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Algebraic operations of virtual fuzzy parameterized soft sets and their application in decision-making

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Abstract

It is a difficult task for decision-makers to accurately state a membership degree in the range [0,1]. The virtual fuzzy parameterized soft sets (VFPSSs) proposed to overcome this problem is an effective mathematical tool constructed, including the abilities of fuzzy sets and soft sets. In this paper, algebraic operations of VFPSSs are defined and examined these operations' properties. Then, a decision-making algorithm is proposed by the aforesaid operations. Finally, the proposed algorithm is succesfully applied to a decision-making problem.

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1. Introduction

It is almost impossible with classical mathematics to express the uncertainty of available data for problems encountered in many fields. Moreover; it is essential to accurately express these data for these areas to obtain accurate results close to the ideal. For this reason, many researchers have tried to construct a mathematical model for fuzzy data encountered in uncertain environments. Two of these studies is the fuzzy sets [1] and rough sets [2]. However; as pointed out by Molodtsov [3], the aforesaid concepts have certain difficulties expressing uncertainty. Molodtsov stated that these difficulties resulted from the lack of a parameterization tool and proposed a new approach, i.e., soft sets, free from these difficulties. Thanks to its success in expressing the uncertainty problems encountered, this concept has been successfully applied to many areas [4-8].

Many hybrid sets [9-17] have been developed using soft sets and the others [1,2,18]. One of them is fuzzy parameterized soft sets [19]. Unlike soft sets, the parameter set of this set theory is a fuzzy set that can express uncertainty better. Therefore, it is preferable in the solution of problems containing such uncertainties. However, it is a difficult task for a decision-maker to express the membership of a parameter fully. In other words, it is uneasy about expressing a value in the range [0,1] correctly. The virtual fuzzy parameterized soft sets (VFPSSs) [20], which generalize fuzzy parameterized soft sets, overcome the uncertainties. The most important advantages of this concept are as follows:

- The ability of decision-makers to express a decision-making problem by considering their margins of error.
- To be able to observe this change in the case that the margins of error may vary (see ref [20]).

In VFPSSs, the lower and upper parameter sets of a parameter set are defined to express each parameter's membership degrees more clearly. With these advantages, it is a better mathematical model than fuzzy parameterized soft sets for uncertainty. The algebraic operators studied for fuzzy parameterized soft sets are quite successful in constructing a better decision-making process for uncertainty [22]. In this paper, since the virtual fuzzy parameterized soft set theory is still very new, t-norm and s-norm products of this theory are defined. In addition, when we compare the success of the algebraic operators examined for both mathematical models in decision-making processes, it is clear that the algebraic operators proposed in this paper offer better approaches. Because the membership degrees expressed by the decision-makers are given independently in the definition of virtual fuzzy

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parameterized soft sets, which improves the data transfer received from the decision-makers in terms of increasing the reliability of the results in uncertain environments. Since the mathematical model discussed in our paper makes the decision-makers independent, it is an advantage that makes it more dominant than the contribution of other mathematical models to the decision-making processes. Moreover, beyond this situation, it is aimed to build a better approach with the contribution of algebraic operators. To this end, a decision-making method that can be used in expressing uncertain situations is proposed by using these products. Finally, using the proposed algorithm, the solution to an uncertainty problem is exemplified.

2. Materials and Methods

In this section, some definitions and results are reminded. Detailed explanations related to virtual fuzzy parameterized soft sets can be found in [20].

Throughout this paper, let $U = \{u_1, u_2, \dots, u_n\}$ be a universe set, $P = \{p_1, p_2, \dots, p_m\}$ be a set of parameters and X be a fuzzy set over P. In this case, the lower virtual parameter set and the upper virtual parameter set are expressed as $\underline{P} = \left\{p_1^{\underline{\alpha_1}}, p_2^{\underline{\alpha_2}}, \dots p_m^{\underline{\alpha_m}}\right\}$ and $\overline{P} = \left\{p_1^{\overline{\alpha_1}}, p_2^{\overline{\alpha_2}}, \dots, p_m^{\overline{\alpha_m}}\right\}$ respectively. Also, let 2^U denotes the power set of U and $\emptyset \neq A \subseteq P$.

