C}(n)$, $(i) - gH$ are differentiable

$$
\begin{aligned}
\tilde{S}_1(n+1,\alpha) &= \tilde{S}_2(n,\alpha) + \vartheta \tilde{S}_2(n,\alpha) + \vartheta(1-\rho)\left(\widetilde{D}_1(n,\alpha) + \tilde{C}_1(n,\alpha)\right) - \frac{\beta \tilde{S}_2(n,\alpha)\widetilde{D}_1(n,\alpha)}{N} - \mu \tilde{S}_2(n,\alpha)\\
\tilde{S}_2(n+1,\alpha) &= \tilde{S}_1(n,\alpha) + \vartheta \tilde{S}_1(n,\alpha) + \vartheta(1-\rho)\left(\widetilde{D}_2(n,\alpha) + \tilde{C}_2(n,\alpha)\right) - \frac{\beta \tilde{S}_1(n,\alpha)\widetilde{D}_2(n,\alpha)}{N} - \mu \tilde{S}_1(n,\alpha)\\
\widetilde{D}_1(n+1,\alpha) &= \widetilde{D}_2(n,\alpha) + \frac{\beta \tilde{S}_1(n,\alpha)\widetilde{D}_2(n,\alpha)}{N} + \vartheta\rho\left(\widetilde{D}_2(n,\alpha) + \tilde{C}_1(n,\alpha)\right) - (\lambda+\mu)\widetilde{D}_2(n,\alpha) + \gamma \tilde{C}_1(n,\alpha)\\
\widetilde{D}_2(n+1,\alpha) &= \widetilde{D}_1(n,\alpha) + \frac{\beta \tilde{S}_2(n,\alpha)\widetilde{D}_1(n,\alpha)}{N} + \vartheta\rho\left(\widetilde{D}_1(n,\alpha) + \tilde{C}_2(n,\alpha)\right) - (\lambda+\mu)\widetilde{D}_1(n,\alpha) + \gamma \tilde{C}_2(n,\alpha)\\
\tilde{C}_1(n+1,\alpha) &= \tilde{C}_2(n,\alpha) + \lambda \widetilde{D}_1(n,\alpha) - (\gamma+\delta+\mu)\tilde{C}_2(n,\alpha)\\
\tilde{C}_2(n+1,\alpha) &= \tilde{C}_1(n,\alpha) + \lambda \widetilde{D}_2(n,\alpha) - (\gamma+\delta+\mu)\tilde{C}_1(n,\alpha)
\end{aligned}
\tag{3}
$$

The above difference equations are crisp difference equations [24].

Stability analysis

The fuzzy SDC model indicated by (3), in matrix form, is as follows:

$$
\begin{bmatrix}
\tilde{S}_1(n+1,\alpha)\\
\tilde{S}_2(n+1,\alpha)\\
\widetilde{D}_1(n+1,\alpha)\\
\widetilde{D}_2(n+1,\alpha)\\
\tilde{C}_1(n+1,\alpha)\\
\tilde{C}_2(n+1,\alpha)
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
0 & 1+\vartheta-\frac{\beta \widetilde{D}_1(n,\alpha)}{N}-\mu & \vartheta(1-\rho)-\frac{\beta \tilde{S}_2(n,\alpha)}{N} & 0 & \vartheta(1-\rho) & 0\\
1+\vartheta-\frac{\beta \widetilde{D}_2(n,\alpha)}{N}-\mu & 0 & 0 & \vartheta(1-\rho)-\frac{\beta \tilde{S}_1(n,\alpha)}{N} & 0 & \vartheta(1-\rho)\\
\frac{\beta \widetilde{D}_2(n,\alpha)}{N} & 0 & 0 & 1+\frac{\beta \tilde{S}_1(n,\alpha)}{N}+\vartheta\rho-(\lambda+\mu) & \vartheta\rho+\gamma & 0\\
0 & \frac{\beta \widetilde{D}_1(n,\alpha)}{N} & 1+\frac{\beta \tilde{S}_2(n,\alpha)}{N}+\vartheta\rho-(\lambda+\mu) & 0 & 0 & \vartheta\rho+\gamma\\
0 & 0 & \lambda & 0 & 0 & 1-(\gamma+\delta+\mu)\\
0 & 0 & 0 & \lambda & 1-(\gamma+\delta+\mu) & 0
\end{bmatrix}
$$

