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Effect of Ring Radius and Electric Field on the Relative Refractive Index of a GaAs Quantum Ring

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ABSTRACT

The influence of inner ring radius and in-plane electric field on the relative refractive index of a GaAs-AlGaAs single circular quantum ring is theoretically studied. The energy levels and corresponding wave functions are obtained by solving the Schrödinger equation within effective mass and envelope wave function approximations. The changes in the intraband transition energies are presented in terms of varying ring radius and external electric fields. Relative refractive index changes are calculated through the compact-density matrix approach. The results show that both ring radius and electric field significantly affect the location and also the peak intensities of relative refractive index changes on the incident photon energy.

Keywords: Quantum ring, Refractive index, Electric field.

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Introduction

Constraining charge carriers in low-dimensional structures and then modifying the electronic and optical properties have reached great attention due to improvements in technological devices. Semiconductor quantum rings (QR) are one of the topological geometries that confine charge carriers within different shapes such as circular, elliptic or oval. The shape of QR together with the geometrical parameters and the applied external fields lead the changes in the potential profile of QR and so the optical properties. The effect of eccentricity on the electronic spectrum of QRs is handled in some studies [1-8]. In addition to deviations from circular curves of QRs, the exposure to electric field [1-5] and magnetic field [6-8] have been impressively used to tailor the electronic properties. Chakraborty and co-workers present the possibility of tuning the electronic spectrum by adjusting the ring radius, ring shape and as well as exposing the QR intense laser fields and external magnetic fields [9]. The influence of ring geometry on the electronic properties of QR is investigated by Vinasco and co-authors [10]. Xie has studied the influence of the ring radius of QR on the optical absorptions within subbands for two-electron [11]. In another study performed by Barticevic et al., the role of ring radius and the magnetic field is investigated for a QR [12]. Radu and co-workers have examined the responses of electronic spectra and absorption properties to the intense laser field for a single QR [13]. The simultaneous effect of laser radiation and electric field to the electronic and optical absorption properties of the $GaAs/AlGa_{0.7}As_{0.3}$ QR has been studied for single QR again by Radu and co-workers [14] and for double QR by Baghramyan et al. [15]. The influence of lateral electric field on the variations of intraband optical absorption changes for a concentric double QR is handled by Baghramyan et al. [16] and to the possibility of controlling the optoelectronic properties with the applied electric field and as well as light polarization are reported [16]. The effect of electric field on the optical responses in the presence of donor impurity is presented by Restrepo and co-workers [17] and also by Duque and co-workers [18]. The impact of donor impurity together with the hydrostatic pressure on the nonlinear optical properties of GaAs QR is analyzed in the other study of Retrepo and his group [19].

Apart from circular QRs, the optical properties are examined in a triangular QR designed by Nasri and Bettahar [20]. In that study, they conclude that both transition energies between the first two excited states and ground state and optical matrix are significantly influenced by the changes in the side length of the inner triangle [20]. Due to technological applications of low-dimensional quantum systems, theoretical studies are densely performed with low-dimensional systems to support or encourage experimental studies. Here only some of these studies which are carried out on the relative refractive index changes in quantum dots will be referred. The changes in the relative refractive index are investigated for spherical quantum dots in [21-22] and cylindrical quantum dots in [23-25]. In the study with the spherical quantum dot performed by Karabulut and Baskoutas, the influences of impurities, electric field, size, and optical intensity on the change in refractive index as well as absorption coefficient are analyzed [25]. The influence of size variation and donor position on linear, nonlinear and total refractive index changes of a spherical quantum dot is handled by Al [26]. His results indicate the significant changes on these coefficients for donor intersubband transitions [26].

This study was inspired by the studies [13, 14] that investigate the effect of electric field and intense laser field on the absorption coefficients for a single GaAs ring. The novelty of the present study lies in the investigations on the first-order relative refractive index changes with varying electric field and inner ring radius. The theoretical description of the problem is given in Section 2. Obtained results and related discussion are presented in Section 3 and the main conclusion of the study is given in Section 4..

