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On the Lyapunov Time Estimations For Comet 1/P Halley

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ABSTRACT

In three consecutive articles published in recent years, quite different estimates were made for the Lyapunov time of comet 1/P Halley, whose orbit is known to have high precision. In this work, we examined the Lyapunov time of the comet 1/P Halley using the MEGNO method and compared our results with previous studies. To investigate the effects of numerical overflows on the results that may have occurred during the calculations, we conducted tests with and without the renormalization procedure. We used various renormalization intervals to see their possible effects on the results and to avoid improper ones. We reached the maximum Lyapunov exponents at renormalization times for 2250 yr, 2265 yr, and 3000 yr. In both cases where renormalization is used and not used, the Lyapunov time is calculated as 119 yr and 190 yr, respectively. Besides, we performed orbital integrations for ∓ 10 kyr for comet 1/P Halley with the clone orbits produced by the MCCM method and compared the standard errors of the means of the orbital parameters with the Lyapunov times. We conclude that calculated different Lyapunov times correspond to different levels of the standard errors of the means.

Keywords: N-body integration, Lyapunov time, Comets.

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Introduction

For the first time in history, Edmund Halley [1] found that some historical comet observations belonged to the same object, as it is known today, comet 1/P Halley. Since that time, comet 1/P Halley is one of the most well-known objects of both popular and dynamical astronomy. It has been observed repeatedly by different civilizations since 240 BC in its every visit. Edmund Halley calculated its orbit and predicted for the next apparition as late 1758 or the beginning of 1759 [2, 3]. It turned back with a perihelion passage in 1759 March 13.1. It was the first solar system object whose periodicity was discovered other than planets and their natural satellites. During its last apparition in 1986, it was observed by seven spacecraft [4]. It is the first comet observed by spacecraft. Today, short-period comets with a period between 20 yr and 200 yr are also called Halley type comets (HTC). There are 14 numbered, 80 unnumbered HTC listed (JPL's SBDB [5-7]) as of 27/12/2021. Unnumbered comets were observed only in 1 apparition. Among the numbered HTCs, 1/P Halley is the second object with the smallest perihelion distance (0.586 au).

Although comet 1/P Halley perhaps is the best known, long studied, and most observed comet, its orbit continues to attract attention in terms of dynamical astronomy. One of the most important reasons for this is that it is in a chaotic orbit, as it has been known since the work of [8]. The future trajectory of 1/P Halley cannot be determined with great accuracy, even if non-gravitational and relativistic effects are well known or can be calculated [9]. The measure of the dynamical predictability of a chaotic trajectory is given by the Lyapunov time calculated by taking the inverse of the maximum Lyapunov exponent. No matter how well the trajectory of the object is known and how advanced the computing tools at hand are, long term evolution of the orbit cannot be predicted for longer than Lyapunov time. Therefore, statistical methods should be preferred for trajectory calculations that go beyond Lyapunov time.

The oldest estimate we can reach for the Lyapunov time of comet 1/P Halley is in [10] and is given as approximately 34 yr, indicating the lower limit. Three consecutive publications [11-13] in recent years show that the dynamic study of 1/P Halley's orbit still deserves attention. In these three articles, different Lyapunov time estimations were made varying between 70 yr and 562 yr. In [11], unlike previous studies, indirect numerical integration was used for the first time to calculate the Lyapunov exponent. A total of 30 simulations were run for 3000 yr. All planets except Mercury, all dwarf planets except Sedna and 5 dwarf planet candidates were included in the simulation. Integrated ghost particles were produced by applying $\mp 10^{-6}$ perturbations to the position vectors. A total of 30 simulations were run. In [12], ghost particles were produced similar to [11], but initial perturbations were added to the velocity vectors in addition to the position vectors and 13 initial conditions were used together with the nominal orbit. However, the integration time was kept longer (10 kyr). As a result of the 3-body tests, it was stated that the influence of Mercury, Uranus and Neptune is negligible. In [13], first-order variational equations were used instead of the ghost particle approach in previous studies. In the simulations, test objects were integrated with the major planets and the Moon. Integration time was quite long compared to previous studies (2 x10^5yr).