$$
\begin{bmatrix}
\tilde{S}_1(n,\alpha)\\
\tilde{S}_2(n,\alpha)\\
\widetilde{D}_1(n,\alpha)\\
\widetilde{D}_2(n,\alpha)\\
\tilde{C}_1(n,\alpha)\\
\tilde{C}_2(n,\alpha)
\end{bmatrix}
$$

The stability of the solution is shown with the help of numerical simulations.

Numerical Solutions

Let us take into account the model, which includes the following fuzzy values. At $n = 0$, the number of susceptible individuals is $\tilde{S}_0 = (280,290,300)$, $\tilde{D}_0 = (7,9,11)$ for non-complicated diabetic, and $\tilde{C}_0 = (9,11,13)$. The values of other parameters are $\gamma = 0.37141, \vartheta = 0.01623, \delta = 0.0068, \lambda = 0.67758, \mu = 0.00764, \rho = 0.077, \beta = 0.16263$.

Numerical simulation: α-cut of initial conditions

$$\tilde{S}_1(0,\alpha) = 280 + 10\alpha$$
$$\tilde{S}_2(0,\alpha) = 300 - 10\alpha$$
$$\tilde{D}_1(0,\alpha) = 7 + 2\alpha$$
$$\tilde{D}_2(0,\alpha) = 11 - 2\alpha$$
$$\tilde{C}_1(0,\alpha) = 9 + 2\alpha$$
$$\tilde{C}_2(0,\alpha) = 13 - 2\alpha$$

Now the solutions for a patient for the fuzzy initial conditions constant n are given as follows tables (see table 2, see table 3, see table 4) and figures (Figure 1, Figure 2, Figure 3):

Table 2. $S1(n,\alpha), S2(n,\alpha)$ $n = 10$ value

α	$S_1(n,\alpha)$	$S_2(n,\alpha)$
0	312.0297	294.1509
0.1	311.1297	295.0382
0.2	310.2303	295.9263
0.3	309.3315	296.8151
0.4	308.4333	297.7047
0.5	307.5357	298.5950
0.6	306.6387	299.4860
0.7	305.7423	300.3778
0.8	304.8466	301.2702
0.9	303.9515	302.1633
1	303.0571	303.0571

Table 3. $D_1(n,\alpha), D_2(n,\alpha)$ $n = 10$ value

α	$D_1(n,\alpha)$	$D_2(n,\alpha)$
0	14.3227	9.6775
0.1	14.0931	9.9128
0.2	13.8634	10.1477
0.3	13.6333	10.3823
0.4	13.4030	10.6165
0.5	13.1724	10.8504
0.6	12.9415	11.0840
0.7	12.7104	11.3173
0.8	12.4789	11.5502
0.9	12.2472	11.7828
1	12.0152	12.0152

Table 4. $C1(n,\alpha), C2(n,\alpha)$ $n = 10$ value

α	$C_1(n,\alpha)$	$C_2(n,\alpha)$
0	22.4534	15.1063
0.1	22.0892	15.4772
0.2	21.7248	15.8477
0.3	21.3601	16.2178
0.4	20.9950	16.5874
0.5	20.6296	16.9567
0.6	20.2639	17.3257
0.7	19.8978	17.6942
0.8	19.5314	18.0624
0.9	19.1647	18.4302
1	18.7976	18.7976

