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A new Lifetime Distribution Based on the Transmuted First Two Lower Records

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ABSTRACT

This article introduces a new lifetime distribution by merging the first two lower records based on exponential distribution and discusses the different features of the distribution. Statistical inferences about the distribution parameters are discussed with three estimation methods, namely maximum likelihood, least squares, and weighted least squares. Monte Carlo simulation study is performed to evaluate of these estimators based on mean square errors estimation, mean absolute deviation, and mean relative errors of estimation for a sample of different sizes. A distribution simulation analysis based on real data is provided to demonstrate the adaptability of the proposed model.

Keywords: Lower record value, Lifetime distribution, Monte Carlo simulation, Estimation.

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Introduction

Over the past two decades, many discrete and continuous statistical distributions have been introduced into the literature. These distributions are saved as members of a distribution family. Some distribution families in the literature can be ordered as follows: Azzalini [1] obtained the skew normal distribution, Mudholkar and Srivastava [2] described exponential distribution family, Marshall and Olkin [3] proposed a new family of continuous distribution, namely the Marshall and Olkin family. Eugene et al. [4] proposed another family of distribution titled beta-generated family. Recently, Mahdavi and Kundu [5] introduced a new family titled α−power transformation (APT) and Karakaya et. al. [6] obtained alpha logarithmic transformation (ALT) family.

An example of a distribution family is transmuted families. Generally, transmuted families are reported based on order statistics. Transformed distributions were introduced by Shaw and Buckley [7,8] using a quadratic transformation. The order statistics of the transformed distributions can be sorted as [9,10]. Recently, Balakrishnan [11] proposed a new family of transformed distributions based on datasets. Tanış and Saraçoğlu [12] studied a special model based on the Weibull distribution of a family of transformed record-based distributions. Both in terms of distribution properties and statistical inference parallel to the study [11]. Tanış et al. [13], introduced a transmutation lower record type and suggested a sub-model for Fréchet distribution, moreover Tanış [14], suggested transmutation lower record type inverse Rayleigh and Tanış [15], suggested transmutation lower record type power function distribution.

Let $X_{L(1)}$ and $X_{L(2)}$ be the first two lower record values from a population with cumulative distribution function (cdf) $F(x)$.

$$\begin{aligned} G(x) &= pP\left(X_{L(1)} \le x\right) + (1-p)P\left(X_{L(2)} \le x\right) \\ &= pF(x) + (1-p)\left[F(x)\left(1 - log\left(F(x)\right)\right)\right] \\ &= F(x)\left[1 - p\, log\left(F(x)\right)\right] \end{aligned} \quad (1)$$

The distribution family with cdf in Equation (1) is called transmuted lower record type (TLRT) and using TLRT, the probability density function (pdf) of distribution is given by

$$g(x) = f(x)\left[1 - p\left(1 + log\left(F(x)\right)\right)\right]. \quad (2)$$

In this paper, we obtained the TLRT version of the exponential distribution in Section 2. In Section 3, the unknown parameters are estimated by estimation methods. A simulation study is performed in order to compare the performance of these estimators in terms of mean squared errors (MSEs), mean absolute deviations (ABBs) and mean relative errors estimates (MREs). Two applications with real data are made to show the applicability of introduced distribution.

Transmuted Lower Record Type Exponential Distribution

Let X be a random variable having exponential distribution. The cdf and pdf of X as follows,

$$F(x) = 1 - exp(-\lambda x), \quad (3)$$

and

$$f(x) = \lambda\, exp(-\lambda x) \quad (4)$$

where $\lambda > 0$. Substituting the cdf (3) and pdf (4) into TLRT family the following cdf and pdf are obtained as

$$F(x;\lambda) = (1 - exp(-\lambda x))(1 - p\, log(1 - exp(-\lambda x))) \quad (5)$$

and

$$f(x;\lambda) = \lambda\, exp(-\lambda x)\,(1 - p\, log(1 - exp(-\lambda x))) - p\lambda\, exp(-\lambda x) \quad (6)$$

where $\lambda > 0, p \in (0,1)$. The distribution with cdf is called $\mathrm{TLRT} - \mathrm{Exp}(p,\lambda)$ distribution. Figure 1 presents the plots of the $\mathrm{TLRT} - \mathrm{Exp}(p,\lambda)$ pdf for some choices of parameters. From Figure 1, we observe that the probabilities are decreasing when x is increasing.

