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Evaluating the Goodness of Fit of Generalized Nakagami Distribution

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ABSTRACT

The Generalized Nakagami distribution is a popular distribution in wireless communication. This distribution includes the Nakagami distribution as a special case. Likelihood ratio, score, and two C(α) tests are developed to evaluate the fit of Nakagami distribution against Generalized Nakagami distribution. A Monte Carlo simulation study is performed in order to investigate the performance of these tests with regard to Type I errors and powers of tests. Finally, two data sets are analyzed using the proposed goodness of fit tests.

Keywords: C(α) test, Generalized Nakagami Distribution, Likelihood ratio test, Power of test, Score test.

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Introduction

Modeling of wireless channels is crucial in many applied sciences such as engineering, medicine, and hydrology. In wireless communications, signals usually do not reach the receiver directly during transmission, and degradation of signal quality known as fading occurs. In the literature, various models such as Rayleigh, Rician, Nakagami, and K-models are used to model the fading in wireless channels. Since the modeling of the statistical properties of the fading channels is flexible, differences arise among these models. These models also differ in terms of parameter numbers. Among these models, the Nakagami model can model fading of signals reasonably well. The Generalized Nakagami distribution (GND) is produced by adding a new parameter that takes into account the tails of the density function of the Nakagami distribution [1]. The probability density function (pdf) of the GND is given as follows:

$$f(y;s,m,\Omega)=\frac{2sm^m}{\Gamma(m)\Omega^m}y^{2ms-1}e^{-\left(\frac{m}{\Omega}\right)y^{2s}},\ \ y>0, \qquad (1)$$

where $m \geq 0.5$ and $s>0$ are shape parameters and $\Omega>0$ is scale parameter. Note that the GND is related to the generalized gamma distribution introduced by Stacy [2]. It can be noted that the Generalized Nakagami random variable is generated by taking the square root of the generalized gamma random variable.

The probability densities of GND are plotted for various parameter combinations in Figure 1.

The GND converts to some special distributions such as when $s=1$, the GND becomes the ND; when $m=s=1$ the GND reduces to the Rayleigh distribution and when $m=1, s=1/2$ the GND reverts to exponential distribution.

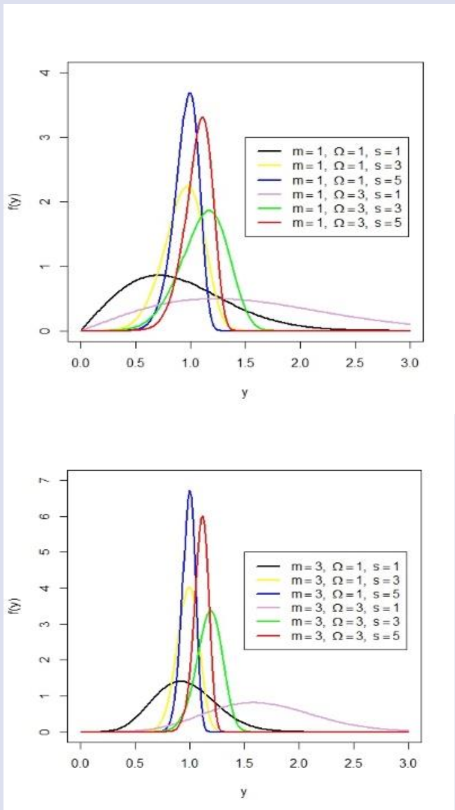

Figure 1. Generalized Nakagami densities for various parameter combinations.

It is essential to identify an appropriate distribution to be fitted for a given data. Increasing of number of parameters can cause some complications such as estimations of parameters depend on each other. In real applications, distributions with fewer parameters may be sufficient to describe data or provide better fit than distribution which have more parameters or complete opposite of this situation may occur. For example, Shankar [1] has reported that the GND is likely to match ultrasound data much better than the ND. Hence, in this study the ND is considered as a special case of the GND, and some goodness of fit tests are constructed to decide which distribution will have better fitting.

The ND was proposed by Nakagami to model fading of radio signals [3]. Although many statistical distributions describe the fading of signals such as Weibull, Rician, Rayleigh, and lognormal distributions, the ND matches some empirical data better than the other distributions [4].