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Materials and Methods

We have investigated the influence of varying ring radius and electric field on the intraband transitions, dipole moment matrix elements as well as relative refractive index changes for a single circular GaAs-GaAlAs QR. For the determination of electronic states and related wavefunctions of structure, the Schrödinger equation is solved within the framework of effective mass and envelope wave function approximation. Time-independent Schrödinger equation for the confined electron in a QR in the presence of an *x*-oriented external electric field has the following form:

$$E\,\phi(x,y) = \left[-\frac{\hbar^2}{2m^*}\nabla^2 + V(x,y) - eFx\right]\phi(x,y), \quad (1)$$

where $\hbar$ is the Planck's constant, m^* is the effective electron mass, e is the charge of electron and F is the applied static electric field along the negative x direction. Here E stands for the Eigen energies of energy levels and $\phi(x,y)$ denotes the corresponding wave functions of these levels. $V(x,y)$ is the potential term and for a circular single ring and defined as below:

$$V(x,y) = \begin{cases} 0\,, & if\ R_1 \le \sqrt{x^2+y^2} \le R_2\,, \\ V_0\,, & if\ \sqrt{x^2+y^2} < R_1,\ or\ \sqrt{x^2+y^2} > R_2. \end{cases} \quad (2)$$

where R_1 and R_2 are the inner and outer radii of circular ring and V_0 is the maximum value of potential. The solution of Eq. (1) with the potential given in Eq. (2) will lead the energies of levels, E and the corresponding wave functions $\phi(x,y)$ and thereby the electronic spectrum of QR will be obtained. After the determination of the electronic structure of QR, the first order coefficient of relative refractive index changes can be calculated with the help of the density-matrix approach and the analytical expression for it is given as follows [27-29],

$$\frac{\Delta n^{(1)}(\omega)}{n_r} = \frac{\sigma_v |M_{ij}|^2}{2n_r^2\varepsilon_0}\left[\frac{\Delta E - \hbar\omega}{(\Delta E-\hbar\omega)^2 + (\hbar\Gamma)^2}\right] \quad (3)$$

Here $n_r = \sqrt{\varepsilon_r}$ is the relative refractive index of the system, ε_0 is the permittivity of vacuum, σ_v is the carrier density of the system and Γ is the relaxation rate. ω is the angular frequency of incident photon and ΔE is the energy difference between excited states and lowest-lying state. $M_{ij} = \langle\psi_i(z)|ez|\psi_j(z)\rangle$ stands for the dipole moment matrix element for the transitions between excited states and ground state.

Results and Dicussion

This numerical study represents the first order relative refractive index changes of a single circular GaAs QR for varying inner ring radius and electric field application. In the calculations the used parameters are as follows: $m^* = 0.067m_0$ (where m_0 is the free electron mass), $\sigma_v = 3\times10^{22}\ \mathrm{m}^{-3}$, $n_r = 3.2$, $\varepsilon_r = 12.58$, $\Gamma = 0.14\ \mathrm{ps}^{-1}$ and $V_0 = 228\ \mathrm{meV}$. First, we present the probability densities and energy eigenvalues and then relative refractive index changes for different values of inner ring radius and electric field.

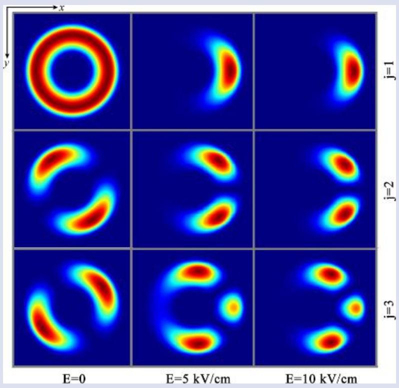

Figure 1. 2D probability distributions of the first three states of the quantum ring for three different electric field values. The results are displayed for $R_1 = 10$ nm and $R_2 = 25$ nm

Figure 1 displays the electron probability densities for the first three states of the circular quantum ring for the inner radius $R_1 = 10$ nm and outer radius $R_2 = 25$ nm. The densities of *j*=1, 2, and 3 electron states are displayed through the rows of Figure 1, respectively. Influences of three in-plane electric field values on the probability densities are shown through the columns of Figure 1. *x*-oriented electric field disturbs the densities of states by changing the probabilities of wave functions. Electric field application through the $-x$ direction destroys the axial symmetry of the confining potential. Electrons begin to accumulate along the $+x$ direction and therefore the right side of the well get deepen. Thus, the wave functions are aimed to localize to the right side of the ring as seen in Figure 1.