The mean of the $\mathrm{TLRT} - \mathrm{Exp}(p,\lambda)$ distribution is obtained as

$$E(X) = -\frac{6p - 6 + p\pi^2}{6\lambda} \quad (7)$$

The second moment of distribution cannot be obtained. It is a rare distribution whose variance and other moments cannot be obtained due to the non-finite integral solution. The survival function and hazard function $\mathrm{TLRT} - \mathrm{Exp}(p,\lambda)$ distribution is given by respectively

$$S(x) = 1 - F(x) = -p(-1 + exp(-\lambda x))\; log(1 - exp(-\lambda x)) + exp(-\lambda x) \quad (8)$$

And

$$h(x) = \frac{f(x)}{S(x)} \quad (9)$$

$$= \frac{\lambda\, exp(-\lambda x)\,(-1 + p\, log(1 - exp(-\lambda x)) + p)}{p(-1 + exp(-\lambda x))\, log(1 - exp(-\lambda x)) - exp(-\lambda x)}.$$

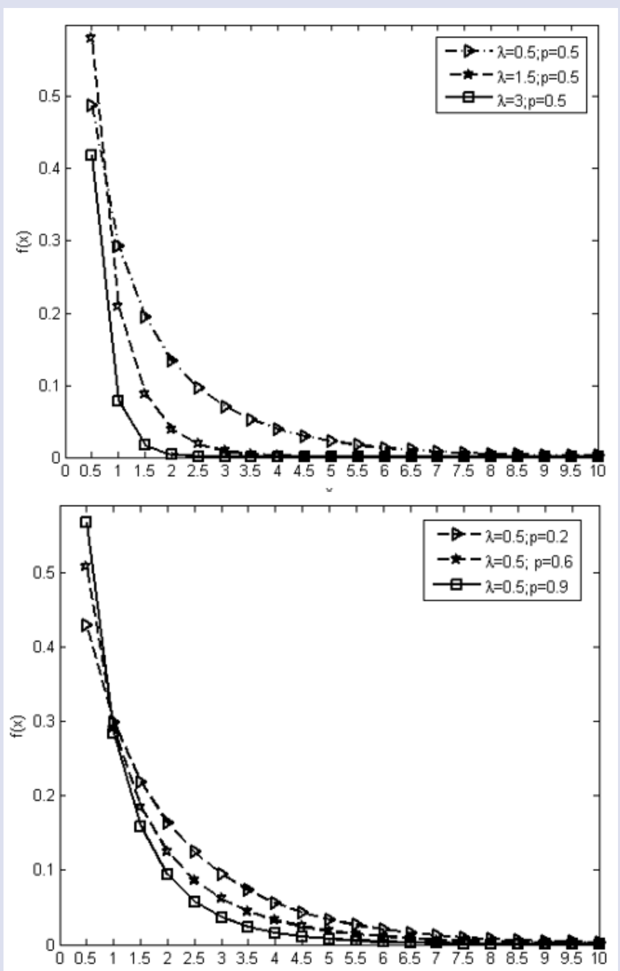

Figure 1. The pdf of $\mathrm{TLRT} - \mathrm{Exp}(p,\lambda)$ distribution for some choices of p and λ.