Applications of the ND have been carried out in many scientific fields such as modelling of high-frequency seismogram envelopes, communications engineering, medical imaging studies etc. The usefulness of ND for dealing with high frequency seismogram envelopes has been demonstrated [5]. Sarkar et al. [6, 7] have examined performance of the ND to derivation of unit hydrographs in hydrology. Datta et al. [8] have modelled ultrasound kidney images with the ND. Alavi et al. [9] have analyzed performance of the ND for wind speed data. Since the ND has memoryless property, Ahmad et al. [10] have used the distribution to model hazard rate in reliability studies. Ramos et al. [11,12] have proposed new estimator for the ND and presented Bayesian inference considering objective priors for the ND parameters. Kumar et al. [13] have used the ND as a lifetime model. Ozonur et al. [14] have adapted some tests to evaluate Rayleigh distribution against the ND. Ozonur and Paul [15] have developed tests of fit of the generalized gamma distribution.

Goodness of fit tests are statistical tools evaluating adequacy of a distribution. Although goodness of fit of the ND is examined in the literature, asymptotically optimal goodness of fit tests such as the Neyman's $C(\alpha)$, Rao's score and likelihood ratio tests are firstly developed in this study to check whether the GND is statistically superior to the ND for a given data set. In order to decide whether a random sample has been taken from the ND or GND, null and alternative hypotheses can be constructed as below:

$$H_0 : s = 1 \qquad H_1 : s \neq 1. \tag{2}$$

The Neyman's $C(\alpha)$, Rao's score and likelihood ratio tests are asymptotically optimal, and they provide tests with good properties in large samples [16]. Although there are various studies including these goodness of fit tests [17,18], these tests have not been taken into consideration for the GND. Goodness of fit problem of the distribution is considered in the study due to pervasive usage in many scientific areas. In this context, the main focus of this study is to test goodness of fit of the ND against the GND.

The rest of the article is designed as follows. Firstly, the goodness of fit tests such as the likelihood ratio, score, and two $C(\alpha)$ tests are proposed to test the null hypothesis against the alternative hypothesis. Secondly, simulation study is performed to evaluate the performance of the tests with regard to Type I errors and powers of tests. In the subsequent section, two real data sets are analyzed and finally, some conclusions are provided.

Materials and Methods

Goodness of Fit Tests

$Y_1,\ldots,Y_n$ is a random sample from the GND with pdf given in Equation (1) with the parameter vector $\gamma=(s,\beta^T)^T$ where $\beta=(m,\Omega)^T$. The aim of this study is to test the $H_0 : s=1$ against the $H_1 : s\neq 1$ treating the $\beta=(m,\Omega)^T$ as nuisance parameter.

Likelihood ratio test

The log-likelihood function of the GND is given by

$$l(\gamma;Y)=n\left[\log(2s)+m\log(m)-\log\Gamma(m)-m\log(\Omega)\right]+(2ms-1)\sum_{i=1}^{n}\log(y_i)-\frac{m}{\Omega}\sum_{i=1}^{n}y_i^{2s}. \tag{3}$$

The maximum likelihood estimate (MLE) of γ under the full model, say $\tilde{\gamma}=\left(\tilde{s},\tilde{m},\tilde{\Omega}\right)^T$, is a simultaneous solution of the following equations:

$$\frac{n}{s}+2m\sum_{i=1}^{n}\log(y_i)-\frac{m}{\Omega}\sum_{i=1}^{n}2y_i^{2s}\log(y_i)=0$$

$$n\left[\log(m)+1-\Psi(m)-\log(\Omega)\right]+2s\sum_{i=1}^{n}\log(y_i)-\frac{1}{\Omega}\sum_{i=1}^{n}y_i^{2s}=0$$

$$\frac{-nm}{\Omega}+\frac{m}{\Omega^2}\sum_{i=1}^{n}y_i^{2s}=0$$

where $\Psi(m)$ is digamma function, i.e., $\Psi(m)=\frac{\partial \log\Gamma(m)}{\partial m}$. Similarly, the MLE of γ under the reduced model, say $\hat{\gamma}=\left(1,\hat{m},\hat{\Omega}\right)^{T}$, can be obtained by solving the following equations :

$$n\log(m)+n-n\Psi(m)-n\log\Omega+2\sum_{i=1}^{n}\log y_i-\frac{1}{\Omega}\sum_{i=1}^{n}y_i^2=0$$

$$\frac{-nm}{\Omega}+\frac{m}{\Omega^2}\sum_{i=1}^{n}y_i^2=0.$$

Since the MLE equations of the full and reduced models have no direct solutions, iterative methods must be used to solve the equations. The MLEs of γ under the full and reduced models are derived using the mle2 function in R 3.4.1.