Definition 2.1. [1] A fuzzy set X over U is a set defined by $\mu_X : U \to [0,1]$. μ_X is called the membership function of X, and the value $\mu_X(u)$ is called the grade of membership of $u \in U$. Thus, a fuzzy set X over U can be represented as follows:

$X = \{(\mu_X(u)/u) : u \in U, \mu_X(u) \in [0,1]\}$

Definition 2.2. [2] A pair (F, P) is called a soft set over U, where F is a mapping given by $F: P \to 2^U$. In other words, a soft set over U is a parameterized family of subsets of U for $p \in P$, $F(p)$ may be considered as the set of p-approximate elements of (F, P).

Definition 2.3. [21] t-norms are associative, monotonic and commutative two valued functions t that map from $[0, 1] \times [0, 1]$ to $[0, 1]$. These properties are formulated with the following conditions:

i. $t(0,0) = 0$ and $t(\mu_X(p), 1) = t(1, \mu_X(p)) = \mu_X(p)$, $p \in P$,

ii. If $\mu_{X_1}(p) \le \mu_{X_3}(p)$ and $\mu_{X_2}(p) \le \mu_{X_4}(p)$, then $t\left(\mu_{X_1}(p), \mu_{X_2}(p)\right) \le t\left(\mu_{X_3}(p), \mu_{X_4}(p)\right)$,

iii. $t\left(\mu_{X_1}(p), \mu_{X_2}(p)\right) = t\left(\mu_{X_2}(p), \mu_{X_1}(p)\right)$,

iv. $t\left(\mu_{X_1}(p), t\left(\mu_{X_2}(p), \mu_{X_3}(p)\right)\right) = t\left(t\left(\mu_{X_1}(p), \mu_{X_2}(p)\right), \mu_{X_3}(p)\right)$.

Definition 2.4. [21] t-conorms (s-norms) are associative, monotonic and commutative two placed functions s which map from $[0, 1] \times [0, 1]$ to $[0, 1]$. These properties are formulated with the following conditions:

i. $s(1,1) = 1$ and $s(\mu_X(p), 0) = s(0, \mu_X(p)) = \mu_X(p)$, $p \in P$,

ii. If $\mu_{X_1}(p) \le \mu_{X_3}(p)$ and $\mu_{X_2}(p) \le \mu_{X_4}(p)$, then $s\left(\mu_{X_1}(p), \mu_{X_2}(p)\right) \le s\left(\mu_{X_3}(p), \mu_{X_4}(p)\right)$,

iii. $s\left(\mu_{X_1}(p), \mu_{X_2}(p)\right) = s\left(\mu_{X_2}(p), \mu_{X_1}(p)\right)$,

iv. $s\left(\mu_{X_1}(p), s\left(\mu_{X_2}(p), \mu_{X_3}(p)\right)\right) = s\left(s\left(\mu_{X_1}(p), \mu_{X_2}(p)\right), \mu_{X_3}(p)\right)$.

t-norms and t-conorms are related in a sense of logical duality. Typical dual pairs of non parameterized t-norm and t-conorm are complied below:

i. Drastic product:

$$t_w\left(\mu_{X_1}(p), \mu_{X_2}(p)\right) = \begin{cases} min\{\mu_{X_1}(p), \mu_{X_2}(p)\}, & max\{\mu_{X_1}(p), \mu_{X_2}(p)\} = 1, \\ 0, & otherwise \end{cases} \tag{1}$$

ii. Drastic sum:

$$s_w\left(\mu_{X_1}(p),\mu_{X_2}(p)\right)=\begin{cases} max\{\mu_{X_1}(p),\mu_{X_2}(p)\}, & min\{\mu_{X_1}(p),\mu_{X_2}(p)\}=0, \\ 1, & otherwise \end{cases} \quad (2)$$

iii. Bounded product:

$$t_1\left(\mu_{X_1}(p),\mu_{X_2}(p)\right)= max\{0,\mu_{X_1}(p)+\mu_{X_2}(p)-1\} \quad (3)$$

iv. Bounded sum:

$$s_1\left(\mu_{X_1}(p),\mu_{X_2}(p)\right)= min\{1,\mu_{X_1}(p)+\mu_{X_2}(p)\} \quad (4)$$

v. Einstein product:

$$t_{1.5}\left(\mu_{X_1}(p),\mu_{X_2}(p)\right)= \frac{\mu_{X_1}(p)\mu_{X_2}(p)}{2-[\mu_{X_1}(p)+\mu_{X_2}(p)-\mu_{X_1}(p)\mu_{X_2}(p)]} \quad (5)$$

vi. Einstein sum:

$$s_{1.5}\left(\mu_{X_1}(p),\mu_{X_2}(p)\right)= \frac{\mu_{X_1}(p)+\mu_{X_2}(p)}{1+\mu_{X_1}(p)\mu_{X_2}(p)} \quad (6)$$

vii. Algebraic product:

$$t_2\left(\mu_{X_1}(p),\mu_{X_2}(p)\right)= \mu_{X_1}(p)\mu_{X_2}(p) \quad (7)$$

viii. Algebraic sum:

$$s_2\left(\mu_{X_1}(p),\mu_{X_2}(p)\right)= \mu_{X_1}(p)+\mu_{X_2}(p)-\mu_{X_1}(p)\mu_{X_2}(p) \quad (8)$$

ix. Hamacher product:

$$t_{2.5}\left(\mu_{X_1}(p),\mu_{X_2}(p)\right)= \frac{\mu_{X_1}(p)\mu_{X_2}(p)}{\mu_{X_1}(p)+\mu_{X_2}(p)-\mu_{X_1}(p)\mu_{X_2}(p)} \quad (9)$$

x. Hamacher sum:

$$s_{2.5}\left(\mu_{X_1}(p),\mu_{X_2}(p)\right)= \frac{\mu_{X_1}(p)+\mu_{X_2}(p)-2\cdot\mu_{X_1}(p)\mu_{X_2}(p)}{1-\mu_{X_1}(p)\mu_{X_2}(p)} \quad (10)$$

xi. Minimum:

$$t_3\left(\mu_{X_1}(p),\mu_{X_2}(p)\right)= min\{\mu_{X_1}(p),\mu_{X_2}(p)\} \quad (11)$$

xii. Maximum:

$$s_3\left(\mu_{X_1}(p),\mu_{X_2}(p)\right)= max\{\mu_{X_1}(p),\mu_{X_2}(p)\} \quad (12)$$

Definition 2.5. [20] Let $\underline{X}$, X, $\overline{X}$ be a fuzzy set over $\underline{P}$, P, $\overline{P}$, respectively. A virtual fuzzy parameterized soft set Ψ_X over U is defined as follows:

$$\Psi_X=\underline{\Upsilon_X}\cup\Upsilon_X\cup\overline{\Upsilon_X}$$

such that

$$\underline{\Upsilon_X}=\left\{\left(\frac{\mu_X(p)-\underline{\alpha}}{p},\underline{\psi_X}(p^{\underline{\alpha}})\right):p^{\underline{\alpha}}\in\underline{P},p\in P,\mu_X(p)\in[0,1],0\le\underline{\alpha}<\mu_X(p)\right\},$$

$$\Upsilon_X=\left\{\left(\frac{\mu_X(p)}{p},\psi_X(p)\right):p\in P,\mu_X(p)\in[0,1]\right\},$$

$$\overline{\Upsilon_X}=\left\{\left(\frac{\mu_X(p)+\overline{\alpha}}{p},\overline{\psi_X}(p^{\overline{\alpha}})\right):p\in P,p^{\overline{\alpha}}\in\overline{P},\mu_X(p)\in[0,1],0\le\overline{\alpha}\le 1-\mu_X(p)\right\},$$

where the functions $\underline{\psi_X}:\underline{P}\to 2^U$, $\psi_X:P\to 2^U$, $\overline{\psi_X}:\overline{P}\to 2^U$ are called lower approximate function, approximate function, upper approximate function, respectively, and the function $\mu_X:P\to[0,1]$ is called membership function of the set X. Here, $\psi_X(p)=\emptyset$ if $\mu_X(p)=0$. Moreover, $\underline{\psi_X}(p^{\underline{\alpha}})=\emptyset$ if $\mu_X(p)-\underline{\alpha}=0$ and $\overline{\psi_X}(p^{\overline{\alpha}})=\emptyset$ if $\mu_X(p)+\overline{\alpha}=0$.

From now on, $VFPS(U)$ denotes the family of all virtual fuzzy parameterized soft sets over U with P as the set of parameters.