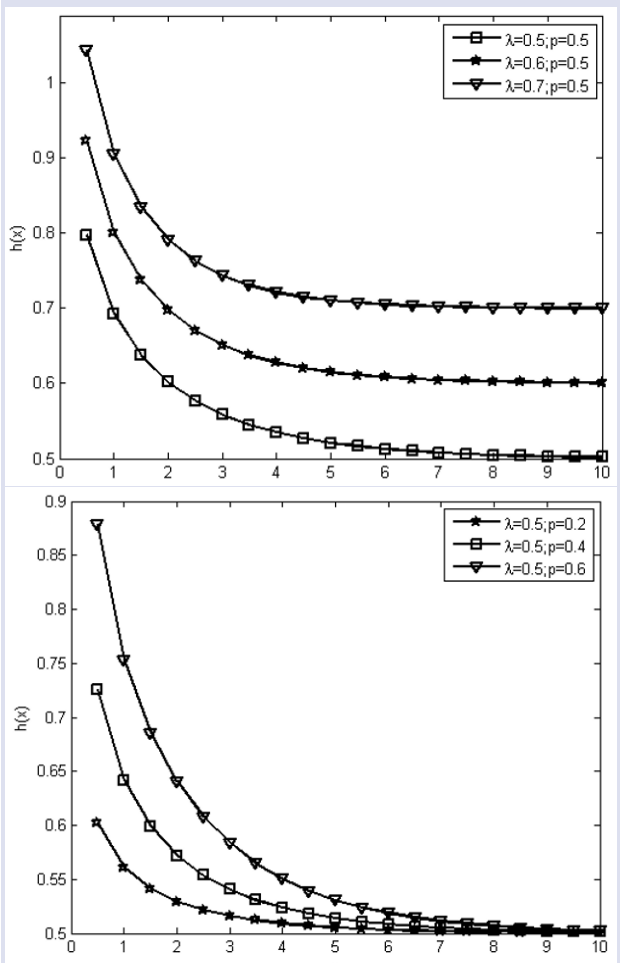

Figure 2. The hf of $\mathrm{TLRT} - \mathrm{Exp}(p,\lambda)$ distribution for some choices of p and λ

Estimation

In this section, estimations of TLRT − Exp parameters have been examined using some classical methods.

Method of Maximum Likelihood

Let $x_1; x_2; \dots; x_n$ be the observations of n independent and identically random variable $X_1; X_2; \dots; X_n$ from the TLRT − Exp distribution. Such that, the log likelihood function has the following formula:

$$\ell_n(p,\lambda) = n\,log(\lambda) - \lambda\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\log\{1 - p\log(1 - exp(\lambda x_i)) - p\}. \quad (10)$$

By differentiating Equation (9) with respect to p and λ respectively, and equating to zero, we have

$$\frac{\partial \ell_n(p,\lambda)}{\partial p} = \frac{log(1 - exp(-\lambda x_i))}{(-1 + p\,log(1 - exp(-\lambda x_i)))} - 1 = 0 \quad (11)$$

and

$$\frac{\partial \ell_n(p,\lambda)}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\frac{p x_i exp(-\lambda x_i)}{(1 - exp(-\lambda x_i))(1 - p\log(1 - exp(-\lambda x_i)))} = 0 \quad (12)$$

We have the maximum likelihood (ML) estimators of the TLRT − Exp parameters p and λ by maximizing the Equations 10-11. These equations cannot be solved analytically for p and λ. Therefore, they can be obtained by numerical methods. The "optim" command in R is used for this purpose.

Method of least square and weighted least square

Let $x_{1:n}; x_{2:n}; \dots; x_{n:n}$ be the order statistics of a random sample from the TLRT − Exp distribution. Hence, we have the least square (LS) estimators of the TLRT − Exp parameters p and λ by minimizing the following equation:

$$S(p,\lambda) = \sum_{j=1}^{n}\left[F(x_{j:n}|p,\lambda) - \frac{j}{n+1}\right]^2$$

with respect to p and λ, where F(.) is the cdf in Equation (5). Equivalently, they can be obtained by solving:

$$\sum_{j=1}^{n}\left[F(x_{j:n}|p,\lambda)\frac{j}{n+1}\right]\eta_1(x_{j:n}|p,\lambda) = 0,$$
$$\sum_{j=1}^{n}\left[F(x_{j:n}|p,\lambda)\frac{j}{n+1}\right]\eta_2(x_{j:n}|p,\lambda) = 0,$$

where,

$$\eta_1(x_{j:n}|p,\lambda) = \frac{\partial F(x_{j:n})}{\partial p} = (-1 + exp(\lambda))\,log(1 - exp(-\lambda))$$

and

$$\eta_2(x_{j:n}|p,\lambda) = \frac{\partial F(x_{j:n})}{\partial \lambda} = -\,exp(-\lambda)\,(-1 + p\,log(1 - exp(-\lambda)) + p)$$