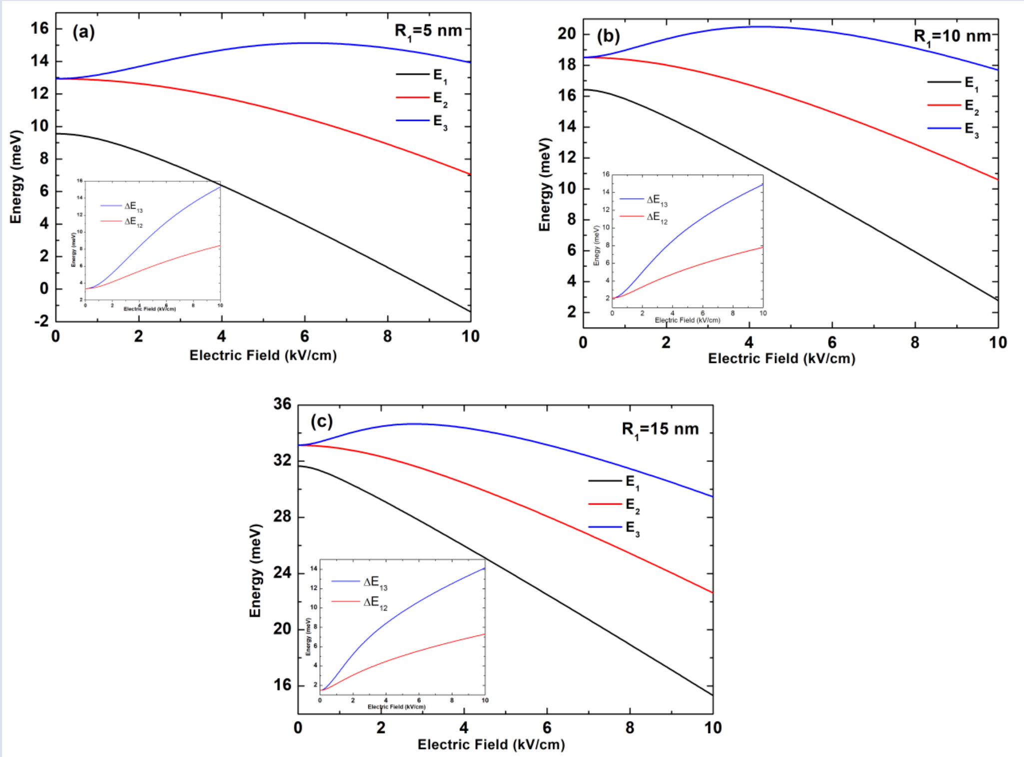

Figure 2. The variation of eigenvalues of lowest-lying three states with respect to the electric field for R1=5 nm (a), R1=10 nm (b), R1=15 nm (c) while outer ring radius is R2=25 nm. The insets display the energy differences between the first excited state and ground state (1 → 2; red lines) and between the second excited state and ground state (1 → 3; blue lines).

In figure 2, we display the energies of the first three states as a function of the electric field for three different inner ring radius. The first excited state has double degeneracy in the absence of an external field. The presence of electric field removes that degeneracy by breaking the axial symmetry. The responses to the increase in the electric field have similar behaviors in terms of energy variations and the energy differences between first/second excited states and ground state while inner ring radius is varying. The energies of states go to higher values as a result of widening inner ring in other words narrowing well widths. While inner ring radius is increasing, although the state energies remarkably soars up, the energy differences between excited states and the ground state are slightly scales down. The augmentation in the electric fields lowers the energies of the first three lowest-lying states as a consequence of more confinement.

Breaking the symmetry of QR by applying an in-plane electric field requires taking into consideration of selection rules between states. The allowed transition for the x-polarization occurs between the second excited state and ground state (1→3). For the y-polarization, it is between the first excited state and ground state (1→2) [13, 14].