Example 2.1. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ be a universe set, $P = \{p_1, p_2, p_3, p_4\}$ be the set of parameters and $X = \{0.5/p_2, 0.7/p_4\}$ be a fuzzy set over P. If $\underline{X} = \{0.4/p_2, 0.35/p_4\}$, $\overline{X} = \{0.75/p_2, 0.9/p_4\}$, and

$$\underline{\psi_X}(p_2^{0.1}) = \{u_1, u_3, u_4, u_6, u_7\}, \qquad \underline{\psi_X}(p_4^{0.35}) = \{u_2, u_3, u_4, u_5, u_7\},$$

$$\psi_X(p_2) = \{u_1, u_4, u_6, u_7\}, \qquad \psi_X(p_4) = \{u_2, u_4, u_5, u_7\},$$

$$\overline{\psi_X}(p_2^{0.25}) = \{u_1, u_6, u_7\}, \qquad \overline{\psi_X}(p_4^{0.2}) = \{u_2, u_5, u_7\},$$

then the virtual fuzzy parameterized soft set Ψ_X is written by

$$\Psi_X = \begin{Bmatrix} (0.4/p_2, \{u_1, u_3, u_4, u_6, u_7\}), (0.35/p_4, \{u_2, u_3, u_4, u_5, u_7\}) \\ (0.5/p_2, \{u_1, u_4, u_6, u_7\}), (0.7/p_4, \{u_2, u_4, u_5, u_7\}) \\ (0.75/p_2, \{u_1, u_6, u_7\}), (0.9/p_4, \{u_2, u_5, u_7\}) \end{Bmatrix}$$

where

$$\underline{\Upsilon_X} = \{(0.4/p_2, \{u_1, u_3, u_4, u_6, u_7\}), (0.35/p_4, \{u_2, u_3, u_4, u_5, u_7\})\},$$

$$\Upsilon_X = \{(0.5/p_2, \{u_1, u_4, u_6, u_7\}), (0.7/p_4, \{u_2, u_4, u_5, u_7\})\}$$

and

$$\overline{\Upsilon_X} = \{(0.75/p_2, \{u_1, u_6, u_7\}), (0.9/p_4, \{u_2, u_5, u_7\})\}.$$

Definition 2.6. [20] Let $\Psi_X \in VFPS(U)$.

(i) If $\underline{\psi_X}(p^{\underline{\alpha}}) = \psi_X(p) = \overline{\psi_X}(p^{\overline{\alpha}}) = \emptyset$ for all $p^{\underline{\alpha}} \in \underline{P}$, $p \in P$, $p^{\overline{\alpha}} \in \overline{P}$, then virtual fuzzy parameterized soft set Ψ_X is called an X-empty VFPSS, denoted by $\Psi_{\emptyset_X}$. If $X = \emptyset$, then Ψ_X is called an empty virtual fuzzy parameterized soft set, denoted by $\Psi_\emptyset$.

(ii) If $\underline{X}$, X, $\overline{X}$ are crisps subset of $\underline{P}$, P, $\overline{P}$, respectively, and $\underline{\psi_X}(p^{\underline{\alpha}}) = \psi_X(p) = \overline{\psi_X}(p^{\overline{\alpha}}) = U$ for all $p^{\underline{\alpha}} \in \underline{P}$, $p \in P$, $p^{\overline{\alpha}} \in \overline{P}$, then virtual fuzzy parameterized soft set Ψ_X is called an X-universal VFPSS, denoted by $\Psi_{\tilde{X}}$. If $X = P$, then the X-universal virtual fuzzy parameterized soft set is called universal virtual fuzzy parameterized soft set, denoted by $\Psi_{\tilde{P}}$.

Definition 2.7. [20] Let $\Psi_X, \Psi_Y \in VFPS(U)$. Then, Ψ_X is a virtual fuzzy parameterized soft subset of Ψ_Y, denoted by $\Psi_X \,\tilde{\sqsubseteq}\, \Psi_Y$, if

i. $\mu_X(p) - \underline{\alpha} \le \mu_Y(p) - \underline{\beta}$ and $\underline{\psi_X}(p^{\underline{\alpha}}) \subseteq \underline{\psi_Y}(p^{\underline{\beta}})$ for all $p^{\underline{\alpha}}, p^{\underline{\beta}} \in \underline{P}$,

ii. $\mu_X(p) \le \mu_Y(p)$ and $\psi_X(p) \subseteq \psi_Y(p)$ for all $p \in P$,

iii. $\mu_X(p) + \overline{\alpha} \le \mu_Y(p) + \overline{\beta}$ and $\overline{\psi_X}(p^{\overline{\alpha}}) \subseteq \overline{\psi_Y}\left(p^{\overline{\beta}}\right)$ for all $p^{\overline{\alpha}}, p^{\overline{\beta}} \in \overline{P}$.