The weighted least square (WLS) estimators, $\hat{p}_{WLS}$ and $\hat{\lambda}_{WLS}$, can be obtained by minimizing

$$W(p,\lambda) = \sum_{j=1}^{n}\frac{(n+1)^2(n+2)}{i(n-j+1)}\left[F(x_{j:n}|p,\lambda) - \frac{j}{n+1}\right]^2$$

These estimators can also be obtained by solving:

$$\sum_{j=1}^{n}\frac{(n+1)^2(n+2)}{j(n-j+1)}F\left[(x_{j:n}|p,\lambda)\frac{j}{n+1}\right]\eta_1(x_{j:n}|p,\lambda) = 0$$

And

$$\sum_{j=1}^{n}\frac{(n+1)^2(n+2)}{j(n-j+1)}F\left[(x_{j:n}|p,\lambda)\frac{j}{n+1}\right]\eta_2(x_{j:n}|p,\lambda) = 0$$

Simulation Study

To obtain information about the performance of estimators, we conducted an appropriate simulation study. The results are given in the Tables 1-6. We calculated the average absolute biases (ABBs), mean square errors (MSEs), and mean relative errors of the estimates (MREs) for all methods. The ABBs, MREs, and MSEs are calculated by

$$\widehat{ABBs}_{\varphi} = \frac{1}{N}\sum_{i=1}^{N}|\hat{\varphi}_i - \varphi|$$

$$\widehat{MSEs}_{\varphi} = \frac{1}{N}\sum_{i=1}^{N}(\hat{\varphi}_i - \varphi)^2$$

And

$$\widehat{MREs}_{\varphi} = \frac{1}{N}\sum_{i=1}^{N}|\hat{\varphi}_i - \varphi|/_{\varphi},$$

where, $\varphi = (p,\lambda)$ and $\hat{\varphi} = (\hat{p},\hat{\lambda})$. The "optim" BFGS routine in the R program were adopted to generate 5000 trials to estimate these indices of the ML, LS and WLS estimates. The sample sizes are considered as n = 50, 100, 250, 500 and two-parameter settings were considered, ($p = 0.5$, $\lambda = 0.5$), ($p = 0.5$, $\lambda = 1.5$), ($p = 0.5$, $\lambda = 3$), ($p = 0.6$, $\lambda = 1.5$), ($p = 0.9$, $\lambda = 1.5$).

Table 1. The averages of estimates, MSEs, ABBs, and MREs of the TLRT-Exp model for $\lambda = 0.5$ and $p = 0.5$

		MLE estimate		EKK estimate		WEKK estimate	
n		$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$
50	Averages	0.5255	0.4511	0.5226	0.5355	0.5191	0.5415
	MSEs	0.0100	0.0356	0.0099	0.0414	0.0096	0.0416
	ABBs	0.0100	0.1887	0.0997	0.2036	0.0979	0.2041
	MREs	0.2000	0.3774	0.1993	0.4072	0.1957	0.4081
100	Averages	0.5144	0.4596	0.5166	0.4763	0.5166	0.4763
	MSEs	0.0081	0.0292	0.0092	0.0400	0.0925	0.0400
	ABBs	0.0899	0.1709	0.0961	0.2000	0.0962	0.2000
	MREs	0.1799	0.3419	0.1922	0.3999	0.1923	0.4000
200	Averages	0.5093	0.4761	0.5075	0.4834	0.5077	0.4832
	MSEs	0.0052	0.0181	0.0074	0.0342	0.0075	0.0344
	ABBs	0.0721	0.1347	0.0865	0.1852	0.0868	0.1854
	MREs	0.1442	0.2694	0.1729	0.3703	0.1736	0.3708
300	Averages	0.5042	0.4862	0.4999	0.4923	0.4998	0.4923
	MSEs	0.0037	0.0130	0.0062	0.0277	0.0062	0.0278
	ABBs	0.0611	0.1140	0.0787	0.1663	0.0788	0.1666
	MREs	0.1221	0.2280	0.1573	0.3326	0.1575	0.3332
500	Averages	0.5018	0.4928	0.4999	0.4977	0.5009	0.4969
	MSEs	0.0023	0.0077	0.0052	0.0180	0.0052	0.0177
	ABBs	0.0480	0.0879	0.0720	0.1341	0.0721	0.1330
	MREs	0.0960	0.1758	0.1441	0.2681	0.1441	0.2660