The likelihood ratio test statistic (LR) is given as follows:

$$LR=2\left(l(\tilde{\gamma};Y)-l(\hat{\gamma};Y)\right),$$

where $\tilde{\gamma}=\left(\tilde{s},\tilde{m},\tilde{\Omega}\right)^{T}$ and $\hat{\gamma}=\left(1,\hat{m},\hat{\Omega}\right)^{T}$ are the MLEs of γ under the full model and reduced model, respectively. Then the statistic LR asymptotically follows χ_1^2 under the null hypothesis.

C(α) and score tests

Rao proposed the score test as an alternative to the likelihood ratio test and Neyman introduced the $C(\alpha)$ test as a generalization of the Rao's score test [19, 20]. The $C(\alpha)$ and score test statistics are based on score functions. The score functions are the partial derivatives of the log-likelihood function with respect to the parameter of interest and nuisance parameter evaluated under the null hypothesis.

Consider testing $H_0: s=1$ against $H_1: s\neq 1$ where the parameter vector $\gamma=(s,\beta^T)^T$ is partitioned into the parameter of interest s and nuisance parameter $\beta=(m,\Omega)^T$. Similarly, score vector and information matrix can be partitioned as $U=\begin{bmatrix}U_s\\U_\beta\end{bmatrix}$ and $I=\begin{bmatrix}I_{ss} & I_{s\beta}\\ I_{\beta s} & I_{\beta\beta}\end{bmatrix}$, respectively. Under the null hypothesis, we obtained elements of the U and I as follows:

$$U_s=\left.\frac{\partial l(\gamma;Y)}{\partial s}\right|_{s=1}=n+2m\sum_i \log y_i-\frac{2m}{\Omega}\sum_i y_i^2\log y_i,$$

$$U_\beta=\frac{\partial l(\gamma;Y)}{\partial \beta}=\left.\left(\frac{\partial l(\gamma;Y)}{\partial m},\frac{\partial l(\gamma;Y)}{\partial \Omega}\right)^T\right|_{s=1}$$

$$=\left(n\left(\log m+1-\Psi(m)-\log\Omega\right)+2\sum_i\log y_i-\frac{1}{\Omega}\sum_i y_i^2,\quad \frac{-nm}{\Omega}+\frac{m}{\Omega^2}\sum_i y_i^2\right)^T,$$

$$I_{ss}=-E\left[\left.\frac{\partial^2 l(\gamma;X)}{\partial s\partial s}\right|_{s=1}\right]=n+\frac{n}{\Gamma(m)}\left[\Gamma(m+1)\log^2\left(\frac{\Omega}{m}\right)+2\Gamma'(m+1)\log\left(\frac{\Omega}{m}\right)+\Gamma''(m+1)\right]$$

$$I_{s\beta}=-E\left[\left.\frac{\partial^2 l(\gamma;X)}{\partial s\partial \beta^T}\right|_{s=1}\right]=\left[\frac{n}{m},-\frac{nm}{\Omega}\left(\log\left(\frac{\Omega}{m}\right)+\Psi(m+1)\right)\right]^T,$$

$$I_{\beta\beta}=-E\left[\left.\frac{\partial^2 l(\gamma;X)}{\partial \beta\partial \beta^T}\right|_{s=1}\right]=\begin{bmatrix}-\frac{n}{m}+n\Psi'(m) & 0\\ 0 & \frac{nm}{\Omega^2}\end{bmatrix},$$

where trigamma function $\Psi'(m)$ is derivative of the digamma function $\Psi(m)$.

Define adjusted score as $R=U_s-BU_\beta$, where B is the matrix of partial regression coefficients. Bartlett [21] showed that $B=I_{s\beta}I_{\beta\beta}^{-1}$ and variance covariance matrix of R is $I_{ss.\beta}=I_{ss}-I_{s\beta}I_{\beta\beta}^{-1}I_{\beta s}$. The Neyman's $C(\alpha)$ statistic is given by

$$C=\frac{R^2}{I_{ss.\beta}}.$$

The main advantage of the $C(\alpha)$ test is that it only requires the estimates under the null hypothesis. Hence, estimate of the nuisance parameter $\beta=(m,\Omega)^T$ is only required and if the nuisance parameter is replaced by $\sqrt{n}$-consistent estimate, then the $C(\alpha)$ statistic is asymptotically distributed as χ_1^2 under the null hypothesis. In addition, if the nuisance parameter