Various methods, initial conditions, integration schemes and indicators were used in each of these studies. Naturally, their approaches are different. Besides, there are fine-tuning points during the application of these methods-e.g., renormalization that can lead to different results. However, calculated various Lyapunov exponents for the same object should give similar time scales regardless of method. So that would be the limit of dynamical computability of the orbit. The fundamental question is: how long can the dynamics of the movement be followed, and which one of the previous estimations for the Lyapunov time are the most accurate?

In this study, we worked on the calculation of Lyapunov time for comet 1/P Halley using the MEGNO (the Mean Exponential Growth factor of Nearby Orbits) method and comparing the calculated Lyapunov times with orbital integrations. In Section 2, the numerical methods and initial conditions are given. In Section 3, Lyapunov time calculations and orbital integration results of clone orbits are discussed and presented. Section 4 summarizes the comments and results.

Materials and Methods

In this work, we used the publicly available REBOUND integrator package [14] with first-order variational equations. We used MEGNO function included in the REBOUND integrator package to calculate Lyapunov time. Besides, we tested renormalization in the calculation of MEGNO (the detail is given in subsection 2.1). We chose high accuracy integrator IAS 15 based on the 15th-order Gauß-Radau quadrature as the integration method [15]. The integrals included dwarf planets Pluto and Ceres in addition to eight major planets. We ignored the masses for the test objects outside these bodies. Also, all kind of non-gravitational effects and relativistic corrections that can be important in comet dynamics are ignored as in previous papers [11-13].

The initial conditions for all small and big Solar system bodies have been obtained using the Jet Propulsion Laboratory's Solar System Dynamics Group Small-Body Database (JPL's SSDG SBDB) and JPL's Horizons ephemeris system [16-18] for the epoch JD 2449400.5 (1994-Feb-17.0) TDB (Barycentric Dynamical Time).

Clone orbits were used to see the dynamically reliable computability time of the orbit. The orbital elements of the clone orbits were produced at the same precision level as the uncertainties of the orbit obtained from observations. For this, MCCM (Monte Carlo using Covariance Matrix) method [19-21] using the covariance matrix of orbital elements was used.

MEGNO Technique

MEGNO technique was first proposed in [22] and [23] publications. Since then it has been applied for various dynamic systems such as irregular satellites of Jupiter [24], double and binary asteroids [25, 26], planetary systems [27-29], and galaxy dynamics [30]. It has been discussed and compared with previously well-known LCE calculations [24, 31]. Compared to other methods, it has been seen that it gives good results with relatively short integration times [27, 31].

The MEGNO technique has been repeatedly presented with similar formulations in various sources. Here, we summarize the method using the notation in [25]. When a dynamic system in the form below is considered;

$$\frac{dx}{dt} = f\big(x(t)\big), with x \in R^{6n} \tag{1}$$

where the solution of the system is $\varphi(t)$. For a defined tangent vector $\delta_\varphi(t)$ along with $\varphi(t)$, the evolution of this vector is given by;

$$\dot{\delta_\varphi} = \frac{df}{x}\big(\varphi(t)\big)\delta_\varphi(t). \tag{2}$$

Here, the MEGNO indicator is defined as;

$$Y_\varphi(t) = \frac{2}{t}\int_{t_0}^{t} \frac{\|\dot{\delta_s}\|}{\|\delta_s\|} s ds, \tag{3}$$

and the time-averaged mean value of the MEGNO is;

$$\overline{Y}_\varphi(t) = \frac{1}{t}\int_{t_0}^{t} Y_\varphi(s) ds. \tag{4}$$

If the orbit is chaotic, the two quantities Y_φ and $\overline{Y}_\varphi$ increase linearly in time and goes to infinity. If the orbit is quasi-periodic, $\overline{Y}_\varphi$ converges to 2, and if the orbit is stable and periodic, it converges to 0. In addition, a linear least-squares fit $\overline{Y}_\varphi$ gives half of the Lyapunov exponent (γ) where the Lyapunov time (T_γ) is $T_\gamma = 1/\gamma$.