Table 2. The averages of estimates, MSEs, ABBs, and MREs of the TLRT-Exp model for $\lambda = 1.5$ and $p = 0.5$

		MLE estimate		EKK estimate		WEKK estimate	
n		$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$
50	Averages	1.5455	0.4610	1.5613	0.5484	1.5699	0.5375
	MSEs	0.0860	0.0347	0.0889	0.0419	0.0928	0.0419
	ABBs	0.2933	0.1864	0.2982	0.2049	0.3047	0.2048
	MREs	0.1955	0.3728	0.1988	0.4098	0.2031	0.4095
100	Averages	1.5335	0.4733	1.5231	0.5039	1.5190	0.0402
	MSEs	0.0680	0.0260	0.0800	0.0400	0.0810	0.0405
	ABBs	0.2600	0.1611	0.2830	0.2002	0.2840	0.2005
	MREs	0.1733	0.3223	0.1890	0.4004	0.1895	0.4011
200	Averages	1.5364	0.4747	1.5155	0.4879	1.5144	0.4884
	MSEs	0.0460	0.0165	0.0672	0.0350	0.0673	0.0351
	ABBs	0.2145	0.1283	0.2593	0.1871	0.2595	0.1874
	MREs	0.1430	0.2567	0.1729	0.3742	0.1729	0.3747
300	Averages	1.5127	0.4873	1.5052	0.4880	1.5055	0.4879
	MSEs	0.0303	0.0117	0.0581	0.0271	0.0582	0.0271
	ABBs	0.1741	0.1083	0.2412	0.1647	0.2413	0.1647
	MREs	0.1160	0.2165	0.1608	0.3293	0.1608	0.3294
500	Averages	1.5168	0.4871	1.4949	0.4959	1.4950	0.4959
	MSEs	0.0185	0.0071	0.0471	0.0175	0.0471	0.0175
	ABBs	0.1361	0.0847	0.2171	0.1325	0.2171	0.1326
	MREs	0.0907	0.1695	0.1448	0.2650	0.1448	0.2652

Table 3. The averages of estimates, MSEs, ABBs, and MREs of the TLRT-Exp model for $\lambda = 3$ and $p = 0.5$

		MLE estimate		EKK estimate		WEKK estimate	
n		$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$
50	Averages	3.0904	0.4595	3.0227	0.6044	3.0384	0.5910
	MSEs	0.3136	0.0321	0.2973	0.0412	0.3022	0.0410
	ABBs	0.5600	0.1793	0.5452	0.2030	0.5497	0.2025
	MREs	0.1867	0.3585	0.1817	0.4059	0.1832	0.4050
100	Averages	3.0425	0.4813	3.0036	0.5246	2.9886	0.5315
	MSEs	0.2503	0.0256	0.2932	0.0395	0.2978	0.0397
	ABBs	0.5003	0.1599	0.5414	0.1987	0.5457	0.1993
	MREs	0.1668	0.3197	0.1805	0.3974	0.1819	0.3986
200	Averages	3.0763	0.4725	3.0461	0.4772	3.0457	0.4773
	MSEs	0.1778	0.0169	0.2580	0.0334	0.2592	0.0336
	ABBs	0.4216	0.1301	0.5079	0.1827	0.5091	0.1834
	MREs	0.1405	0.2602	0.1693	0.3654	0.1697	0.3667
300	Averages	3.0289	0.4874	2.9849	0.5009	2.9848	0.5015
	MSEs	0.1273	0.0124	0.2324	0.0280	0.2331	0.0282
	ABBs	0.3568	0.1112	0.4821	0.1673	0.4828	0.1680
	MREs	0.1189	0.2223	0.1607	0.3346	0.1609	0.3359
500	Averages	3.0083	0.4930	3.0150	0.4917	3.0143	0.4917
	MSEs	0.0794	0.0074	0.1888	0.0167	0.1892	0.0168
	ABBs	0.2818	0.0862	0.4345	0.1294	0.4349	0.1296
	MREs	0.0939	0.1723	0.1448	0.2588	0.1450	0.2592

Table 4. The averages of estimates, MSEs, ABBs, and MREs of the TLRT-Exp model for $\lambda = 1.5$ and $p = 0.2$.