Also, Ψ_X is a virtual fuzzy parameterized soft equal to Ψ_Y, denoted by $\Psi_X = \Psi_Y$, if

i. $\mu_X(p) - \underline{\alpha} = \mu_Y(p) - \underline{\beta}$ and $\underline{\psi_X}(p^{\underline{\alpha}}) = \underline{\psi_Y}(p^{\underline{\beta}})$ for all $p^{\underline{\alpha}}, p^{\underline{\beta}} \in \underline{P}$,

ii. $\mu_X(p) = \mu_Y(p)$ and $\psi_X(p) = \psi_Y(p)$ for all $p \in P$,

iii. $\mu_X(p) + \overline{\alpha} = \mu_Y(p) + \overline{\beta}$ and $\overline{\psi_X}(p^{\overline{\alpha}}) = \overline{\psi_Y}\left(p^{\overline{\beta}}\right)$ for all $p^{\overline{\alpha}}, p^{\overline{\beta}} \in \overline{P}$.

Proposition 2.1. [20] Let $\Psi_X \in VFPS(U)$. $s\left(\overline{\psi_X}(p^{\overline{\alpha}})\right) \leq s(\psi_Y(p)) \leq s\left(\underline{\psi_X}(p^{\underline{\alpha}})\right)$ is valid for all $p^{\underline{\alpha}} \in \underline{P}, p \in P, p^{\overline{\alpha}} \in \overline{P}$.

Definition 2.8. [20] Let $\Psi_X \in VFPS(U)$. Then, complement Ψ_X, denoted by Ψ_X^c, is a virtual fuzzy parameterized soft set defined by the approximate and membership functions as

i. $\mu_{X^c}(p) - \underline{\tilde{\alpha}} = 1 - \left(\mu_X(p) - \underline{\alpha}\right)$ and $\underline{\psi_{X^c}}(p^{\underline{\tilde{\alpha}}}) = U/\underline{\psi_X}(p^{\underline{\alpha}})$ for all $p^{\underline{\alpha}}, p^{\underline{\tilde{\alpha}}} \in \underline{P}$,

ii. $\mu_{X^c}(p) = 1 - \mu_X(p)$ and $\psi_{X^c}(p) = U/\psi_X(p)$ for all $p \in P$,

iii. $\mu_{X^c}(p) + \overline{\tilde{\alpha}} = 1 - (\mu_X(p) + \overline{\alpha})$ and $\overline{\psi_{X^c}}\left(p^{\overline{\tilde{\alpha}}}\right) = U/\overline{\psi_X}(p^{\overline{\alpha}})$ for all $p^{\overline{\alpha}}, p^{\overline{\tilde{\alpha}}} \in \overline{P}$.

Definition 2.9. [20] Let $\Psi_X, \Psi_Y \in VFPS(U)$. Then, union Ψ_X and Ψ_Y, denoted by $\Psi_X \tilde{\cup} \Psi_Y$, is defined by

i. $\mu_{X\cup Y}(p) - \underline{\gamma} = max\left\{\mu_X(p) - \underline{\alpha}, \mu_Y(p) - \underline{\beta}\right\}$ and $\underline{\psi_{X\cup Y}}(p^{\underline{\gamma}}) = \underline{\psi_X}(p^{\underline{\alpha}}) \cup \underline{\psi_Y}(p^{\underline{\beta}})$ for all $p^{\underline{\alpha}}, p^{\underline{\beta}}, p^{\underline{\gamma}} \in \underline{P}$,

ii. $\mu_{X\cup Y}(p) = max\{\mu_X(p), \mu_Y(p)\}$ and $\psi_{X\cup Y}(p) = \psi_X(p) \cup \psi_Y(p)$ for all $p \in P$,

iii. $\mu_{X\cup Y}(p) + \overline{\gamma} = max\{\mu_X(p) + \overline{\alpha}, \mu_Y(p) + \overline{\beta}\}$ and $\overline{\psi_{X\cup Y}}(p^{\overline{\gamma}}) = \overline{\psi_X}(p^{\overline{\alpha}}) \cup \overline{\psi_Y}\left(p^{\overline{\beta}}\right)$ for all $p^{\overline{\alpha}}, p^{\overline{\beta}}, p^{\overline{\gamma}} \in \overline{P}$.