Figure 3 represents the variations in the relative refractive index coefficients with respect to photon energies for three different inner ring radius with the augmentation of the electric field. The changes in the light polarization direction are indicated in Figure 3 by considering the allowed transitions between excited states and ground state. As mentioned above, Figure 3(a-c) represents the changes in relative refractive index coefficients for intraband transition energies between (1→3) and Figure 3(d-f) represent these changes between (1→2) states. The peak position of these coefficients moves to lower photon energies not only for the *x*-polarization but also for the *y*-polarization for increasing inner ring radius at a fixed electric field. This red-shift can be attributed to the reduction in the intraband transition energies while the inner ring is expanding, as mentioned in the explanations of Figure 2. Due to the degenaracy in the energy of first and second excited states in the absence of an electric field, both *x*-polarized and *y*-polarized incident light display the same results. The application of *x*-orientated electric field moves the relative refractive index peak positions to higher energies for both polarization, however the movement is more pronounced for *x*-polarized light concerning the differences of state energies, ΔE_{13} and ΔE_{12}

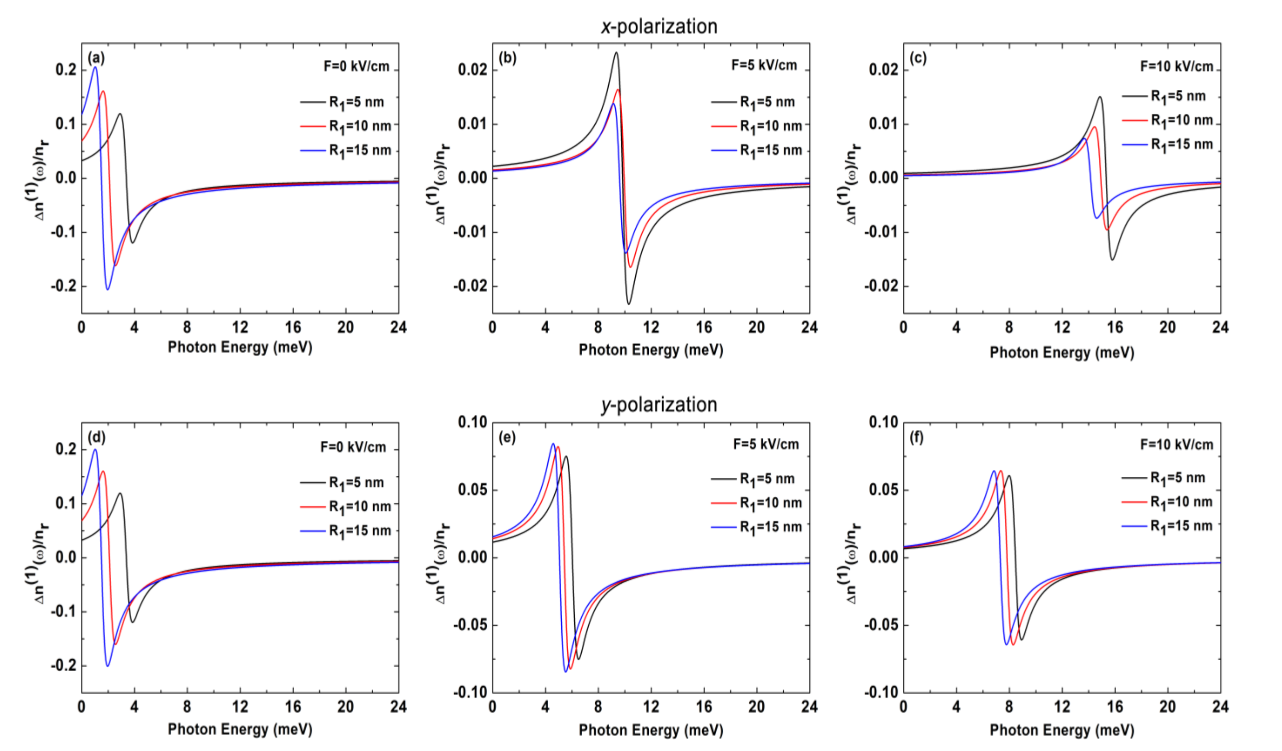

Figure 3. The relative refractive index changes as a function of photon energy with varying inner ring radius for three different electric field values: (a) $F = 0$ kV/cm, (b) $F = 5$ kV/cm, (c) $F = 15$ kV/cm.
The figures (a-c) and (d-f) display the results for x-polarization and y-polarization of the incident light, respectively