		MLE estimate		EKK estimate		WEKK estimate	
n		$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$
50	Averages	1.4239	0.2517	1.3787	0.2944	1.3702	0.2947
	MSEs	0.0459	0.0142	0.0746	0.0205	0.0777	0.0206
	ABBs	0.2143	0.1193	0.2730	0.1433	0.2787	0.1435
	MREs	0.1429	0.5968	0.1820	0.7163	0.1858	0.7175
100	Averages	1.4639	0.2199	1.4445	0.2415	1.4444	0.2415
	MSEs	0.0235	0.0090	0.0387	0.0125	0.0387	0.0124
	ABBs	0.1532	0.0949	0.1968	0.1116	0.1968	0.1116
	MREs	0.1022	0.4748	0.1312	0.5581	0.1312	0.5581
200	Averages	1.4953	0.2004	1.4893	0.2085	1.4893	0.2085
	MSEs	0.0133	0.0057	0.0209	0.0079	0.0209	0.0079
	ABBs	0.1152	0.7553	0.1449	0.0888	0.1449	0.0888
	MREs	0.0768	0.3763	0.0966	0.4442	0.0966	0.4442
300	Averages	1.5004	0.1968	1.4971	0.2005	1.4971	0.2005
	MSEs	0.0092	0.0045	0.0145	0.0056	0.0145	0.0056
	ABBs	0.0959	0.0673	0.1204	0.0747	0.1204	0.0747
	MREs	0.0639	0.3365	0.0803	0.3737	0.0803	0.3737
500	Averages	1.5036	0.1972	1.5025	0.1996	1.5025	0.1996
	MSEs	0.0055	0.0027	0.0087	0.0036	0.0087	0.0036
	ABBs	0.0740	0.0518	0.0931	0.0611	0.0931	0.0601
	MREs	0.0494	0.2592	0.0620	0.3006	0.0620	0.3007

Table 5. The averages of estimates, MSEs, ABBs, and MREs of the TLRT-Exp model for $\lambda = 1.5$ and $p = 0.6$

		MLE estimate		EKK estimate		WEKK estimate	
n		$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$
50	Averages	1.6103	0.5344	1.6363	0.6806	1.6630	0.6622
	MSEs	0.0943	0.0274	0.0769	0.0141	0.0859	0.0140
	ABBs	0.3072	0.1657	0.2772	0.1190	0.2932	0.1184
	MREs	0.2048	0.2762	0.1848	0.1983	0.1954	0.1974
100	Averages	1.5811	0.5519	1.5953	0.6567	1.5884	0.6758
	MSEs	0.0775	0.0251	0.0635	0.0141	0.0668	0.0147
	ABBs	0.2785	0.1585	0.2520	0.1188	0.2584	0.1211
	MREs	0.1856	0.2642	0.1680	0.1979	0.1730	0.2019
200	Averages	1.5618	0.5667	1.5658	0.5908	1.5527	0.6057
	MSEs	0.6111	0.0208	0.0487	0.0129	0.0488	0.0130
	ABBs	0.2472	0.1442	0.2208	0.1136	0.2210	0.1144
	MREs	0.1648	0.2404	0.1472	0.1893	0.1473	0.1907
300	Averages	1.5361	0.5754	1.5366	0.5943	1.5307	0.6105
	MSEs	0.0468	0.0167	0.0432	0.0123	0.0453	0.0126
	ABBs	0.2163	0.1294	0.2078	0.1109	0.2129	0.1122
	MREs	0.1442	0.2156	0.1386	0.1849	0.1419	0.1869
500	Averages	1.5170	0.5889	1.5135	0.5945	1.5042	0.6107
	MSEs	0.0338	0.0116	0.0360	0.0114	0.0387	0.0117
	ABBs	0.1838	0.1079	0.1898	0.1069	0.1968	0.1083
	MREs	0.1226	0.1799	0.1265	0.1782	0.1312	0.1804

Table 6. The averages of estimates, MSEs, ABBs, and MREs of the TLRT-Exp model for $\lambda = 1.5$ and $p = 0.9$