Definition 2.10. [20] Let $\Psi_X, \Psi_Y \in VFPS(U)$. Then, intersection Ψ_X and Ψ_Y, denoted by $\Psi_X \tilde{\cap} \Psi_Y$, is defined by

i. $\mu_{X\cap Y}(p) - \underline{\gamma} = min\left\{\mu_X(p) - \underline{\alpha}, \mu_Y(p) - \underline{\beta}\right\}$ and $\underline{\psi_{X\cap Y}}(p^{\underline{\gamma}}) = \underline{\psi_X}(p^{\underline{\alpha}}) \cap \underline{\psi_Y}(p^{\underline{\beta}})$ for all $p^{\underline{\alpha}}, p^{\underline{\beta}}, p^{\underline{\gamma}} \in \underline{P}$,

ii. $\mu_{X\cap Y}(p) = min\{\mu_X(p), \mu_Y(p)\}$ and $\psi_{X\cap Y}(p) = \psi_X(p) \cap \psi_Y(p)$ for all $p \in P$,

iii. $\mu_{X\cap Y}(p) + \overline{\gamma} = min\{\mu_X(p) + \overline{\alpha}, \mu_Y(p) + \overline{\beta}\}$ and $\overline{\psi_{X\cap Y}}(p^{\overline{\gamma}}) = \overline{\psi_X}(p^{\overline{\alpha}}) \cap \overline{\psi_Y}\left(p^{\overline{\beta}}\right)$ for all $p^{\overline{\alpha}}, p^{\overline{\beta}}, p^{\overline{\gamma}} \in \overline{P}$.

3. t-Norm and t-Conorm Products of Virtual Fuzzy Parameterized Soft Sets

Definition 3.1. Let $\Psi_X, \Psi_Y \in VFPS(U)$. Then, the AND-$t$-norm of Ψ_X and Ψ_Y, denoted by $\Psi_X \tilde{\boxtimes} \Psi_Y$, is the virtual fuzzy parameterized soft set defined as follows:

$$\underline{\mu_{X\tilde{\boxtimes}Y}}: \underline{P} \to [0,1], \quad \underline{\mu_{X\tilde{\boxtimes}Y}}(p^{\underline{\gamma}}) = \left(\mu_X(p) - \underline{\alpha}\right) \cdot \left(\mu_Y(p) - \underline{\beta}\right), \tag{13}$$

$$\mu_{X\tilde{\boxtimes}Y}: P \to [0,1], \quad \mu_{X\tilde{\boxtimes}Y}(p) = \mu_X(p) \cdot \mu_Y(p), \tag{14}$$

$$\overline{\mu_{X\tilde{\boxtimes}Y}}: P \to [0,1], \quad \overline{\mu_{X\tilde{\boxtimes}Y}}(p^{\overline{\gamma}}) = (\mu_X(p) + \overline{\alpha}) \cdot (\mu_Y(p) + \overline{\beta}) \tag{15}$$

and

$$\underline{\psi_{X\tilde{\boxtimes}Y}}: P \to 2^U, \quad \underline{\psi_{X\tilde{\boxtimes}Y}}(p^{\underline{\gamma}}) = \underline{\psi_X}(p^{\underline{\alpha}}) \cap \underline{\psi_Y}(p^{\underline{\beta}}) \tag{16}$$

$$\psi_{X\breve{\boxtimes}Y}: P \to 2^U, \quad \psi_{X\breve{\boxtimes}Y}(p) = \psi_X(p) \cap \psi_Y(p), \tag{17}$$

$$\overline{\psi_{X\breve{\boxtimes}Y}}: P \to 2^U, \quad \overline{\psi_{X\breve{\boxtimes}Y}}(p^{\overline{\gamma}}) = \overline{\psi_X}(p^{\overline{\alpha}}) \cap \overline{\psi_Y}\left(p^{\overline{\beta}}\right) \tag{18}$$

where the functions $\underline{\psi_{X\breve{\boxtimes}Y}}$, $\psi_{X\breve{\boxtimes}Y}$, $\overline{\psi_{X\breve{\boxtimes}Y}}$ are called lower approximate function, approximate function, upper approximate function of $\Psi_X \breve{\boxtimes} \Psi_Y$, respectively, and $\underline{\mu_{X\breve{\boxtimes}Y}}$, $\mu_{X\breve{\boxtimes}Y}$, $\overline{\mu_{X\breve{\boxtimes}Y}}$ are called lower membership function, membership function, upper membership function of $\Psi_X \breve{\boxtimes} \Psi_Y$, respectively.