In many cases, since δ diverges exponentially during integration, the norm of the variational vector grows too much in a short time, causing a numerical overflow. To avoid this situation, it is recommended that the variational vector is renormalized at certain time intervals according to Eq.5 as in [27]. However, there are no specific criteria for determining the length of renormalization intervals. It was shown in [24] that in some cases the choice of renormalization time does not affect the calculation of maximum Lyapunov exponent. However, this may not be the case in all situations as shown in [32]. It should be decided by performing tests at different renormalization ranges.

$$\gamma = \lim_{k\to+\infty} \frac{1}{k\tau}\sum_{i=1}^{k} ln\frac{\|\delta(k\tau)\|}{\|\delta_0(k\tau)\|} \tag{5}$$

Results and Discussion

Lyapunov Time Calculations

In the calculation of the MEGNO indicator, it is suggested to take 10^3 to 10^4 times the period of the largest or outmost planet in the system in [27, 32] as the integration time, which gives the characteristics of the system. Considering Jupiter, the largest planet, this time can be taken in the range of 1.2×10^4 yr to 1.2×10^5 yr for the solar system. In our case, the frequency of interaction of the planets with the targeted body mainly depends on the comet's period, not the major planet Jupiter. Considering the orbital period of comet 1/P Halley, integration time should go up to 7.5×10^5 yr as far as possible. It is stated in various papers that the required minimum integration time for the MEGNO technique to estimate the maximum Lyapunov exponent is $10 - 10^2$ times shorter than any classical method [27, 31].

In [13] the integration time was taken as 2×10^5 yr, and first-order variational equations were preferred to calculate Lyapunov exponents. Lyapunov times corresponding to the calculated positive Lyapunov exponents range from 385 yr to 702 yr. However, it was considered an estimation for the Lyapunov time taking the average value of the maximum and minimum Lyapunov exponents and obtained averaged Lyapunov time as 562 yr. In [12], Lyapunov time was estimated with an integration time of 10 kyr using ghost particles produced by applying $\mp 10^{-6}$ and $\mp 4.4\text{x}10^{-8}$ perturbations to the position and velocity vectors, respectively. Lyapunov time obtained by these approaches is 300 yr. A similar approach was used in [11], but for a quite short (3000 yr) integration time. In this case, they obtained the Lyapunov time as the interval of 70-100 years.

Renormalization is highly recommended not only for the MEGNO method but also Lyapunov exponent calculations for various techniques. Interestingly non of the three papers [11-13] which calculated Lyapunov time for comet 1/P Halley mentioned whether the renormalization is used or not. In cases where short integration times are used, it may be reasonable not to use renormalization. However, short integration time also has other drawbacks as mentioned earlier, therefore it is not recommended in Lyapunov calculations for classical approaches.

Using different renormalization times during the calculation of the maximum Lyapunov exponent from the MEGNO indicator has been examined in various publications [24, 32], and it has been shown that the renormalization period does not change the results in some cases. On the other hand, depending on the initial conditions, it is possible to get incorrect results with improper applications [27].

Here, Lyapunov time is calculated in two different ways using the linear characteristic of the MEGNO indicator, with and without renormalization. Randomly generated 100 variational particle sets were used for each test. In the absence of renormalization, the integrations were allowed to continue until they produced numerical overflows. Various trials were conducted here with different renormalization intervals ranging from 75 yr to 3000 yr.