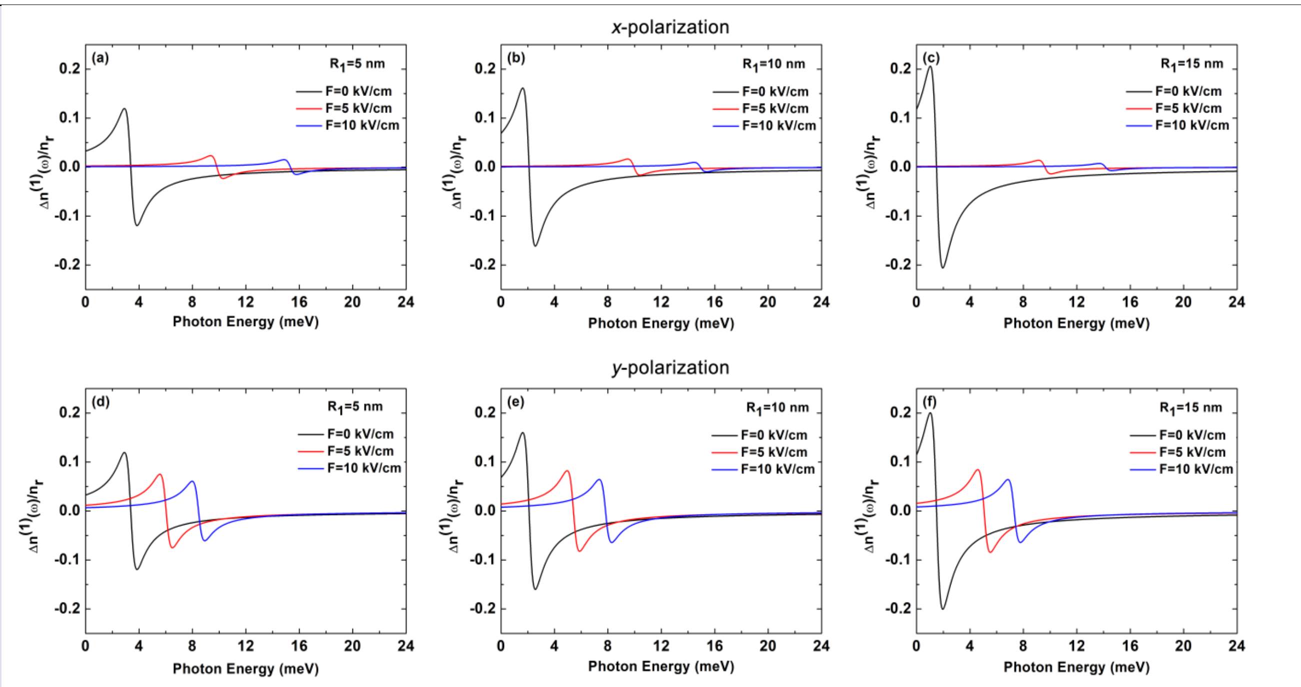

Figure 4. The variations in the relative refractive index coefficients as a function of incident photon energy with three different electric field values. The inner ring radius in (a, d) is $R_1 = 5$ nm, in (b, e) is $R_1 = 10$ nm and in (c, f) $R_1 = 15$ nm. The rows represent the results of x-polarization and y-polarization of the incident light, respectively.

The changes in the relative refractive index coefficients concerning photon energy while the electric field is changing are shown in Figure 4 for three different inner ring radius. The increase in the inner ring radius shifts the relative refractive index peaks to lower energies by increasing the magnitudes of peaks for y-polarized light. For x-polarized light, the increase in the inner ring radius enhances the peak magnitudes when the electric field is absent but slightly diminishes with the augmentation of the electric field. The remarkable reduction in the intensities of relative refractive index changes can be clearly seen for the increasing values of electric field for a fixed ring radius not only for x-polarized light but also y-polarized light. The declines in the peak intensities are consequences of the diminishing overlap between wave functions and as can be noticed here, the subjection of increasing x-oriented electric field has a so strong impact on these peaks for the same light polarization direction.

Conclusion

The energies and corresponding density probabilities of a single circular quantum ring with varying inner ring radius and in-plane static electric field are investigated by solving the Schrödinger equation within the effective mass and envelope band approximation. The obtained electronic spectrum is used to get relative refractive index change coefficients for different inner ring radius as well as for varying electric fields. The increases in the inner ring radius cause the relative refractive index peaks to appear at lower incident photon energies. On the other hand the augmentation in the electric field acts on these peaks by shifting them to blue for both x-polarized and y-polarized light. Both inner ring radius and electric field as well as light polarization lead to significant changes not also in the position of relative refractive index coefficients but also in the intensities of these peaks.

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Conflicts of interest

The author states that did not have a conflict of interest

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