$\beta = \left(m, \Omega\right)^T$ is replaced by its MLE $\hat{\beta} = \left(\hat{m}, \hat{\Omega}\right)^T$, then the $C(\alpha)$ statistic becomes the Rao's score test statistic (S) which is given as follows:

$$S = \frac{U_s^2}{I_{ss.\beta}}.$$

Under the null hypothesis, the MLE $\hat{\beta} = \left(\hat{m}, \hat{\Omega}\right)^T$ can be obtained as mentioned in the reduced model of the likelihood ratio test. In order to calculate the $C(\alpha)$ statistic, two different moment based estimates of the nuisance parameter, say $\breve{\beta}_1 = \left(\breve{m}_{AK}, \breve{\Omega}\right)^T$ and $\breve{\beta}_2 = \left(\breve{m}_{CB}, \breve{\Omega}\right)^T$, are used. Under the null hypothesis k. moment of the data is $\mu_k = E\left[Y^k\right] = \frac{\Gamma(m + k/2)}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^{k/2}$.

The moment estimate of Ω is obtained as $\breve{\Omega} = \mu_2$. Since the moment estimate of m cannot be obtained from first moment equation, Abdi and Kaveh [22] have suggested a moment-based estimator for the m fading parameter as $\breve{m}_{AK} = \frac{\mu_2^2}{\mu_4 - \mu_2^2}$. In addition, Cheng and Beaulieu [23] have reported the estimator of m as $\breve{m}_{CB} = \frac{\mu_1 \mu_2}{2(\mu_3 - \mu_1 \mu_2)}$. In this study, we have used both estimators of m and we have denoted the $C(\alpha)$ statistics by C_{CB} and C_{AK} using the estimators of m as $\breve{m}_{CB}$ and $\breve{m}_{AK}$, respectively.

Asymptotically, it has been shown in the literature that the likelihood ratio test is equivalent to the $C(\alpha)$ or score test and the test statistics C_{CB}, C_{AK} and S are asymptotically follow χ_1^2 under the null hypothesis [24, 25].

Simulation Study

Monte Carlo simulation study is performed to demonstrate quality of the proposed tests LR, C_{CB}, C_{AK} and S in terms of Type I errors and powers of tests by using statistical software R 3.4.1. In the simulation study, critical values of the goodness of fit tests are obtained by simulating 10000 samples of size n from the GND with $s = 1$. Using the critical values, the Type I errors are calculated by generating 5000 random samples from the GND with $s = 1$ for various combinations of levels $(\alpha = 0.10, 0.05, 0.01)$, sample sizes $(n = 20, 30, 50)$ and parameters $(m = 0.75, 1)$. The Type I errors and powers of the tests are not affected as the value of Ω changes. So, all simulation results are obtained based on $\Omega = 1$. The Type I errors of the tests are summarized in Table 1. In addition, we have obtained the powers of the tests under the GND using various combinations of sample sizes n and parameters $(m = 0.75, 1; s = 1,3,5,7,9,11,13,15)$. The power study is performed with 5000 iterations and nominal level of 0.05. The powers of the tests are presented in Table 2.

Table 1. Type I errors of the tests for different m parameters, nominal levels and sample sizes.

Sample Sizes	Level	m	C_{CB}	C_{AK}	S	LR
20	0.10	0.75	0.0970	0.0970	0.0978	0.0954
		1	0.1026	0.1026	0.1070	0.1082
	0.05	0.75	0.0476	0.0450	0.0504	0.0482
		1	0.0524	0.0510	0.0590	0.0546
	0.01	0.75	0.0102	0.0106	0.0100	0.0122
		1	0.0124	0.0122	0.0118	0.0112
30	0.10	0.75	0.1008	0.1026	0.1050	0.1068
		1	0.1038	0.1042	0.0964	0.0960
	0.05	0.75	0.0494	0.0518	0.0518	0.0518
		1	0.0516	0.0530	0.0478	0.0520
	0.01	0.75	0.0082	0.0082	0.0092	0.0104
		1	0.0116	0.0128	0.0060	0.0098
50	0.10	0.75	0.0984	0.0966	0.0928	0.0948
		1	0.0920	0.0936	0.0912	0.0980
	0.05	0.75	0.0462	0.0454	0.0514	0.0484
		1	0.0426	0.0430	0.0442	0.0472
	0.01	0.75	0.0102	0.0110	0.0108	0.0098
		1	0.0056	0.0054	0.0092	0.0052