Figure 1 shows the $log(\gamma)$-$log(yr)$ graph for two methods with and without renormalization. In both graphs, maximum Lyapunov exponents are at the same level. However, without renormalization, numerical overflows are produced in shorter periods for variational particles as predicted in [24, 27, 32]. Therefore, it is necessary to keep the integration times at levels of 10^4 yr even shorter.

We calculated the Lyapunov time as 190 yr by using the linearly increasing characteristic of the MEGNO parameter in the integrations that continued until the numerical overflow. When we take the integration time as 1×10^4 yr, 2×10^4 yr, and 3×10^4 yr to avoid numerical overflows, we obtained the Lyapunov times as 96 yr, 121 yr and 164 yr, respectively.

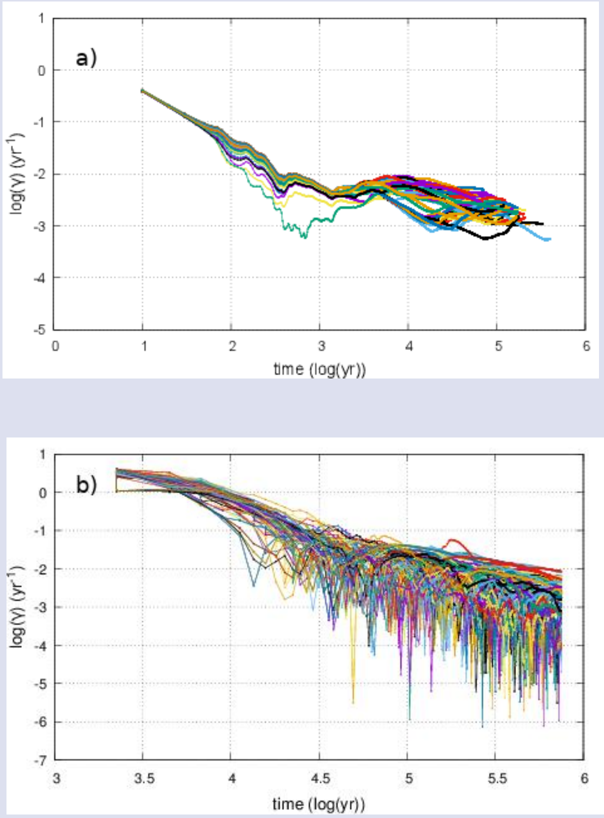

Figure 1. Lyapunov exponents obtained from MEGNO calculations with (panel b) and without (panel a) renormalization

Since we can keep the integration time longer when renormalization is applied, we tried to obtain the maximum convergent Lyapunov exponent. We reached the minimum and quite similar (119 yr, 124 yr, and 123 yr respectively) Lyapunov times with renormalization intervals of 2250 yr, 2265 yr, and 3000 yr.

N-Body Simulations for Comet 1/P Halley with Clon Orbits

It is well-known that long-term simulations made only for the nominal orbits are not very reliable in chaotic regions. Trajectories with very close initial conditions can follow very different paths in a simulation of longer duration than Lyapunov time. Therefore, using clone orbits in long-term dynamic analysis of orbits is a more reliable approach for long simulation times.

The easiest and most general approach to generate clone orbits is to distribute the orbital elements or position and velocity components with small dispersion. Even though the clone orbits obtained in this way are very close to the nominal one, they will not be compatible with the uncertainties of the nominal orbital elements. Such clones have similar but independent orbits near the nominal one. Thus, they may not reflect the change of orbital elements well over time [21].

For these reasons, in this study, clone orbits produced by the MCCM method were used. Thus, all clones produced have the same sensitivity level as the orbital parameters obtained from the observations. In other words, they are not only in the close vicinity of the nominal orbit but also dispersed in the same sensitivity range. Thus, the initial parameters will be as accurate as of the nominal orbital elements. This is a better approach than classical methods to follow the change of clone orbits in time and compare them with the nominal orbit. Besides, the divergence time of the parameters due to chaotic motion can give us a norm for the Lyapunov time.