		MLE estimate		EKK estimate		WEKK estimate	
n		$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$	$\hat{\lambda}$	$\hat{p}$
50	Averages	1.9066	0.7185	2.0446	0.7123	2.0508	0.7112
	MSEs	0.1700	0.0329	0.2968	0.0352	0.3039	0.0353
	ABBs	0.4124	0.1815	0.5448	0.1877	0.5512	0.1880
	MREs	0.2749	0.2017	0.3631	0.2086	0.3675	0.2089
100	Averages	1.8762	0.7339	1.9471	0.7122	1.9535	0.7116
	MSEs	0.1419	0.0276	0.2023	0.0353	0.2085	0.0355
	ABBs	0.3767	0.1661	0.4498	0.1878	0.4566	0.1884
	MREs	0.2511	0.1846	0.2998	0.2087	0.3044	0.2093
200	Averages	1.7546	0.7840	1.9006	0.7143	1.9016	0.7141
	MSEs	0.0724	0.0134	0.1605	0.0345	0.1613	0.0345
	ABBs	0.2690	0.1159	0.4006	0.1857	0.4016	0.1859
	MREs	0.1793	0.1288	0.2670	0.2063	0.2677	0.2065
300	Averages	1.7156	0.8046	1.8707	0.7180	1.8728	0.7174
	MSEs	0.0542	0.0094	0.1379	0.0331	0.1391	0.0333
	ABBs	0.2329	0.0971	0.3714	0.1819	0.3730	0.1826
	MREs	0.1553	0.1079	0.2476	0.2022	0.2487	0.2028
500	Averages	1.6551	0.8313	1.7963	0.7566	1.8031	0.7497
	MSEs	0.0380	0.0067	0.0897	0.0206	0.0940	0.0226
	ABBs	0.1951	0.0816	0.2995	0.1434	0.3066	0.1503
	MREs	0.1300	0.0907	0.1997	0.1593	0.2044	0.1670

From Tables 1-6, it was concluded that the averages estimates, ABBs, MREs and MSEs of all estimates decrease when $\boldsymbol{n}$ increases as expected. The ML, LS and WLS estimates are almost identical in terms of ABBs, MSEs, and MREs criteria.

Real Data Applications

In this section, the $\mathrm{TLRT-Exp}$ distribution is applied to the two real data sets. For the comparison issue, we consider transmuted lower record type Fréchet (TLRT-F), Exponential (Exp), Fréchet (Fr), transmuted log-logistic (TLL), transmuted Weibull (TW) and transmuted exponential (TE) distributions. The pdfs of these distributions are given in the ML estimates, log-likelihood value, Akaike's information criteria (AIC), corrected Akaike information criterion (AICc), Kolmogorov-Smirnov test statistic (KS), p-values based on the statistic for all distributions given in Table 7-8. Different discrimination criterion methods based on log-likelihood function evaluated at the ML estimates were also considered. The discrimination criterion methods are respectively: $AIC = -2l(\hat{\theta}, x) + 2k$, $AICc = AIC + (2k(k+1))/((n-k-1))$ and $BIC = -2l(\theta, x) + k\log(n)$ where k is the number of parameters to be fitted $\hat{\theta}$ and is the estimates of θ.

First data set: These data were reported in by Chouklain and Stephens [16]. The data are $n = 72$ exceedances of flood peaks (in m3/s) of the Wheaton River near Carcross in Yukon Territory, Canada. The 72 exceedances, for the years 1958 to 1984, rounded to one decimal place.

Table 7. MLEs and selection criteria statistics for first data set

	TLRT-Exp	TLRT-F	TLL	TW	TE	Fr	Exp
α	0.2248	0.6521	1.1609	0.9017	24.408	0.6521	12.204
p	0.0703	2.8834	16.405	25.109	-0.1345	2.8790	
λ		0.0016	0.5275	1.000			
$-l_n$	251.23	267.02	257.64	251.50	253.36	267.02	253.66
KS	0.0993	0.1530	0.1286	0.1052	0.1278	0.1532	0.9539
p-value	0.4770	0.0686	0.1846	0.4035	0.3037	0.0682	0.0000
AIC	506.46	540.04	521.28	509.00	510.72	538.04	509.32
AICc	506.63	540.39	521.63	509.35	510.89	538.21	509.38
BIC	506.17	539.61	520.85	508.57	510.43	537.75	509.18