As shown in Table 1, Type I errors of all the four tests close to nominal levels irrespective of values of parameters and sample sizes. As shown in Table 2, as the sample size increases powers of all the tests increase. Also, powers of the tests increase as the s moves away from the null. It can be stated that when s is smaller than 7, the LR is the most powerful test and S provides slightly lower power than the LR. However, the score test S shows better performance among the four tests when $s > 7$ in terms of powers of tests. For all sample sizes and parameter combinations, the C_{AK} is the least powerful test among all these tests. The C_{CB} has better performance than the C_{AK} between the Neyman $C(\alpha)$ tests using moment estimators. The score test S shows generally better performance than the C_{CB} and C_{AK} tests in terms of powers of tests. It is pointed out that for all sample sizes, as m parameter increases, powers of LR, C_{CB} and C_{AK} tests decrease. However, as m parameter increases, the power of S test decreases when $s < 7$, and increases when $s > 7$.

Table 2. Powers of the tests for nominal level 0.05, $m = 0.75, 1$ and $n = 20, 30, 50$.

n	*m*	*Statistics*	*s*							
			1	*3*	*5*	*7*	*9*	*11*	*13*	*15*
20	0.75	C_{CB}	0.0580	0.1540	0.1974	0.2114	0.2070	0.2226	0.2356	0.2256
		C_{AK}	0.0562	0.1198	0.1274	0.1298	0.1172	0.1182	0.1224	0.1110
		S	0.0548	0.1712	0.2606	0.3238	0.3998	0.5688	0.7200	0.8230
		LR	0.0544	0.2086	0.3064	0.3568	0.3808	0.4106	0.4216	0.4206
	1	C_{CB}	0.0580	0.1446	0.1674	0.1790	0.1818	0.1668	0.1830	0.1806
		C_{AK}	0.0560	0.1082	0.1126	0.1114	0.1026	0.0892	0.0904	0.0852
		S	0.0504	0.1472	0.2118	0.3096	0.4884	0.6952	0.8102	0.8450
		LR	0.0542	0.1798	0.2544	0.3186	0.3296	0.3268	0.3520	0.3646
30	0.75	C_{CB}	0.0544	0.2572	0.3156	0.3478	0.3660	0.3924	0.3874	0.3908
		C_{AK}	0.0540	0.2106	0.2340	0.2444	0.2486	0.2512	0.2458	0.2468
		S	0.0572	0.2630	0.3674	0.4436	0.5438	0.6954	0.8442	0.9002
		LR	0.0546	0.2794	0.3960	0.4586	0.5018	0.5214	0.5318	0.5508
	1	C_{CB}	0.0460	0.2188	0.2960	0.3114	0.3256	0.3368	0.3276	0.3444
		C_{AK}	0.0480	0.1774	0.2110	0.2182	0.2234	0.2208	0.2060	0.2188
		S	0.0480	0.2100	0.3244	0.3974	0.6130	0.8144	0.8912	0.9372
		LR	0.0468	0.2250	0.3466	0.3770	0.4128	0.4324	0.4358	0.4556
60	0.75	C_{CB}	0.0496	0.4334	0.5624	0.6050	0.6364	0.6462	0.6662	0.6666
		C_{AK}	0.0484	0.3728	0.4676	0.5002	0.5050	0.4986	0.5294	0.5276
		S	0.0488	0.4110	0.5840	0.6508	0.7188	0.8412	0.9506	0.9604
		LR	0.0488	0.4360	0.6136	0.6762	0.7106	0.7352	0.7554	0.7698
	1	C_{CB}	0.0536	0.4106	0.5114	0.5454	0.5874	0.6102	0.6070	0.6204
		C_{AK}	0.0532	0.3582	0.4294	0.4512	0.4804	0.4940	0.4960	0.4924
		S	0.0514	0.3674	0.5096	0.5808	0.7774	0.9332	0.9606	0.9898
		LR	0.0504	0.3670	0.5218	0.5836	0.6310	0.6542	0.6596	0.6780

Real Data Examples

Notice that all the tests discussed above asymptotically follow chi-square distribution. However, the asymptotic distribution may be poor for small or moderate sample sizes; hence, p-values based on the asymptotic distribution may not always be reliable. Therefore, we obtain parametric bootstrap p-values for the following real data examples. The parametric bootstrap procedure is performed as follows.