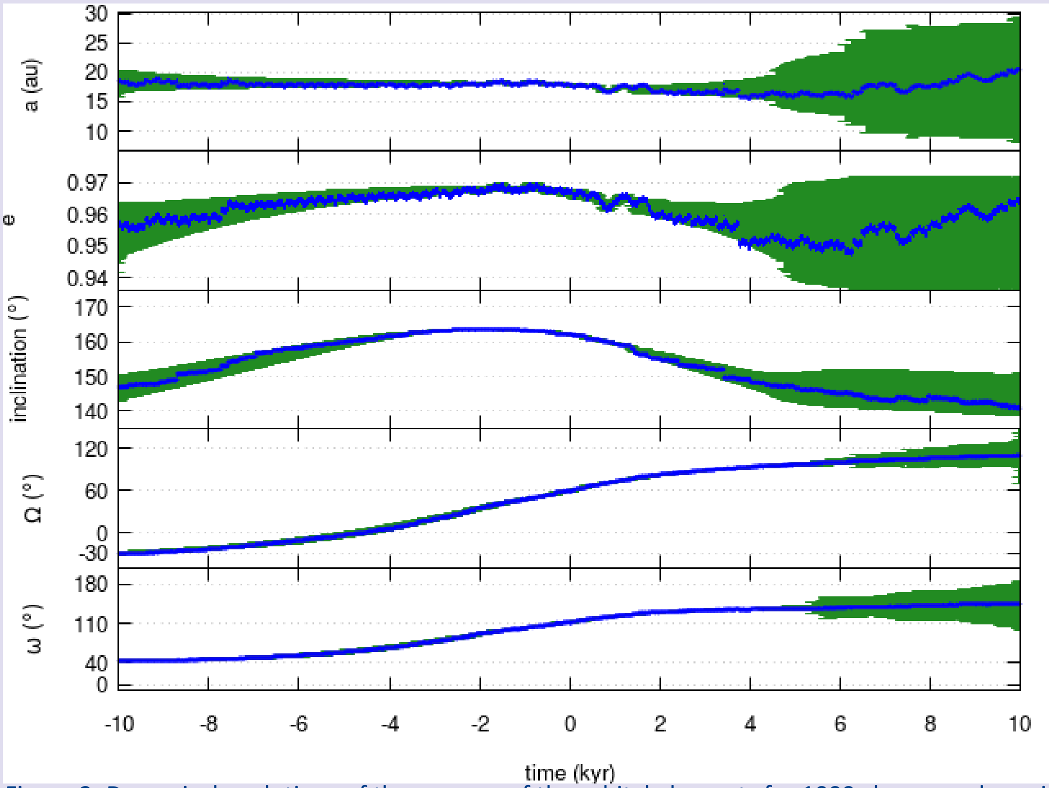

Figure 2. Dynamical evolutions of the average of the orbital elements for 1000 clones, and nominal orbit for ∓10 kyr. The green curves with error bars show the average of clones, and the blue curve shows the nominal orbit. The nominal orbit of the comet for the epoch JD 2449400.5 (1994-Feb-17.0) is used for initial conditions.

In Figure 2, comet 1/P Halley's nominal orbital elements are given comparatively with the averages of the clone orbits. The clone orbits in the approximately ∓1500 yr range are in great coherence with the nominal orbit. In this range, all clone orbits provide almost the same orbit shape, size and orientation. When we look at the error bars of the average of clone orbits, a larger error range is seen in future simulations than in the past. This situation can be interpreted as an indication that the comet's orbit has evolved into a more chaotic orbit in time.

Figure 3 shows the change of basic orbital elements of all clones to time in the ∓2000 yr interval. Similar to Figure 2, all orbital elements support the same orbital shape and orientation with very small changes over the range of about -1000 yr to +1300 yr. However, similar to what is mentioned in [11], the time it takes for the differences between orbital elements to start to be greater than their initial sensitivity ranges is 108 yr. This definition also gives us an approximation for the Lyapunov time.