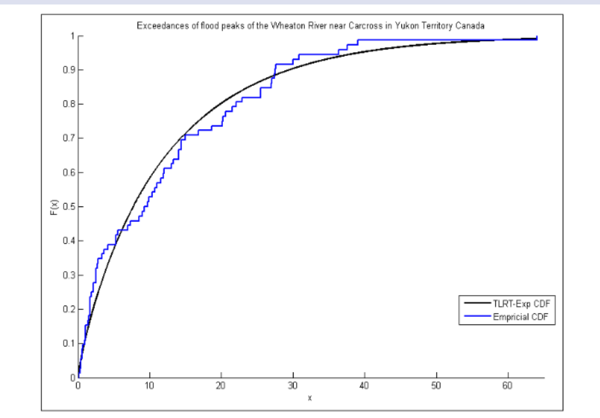

Figure 3. Empirical and TLRT-Exp distributions based on the first data set

Second data set

These data were reported in by Riffi et.al [17]. The data was collected from a group of 46 patients, per years, upon the recurrence of leukemia who received autologous marrow. The data set is listed below which is about leukemia free-survival times (in years) for the 46 autologous transplant patients.

According to the results in Table 7-8, TLRT-Exp has minimum KS and maximum p-value. In addition, when the discrimination criteria are examined, it is seen that it has minimum values in three criteria (AIC, AICc and BIC).

Table 8. MLEs and selection criteria statistics for second data set

Test	TLRT-Exp	TLRT-F	TLL	TW	TE	Fr	Exp
α	0.3884	0.4887	1.0127	0.8403	1.7678	0.7017	1.5172
p	0.4985	1.7313	1.9256	3.1614	0.3206	0.3371	
λ		1.0000	1.0000	1.0000			
$-l_n$	63.72	67.80	65.32	63.99	64.65	69.45	65.18
KS	0.0964	0.1204	0.0972	0.1001	0.1431	0.1399	0.2701
p-value	0.7864	0.5170	0.7781	0.7460	0.5073	0.3288	0.0024
AIC	131.44	141.60	136.64	133.98	133.30	142.90	132.36
AICc	131.72	142.17	137.21	134.55	133.58	143.18	132.45
BIC	130.77	140.59	135.63	132.97	132.63	142.23	132.02

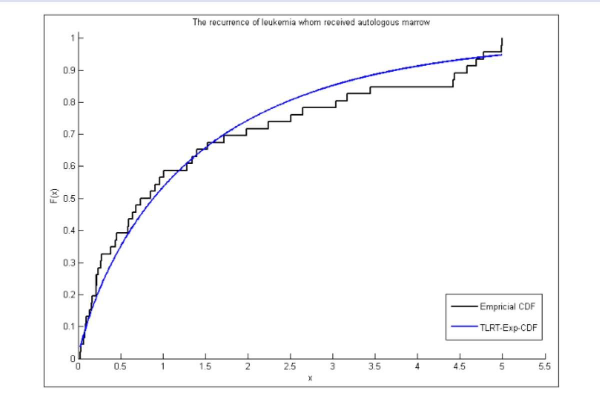

Figure 4. Empirical and TLRT-Exp distributions based on the second data set

From Figure 3-4**,** it can be said that TLRT-Exp can be a good alternative to modeling real data.

Conclusions

In this study, we proposed a new model via the transmuted lower record type family using exponential distribution and examined the properties of the new model such as survival, cumulative distribution, hazard rate functions, and expected value. In the literature review, it can be seen that the family so far have been applied to the Frechet, power function and inverse Rayleigh distributions. Statistical inferences about the distribution parameters are discussed with three estimation methods, namely maximum likelihood, least squares, and weighted least squares. A detailed Monte Carlo simulation study is conducted to examine the performance of given estimation methods. In addition, the new model is examined in two real data sets with regard to discrimination criteria. It was observed that the obtained model is more flexible than the known distributions such as transmuted lower record type Fréchet, exponential, Fréchet, transmuted log-logistic, transmuted Weibull, and transmuted exponential distributions. Considering that the variety of data is increasing, the family used in this study can be applied to other existing and primarily newly obtained continuous distributions for future studies.

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Conflicts of interest

The authors declare that they have no conflicts of interest.

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