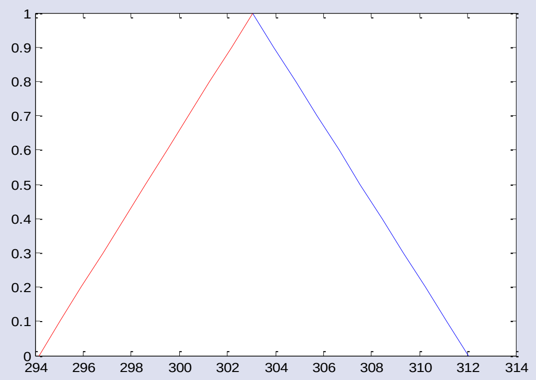

Figure 1. Graph of $S_1\ (n, \alpha), S_2\ (n, \alpha)\ n\ =\ 10$

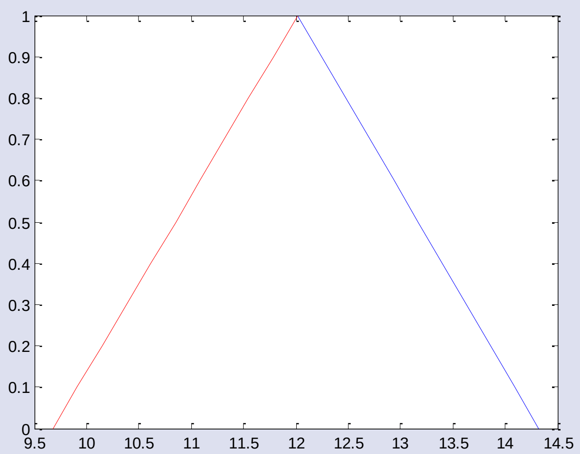

Figure 2. Graph of $D1(n, \alpha), D2(n, \alpha)\ n = 10$

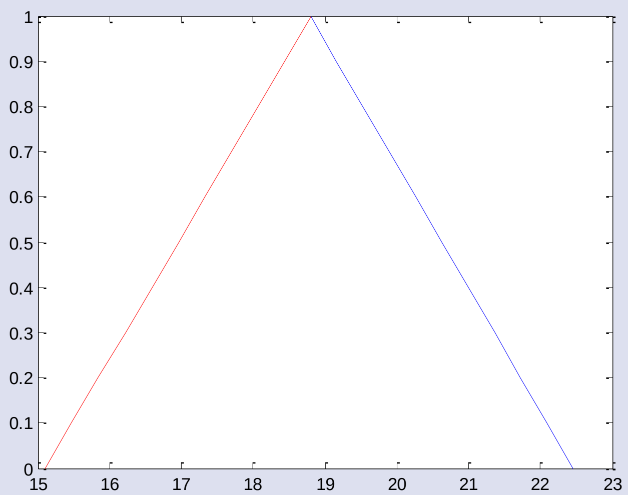

Figure 3. Graph of $C1(n, \alpha), C2(n, \alpha)\ n = 10$

Explanation: From the Table 2, Figure 1 we see that $S_1\,(n,\alpha)$ are decreasing and $S_2\,(n,\alpha)$ are increasing, Table 3, Figure 2 we see that $D_1\,(n,\alpha)$ are decreasing and $D_2\,(n,\alpha)$ are increasing and Table 4, Figure 3, we see that $C_1\,(n,\alpha)$ are decreasing, $C_2\,(n,\alpha)$ are increasing . For $\alpha \in [0,1]$ for $n = 10$. $\tilde{S}(n), \tilde{D}(n)$ and $\tilde{C}(n)$ provide a strong solution. Therefore, the system is stable in these situations.

Conclusions

In this study, we examined the mathematical model of the sensitive diabetes complication (SDC) system to determine the number of likely interactions that cause the interaction between unhealthy individuals and healthy individuals in the fuzzy and crisp environment Approaches to the generalized Hukuhara derivative notion have been applied to explain fuzzy solutions of the given model In addition, demonstration of the sensitive diabetes complication system in the fuzzy environment was used to better analyze the decision-making situation by ensuring that uncertain parameters were met. The gH-derivative approach method, which has a substantial place in fuzzy analysis, makes it likely to obtain the fuzzy solution of the model. Stability analysis was obtained for the model in Fuzzy environment.

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Conflicts of interest

There are no conflicts of interest in this work.

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