Calculate the test statistic value T_0 from original data. Then generate a bootstrap sample according to the null hypothesis using the MLE and compute the test statistic T_0 with this bootstrap sample and call it T_0^*. Replicate this sampling a large number of times, say B times. Thus, the T_0^* values are $T_{01}^*, T_{02}^*, \ldots, T_{0B}^*$. The bootstrap p-value of T_0 is calculated as $p^* = \left\{\#\left(T_{0i}^* > T_0\right)\right\} / B \quad (i = 1, 2, \ldots, B)$. Reject the null hypothesis if the bootstrap p-value is smaller than nominal level of test. In the real data examples, we have considered the nominal level $\alpha = 0.05$ and the replication number $B = 100000$.

Example 1

In the first example, we have considered daily wind speed data analyzed by [26]. This data is presented in Table 3.

Table 3. Daily wind speed data

5.5	4.0	5.3	5.7	4.1	6.7	5.4	3.9	2.8	3.7	2.9	4.7	3.8	3.4	2.5
3.3	3.5	2.6	4.1	3.3	6.9	2.7	2.0	2.5	2.8	2.0	3.2	2.6	3.8	4.0

The MLEs of the parameters for the data are obtained as $\hat{m} = 2.516$ and $\hat{\Omega} =$ 15.973 under the null hypothesis and $\tilde{s} = 0.134$, $\tilde{m} = 131.981$, $\tilde{\Omega} = 1.416$ under the alternative hypothesis. We calculate values of the tests statistics $LR = 2.436$ and $S = 2.072$ with parametric bootstrap p-values 0.040 and 0.116, respectively. Although the ND sufficiently fits the wind speed data according to the p-value of the S statistic, the GND provides a better fit than the ND according to p-the value of the LR statistic.

Example 2

As the second example, we have analyzed a real data of remission times (in months) of patients with bladder cancer [27]. The data is presented in Table 4.

Table 4. Remission times

0.08	2.09	3.48	4.87	6.94	8.66	13.11	23.63	0.20	2.23	3.52	4.98
6.97	9.02	13.29	0.40	2.26	3.57	5.06	7.09	9.22	13.80	25.74	0.50
2.46	3.64	5.09	7.26	9.47	14.24	25.82	0.51	2.54	3.70	5.17	7.28
9.74	14.76	6.31	0.81	2.62	3.82	5.32	7.32	10.06	14.77	32.15	2.64
3.88	5.32	7.39	10.34	14.83	34.26	0.90	2.69	4.18	5.34	7.59	10.66
15.96	36.66	1.05	2.69	4.23	5.41	7.62	10.75	16.62	43.01	1.19	2.75
4.26	5.41	7.63	17.12	46.12	1.26	2.83	4.33	5.49	7.66	11.25	17.14
79.05	1.35	2.87	5.62	7.87	11.64	17.36	1.40	3.02	4.34	5.71	7.93
11.79	18.10	1.46	4.40	5.85	8.26	11.98	19.13	1.76	3.25	4.50	6.25
8.37	12.02	2.02	3.31	4.51	6.54	8.53	12.03	20.28	2.02	3.36	6.76
12.07	21.73	2.07	3.36	6.93	8.65	12.63	22.69				

The MLEs of the parameters for the data are obtained as $\hat{m}=0.503$ and $\hat{\Omega}=193.327$ under the null hypothesis and $\tilde{s}=0.257$, $\tilde{m}=3.875$, $\tilde{\Omega}=2.804$ under the alternative hypothesis. We obtain values of the tests statistics $LR=42.485$ and $S=509.530$ with the bootstrap p-values 0.000 and 0.000, respectively. So, our conclusion is that the GND sufficiently fits the remission times data.

Conclusion

The Generalized Nakagami distribution is popular distribution in engineering, survival analysis, medical studies, and hydrology. The distribution contains the Nakagami distribution as a special case. When the reduced model may be adequate for the considered data, the Generalized Nakagami distribution may be a redundant and complex distribution.

Since it is important to decide whether reduced distribution is sufficient to describe data, four goodness of fit tests, namely, the $C(\alpha)$ tests C_{CB} and C_{AK}; score test S and likelihood ratio test LR are developed to test goodness of fit of the Nakagami distribution against the Generalized Nakagami distribution. These tests are then compared by a Monte Carlo simulation study for various sample size and parameter scenarios. Simulation study suggests that the LR and S tests provide better performance than the C_{CB} and C_{AK} tests. Although, the LR is the most powerful test and the S provides slightly lower power than the LR for small values of m and s, the score test S is the most powerful test for large values of m and s. So, our recommendation is to use the LR and S tests for small m and s values and the S test for large m and s values.

Conflicts of interest

There are no conflicts of interest in this work.

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