Definition 3.2. Let $\Psi_X, \Psi_Y \in VFPS(U)$. Then, the OR-$t$-norm of Ψ_X and Ψ_Y, denoted by $\Psi_X \boxtimes \Psi_Y$, is the virtual fuzzy parameterized soft set defined as follows:

$$\underline{\mu_{X\boxtimes Y}}: \underline{P} \to [0,1], \quad \underline{\mu_{X\boxtimes Y}}(p^{\underline{\gamma}}) = \left(\mu_X(p) - \underline{\alpha}\right) \cdot \left(\mu_Y(p) - \underline{\beta}\right), \tag{19}$$

$$\mu_{X\boxtimes Y}: P \to [0,1], \quad \mu_{X\boxtimes Y}(p) = \mu_X(p) \cdot \mu_Y(p), \tag{20}$$

$$\overline{\mu_{X\boxtimes Y}}: P \to [0,1], \quad \overline{\mu_{X\boxtimes Y}}(p^{\overline{\gamma}}) = (\mu_X(p) + \overline{\alpha}) \cdot (\mu_Y(p) + \overline{\beta}) \tag{21}$$

and

$$\underline{\psi_{X\boxtimes Y}}: P \to 2^U, \quad \underline{\psi_{X\boxtimes Y}}(p^{\underline{\gamma}}) = \underline{\psi_X}(p^{\underline{\alpha}}) \cup \underline{\psi_Y}(p^{\underline{\beta}}) \tag{22}$$

$$\psi_{X\boxtimes Y}: P \to 2^U, \quad \psi_{X\boxtimes Y}(p) = \psi_X(p) \cup \psi_Y(p), \tag{23}$$

$$\overline{\psi_{X\boxtimes Y}}: P \to 2^U, \quad \overline{\psi_{X\boxtimes Y}}(p^{\overline{\gamma}}) = \overline{\psi_X}(p^{\overline{\alpha}}) \cup \overline{\psi_Y}\left(p^{\overline{\beta}}\right) \tag{24}$$

where the functions $\underline{\psi_{X\boxtimes Y}}$, $\psi_{X\boxtimes Y}$, $\overline{\psi_{X\boxtimes Y}}$ are called lower approximate function, approximate function, upper approximate function of $\Psi_X \boxtimes \Psi_Y$, respectively, and $\underline{\mu_{X\boxtimes Y}}$, $\mu_{X\boxtimes Y}$, $\overline{\mu_{X\boxtimes Y}}$ are called lower membership function, membership function, upper membership function of $\Psi_X \boxtimes \Psi_Y$, respectively.

Definition 3.3. Let $\Psi_X, \Psi_Y \in VFPS(U)$. Then, the OR-$t$-conorm of Ψ_X and Ψ_Y, denoted by $\Psi_X \boxplus \Psi_Y$, is the VFPSS defined as follows:

$$\underline{\mu_{X\boxplus Y}}: \underline{P} \to [0,1], \quad \underline{\mu_{X\boxplus Y}}(p^{\underline{\gamma}}) = \left[\left(\mu_X(p) - \underline{\alpha}\right) + \left(\mu_Y(p) - \underline{\beta}\right)\right] - \left(\mu_X(p) - \underline{\alpha}\right) \cdot \left(\mu_Y(p) - \underline{\beta}\right), \tag{25}$$

$$\mu_{X\boxplus Y}: P \to [0,1], \quad \mu_{X\boxplus Y}(p) = \mu_X(p) + \mu_Y(p) - \mu_X(p) \cdot \mu_Y(p), \tag{26}$$

$$\overline{\mu_{X\boxplus Y}}: P \to [0,1], \quad \overline{\mu_{X\boxplus Y}}(p^{\overline{\gamma}}) = \left[(\mu_X(p) + \overline{\alpha}) + \left(\mu_Y(p) + \overline{\beta}\right)\right] - (\mu_X(p) + \overline{\alpha}) \cdot \left(\mu_Y(p) + \overline{\beta}\right) \tag{27}$$

and

$$\underline{\psi_{X\boxplus Y}}: P \to 2^U, \quad \underline{\psi_{X\boxplus Y}}(p^{\underline{\gamma}}) = \underline{\psi_X}(p^{\underline{\alpha}}) \cup \underline{\psi_Y}(p^{\underline{\beta}}) \tag{28}$$

$$\psi_{X\boxplus Y}: P \to 2^U, \quad \psi_{X\boxplus Y}(p) = \psi_X(p) \cup \psi_Y(p), \tag{29}$$