Our primary motivation here is to see footprints of the forward Lyapunov time in the orbital dynamics of clones. Figure 4 portrays the standard deviations of the mean of the clone orbital elements for the time interval 0-2000 yr. The y axis (standard deviations) is in log scaled so that any sudden increase in dispersions can be noticed easily. By definition, the required time where the nearby orbits begin to diverge exponentially is Lyapunov time. That time limit is 108 yr in Figure 4.

After the exponential increase at 108 yr, there is no sudden growth till 1300 yr even though the standard deviation for 1000 clones remains in the same band.

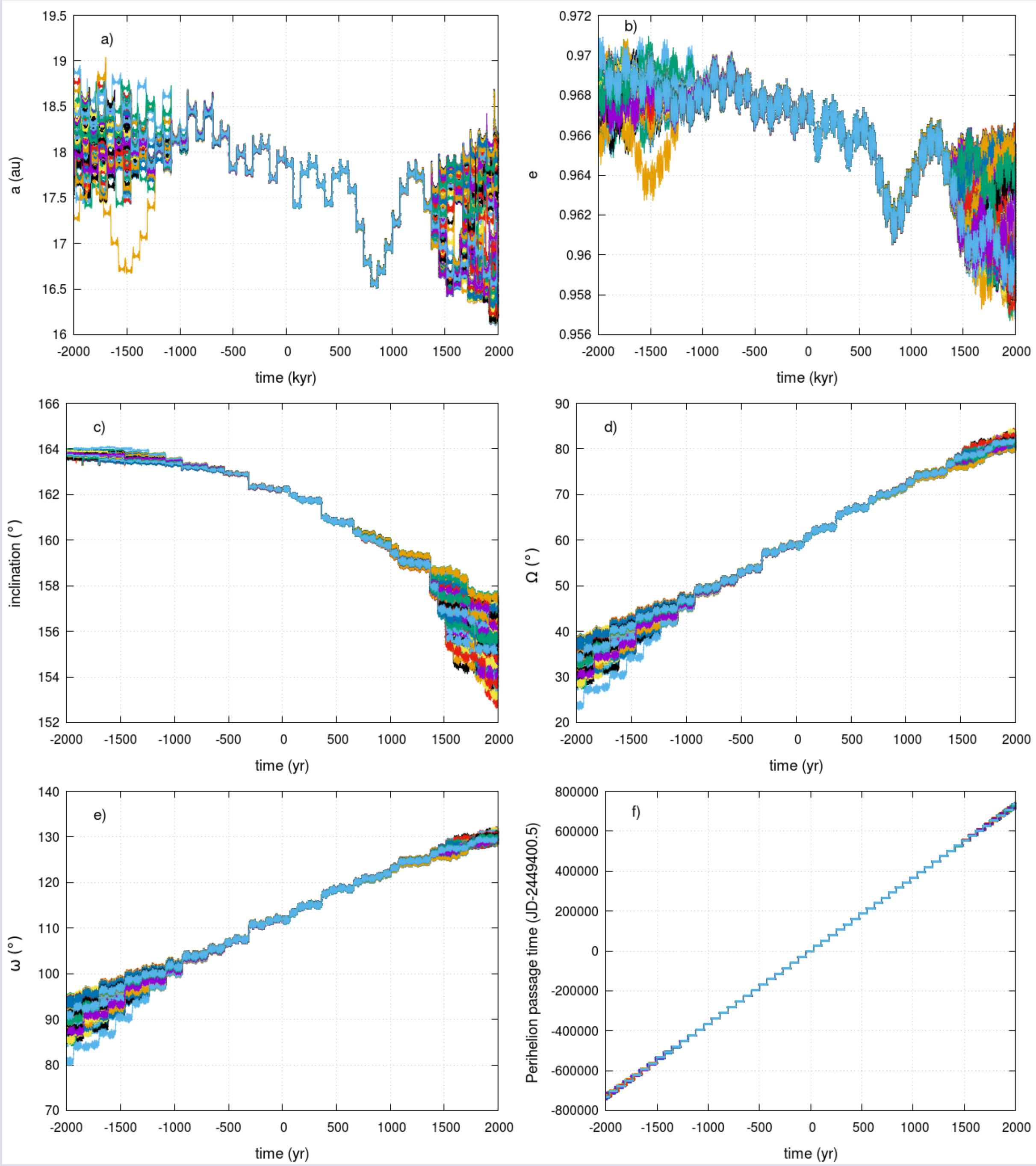

Figure 3. Dynamical evolutions of the orbital parameters (a, e, i, Ω, ω , and Tp (time of perihelion passage) of 1000 clones of comet 1/P Halley for ∓2000 yr.

For almost all orbital elements, the separations that start after about 1300 yr become more evident around 1400 yr, and standard deviations start to increase exponentially one more time. Nevertheless, all clones retain more or less the same trajectory shape between 108 yr and 1400 yr. Although various Lyapunov times can be obtained using different methods for this range, no distinctly different results are seen in terms of dispersion range.

For the 108 yr to 1400 yr range, it seems that it will be easier and much more clear to do a detailed examination on Tp (time of perihelion passage). In Figure 4f, the smallest dispersions for each time interval belongs to the value calculated at the perihelion of the comet. The standard deviations calculated at the aphelion are much higher. However, it should be noted here that the dispersions formed during the calculation of Tp when the body is at perihelion are more determinant and distinct. For this reason, the standard deviations of Tp close to the perihelion are taken as a basis in these analyses. Here, the standard deviation calculated at perihelions between 109 yr and 289 yr is less than 1 day, between 290 yr and 589 yr is less than 5 days. Between 590 yr and 1027 yr, it is still less than 20 days.

Standard deviations start to increase exponentially after 1396 yr.

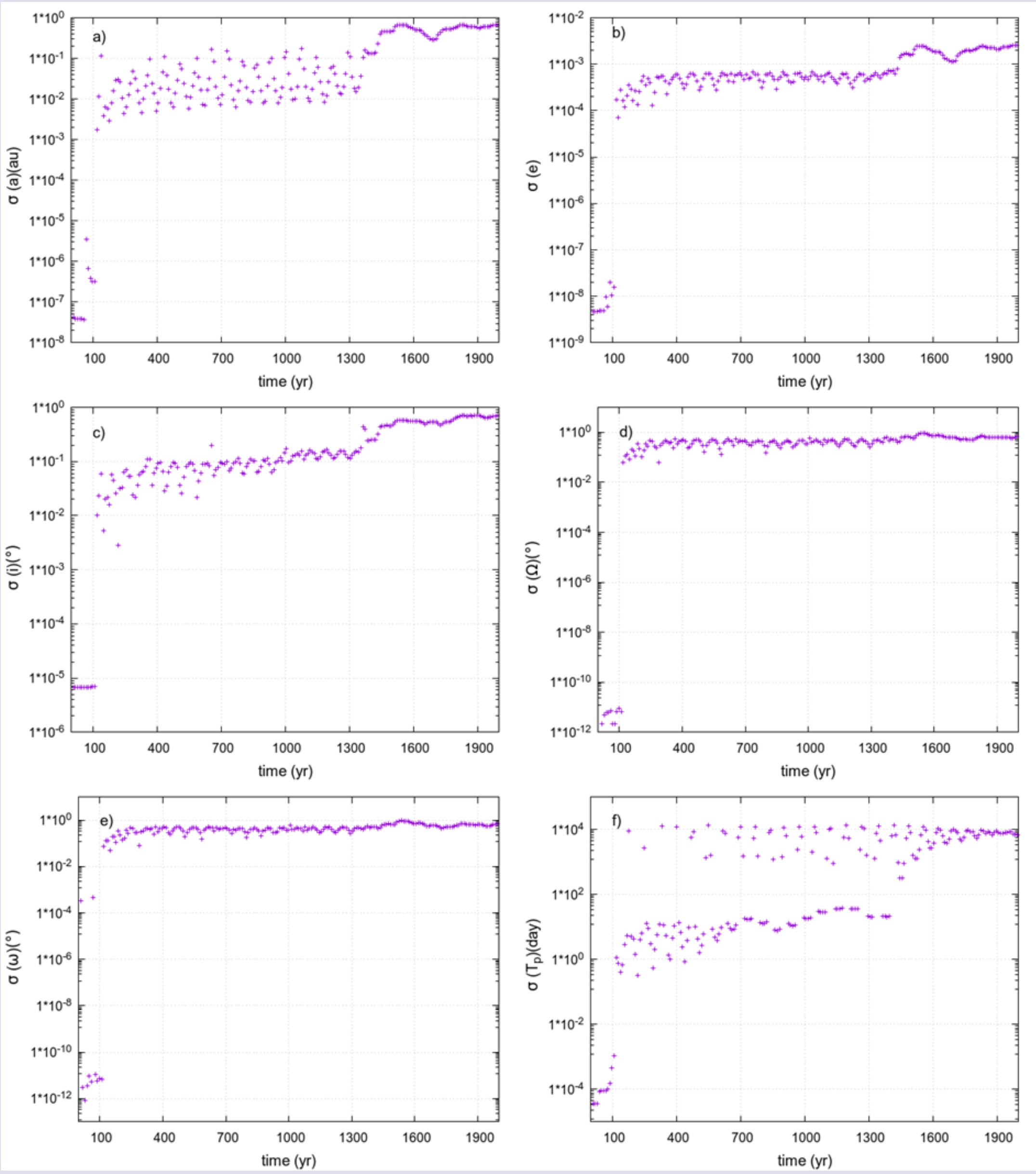

Figure 4. Standard deviations for the mean of the orbital elements (a, e, i, Ω, ω and Tp (time of perihelion passage) of the clones for comet 1/P Halley.

Conclusion

In this study, we calculated the Lyapunov time for comet 1/P Halley using the MEGNO indicator. Estimated Lyapunov times with and without renormalization are 119 yr and 190 yr, respectively. This result showed us the effect of renormalization in calculating the Lyapunov time for a high eccentric orbit. Depending on the different integration levels (1×10^4 yr, 2×10^4 yr, and 3×10^4 yr), varying results were also obtained, such as 96 yr, 121 yr and 164 yr respectively. Besides, to see the reflection of Lyapunov time in dynamic analysis, we performed orbital integrations for ∓10 kyr interval of the 1000 clone orbits produced using the MCCM method. We compared the means of the orbital elements of the clones and the time-dependent variations of their standard deviations with the Lyapunov times in the literature.

These results lead us to conclude that when we calculate different Lyapunov times at different levels, that can give us different scales: if our measure is at the initial precision range, Lyapunov time should not be more than 108 yr (such as 70-100 yr in [11]), for a standard deviation of less than one day at Tp, it should not be more than 289 yr (such as 300 yr in [12]), and for a standard deviation of fewer than five days on Tp , it should not be more than 589 yr (such as 562 yr in [13]). Moreover, for a measure where the shape and orientation of the orbit are still similar, and for a standard deviation of up to 20 days at Tp is acceptable, approximately 1300 yr can be taken as the limit for dynamical studies. It seems that Lyapunov times obtained from different methods using various assumptions can correspond to different levels in the dynamical analysis of the body.

Conflicts of interest

The authors state that did not have conflict of interests

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