$$\overline{\psi_{X\boxplus Y}}: P \to 2^U, \quad \overline{\psi_{X\boxplus Y}}(p^{\overline{\gamma}}) = \overline{\psi_X}(p^{\overline{\alpha}}) \cup \overline{\psi_Y}\left(p^{\overline{\beta}}\right) \tag{30}$$

where the functions $\underline{\psi_{X\boxplus Y}}$, $\psi_{X\boxplus Y}$, $\overline{\psi_{X\boxplus Y}}$ are called lower approximate function, approximate function, upper approximate function of $\Psi_X \boxplus \Psi_Y$, respectively, and $\underline{\mu_{X\boxplus Y}}$, $\mu_{X\boxplus Y}$, $\overline{\mu_{X\boxplus Y}}$ are called lower membership function, membership function, upper membership function of $\Psi_X \boxplus \Psi_Y$, respectively.

Definition 3.4. Let $\Psi_X, \Psi_Y \in VFPS(U)$. Then, the AND-$t$-conorm of Ψ_X and Ψ_Y, denoted by $\Psi_X \breve{\boxplus} \Psi_Y$, is the VFPSS defined as follows:

$$\underline{\mu_{X\breve{\boxplus}Y}}: \underline{P} \to [0,1], \quad \underline{\mu_{X\breve{\boxplus}Y}}(p^{\underline{\gamma}}) = \left[\left(\mu_X(p) - \underline{\alpha}\right) + \left(\mu_Y(p) - \underline{\beta}\right)\right] - \left(\mu_X(p) - \underline{\alpha}\right) \cdot \left(\mu_Y(p) - \underline{\beta}\right), \tag{31}$$

$$\mu_{X\breve{\boxplus}Y}: P \to [0,1], \quad \mu_{X\breve{\boxplus}Y}(p) = \mu_X(p) + \mu_Y(p) - \mu_X(p) \cdot \mu_Y(p), \tag{32}$$

$$\overline{\mu_{X\breve{\boxplus}Y}}: P \to [0,1], \quad \overline{\mu_{X\breve{\boxplus}Y}}(p^{\overline{\gamma}}) = \left[(\mu_X(p) + \overline{\alpha}) + \left(\mu_Y(p) + \overline{\beta}\right)\right] - (\mu_X(p) + \overline{\alpha}) \cdot \left(\mu_Y(p) + \overline{\beta}\right) \tag{33}$$

and

$$\underline{\psi_{X\breve{\boxplus}Y}}: P \to 2^U, \quad \underline{\psi_{X\breve{\boxplus}Y}}(p^{\underline{\gamma}}) = \underline{\psi_X}(p^{\underline{\alpha}}) \cap \underline{\psi_Y}\left(p^{\underline{\beta}}\right) \tag{34}$$

$$\psi_{X\breve{\boxplus}Y}: P \to 2^U, \quad \psi_{X\breve{\boxplus}Y}(p) = \psi_X(p) \cap \psi_Y(p), \tag{35}$$

$$\overline{\psi_{X\breve{\boxplus}Y}}: P \to 2^U, \quad \overline{\psi_{X\breve{\boxplus}Y}}(p^{\overline{\gamma}}) = \overline{\psi_X}(p^{\overline{\alpha}}) \cap \overline{\psi_Y}\left(p^{\overline{\beta}}\right) \tag{36}$$

where the functions $\underline{\psi_{X\breve{\boxplus}Y}}$, $\psi_{X\breve{\boxplus}Y}$, $\overline{\psi_{X\breve{\boxplus}Y}}$ are called lower approximate function, approximate function, upper approximate function of $\Psi_X \boxplus \Psi_Y$, respectively, and $\underline{\mu_{X\breve{\boxplus}Y}}$, $\mu_{X\breve{\boxplus}Y}$, $\overline{\mu_{X\breve{\boxplus}Y}}$ are called lower membership function, membership function, upper membership function of $\Psi_X \breve{\boxplus} \Psi_Y$, respectively.

Example 3.1. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ be an universe set, $P = \{p_1, p_2, p_3, p_4, p_5\}$ be the set of parameters and $X = \{0.64/p_2, 0.55/p_4\}$, $Y = \{0.3/p_2, 0.48/p_3\}$ are two fuzzy sets over P. If

$$\underline{X} = \{0.42/p_2, 0.45/p_4\}, \ \overline{X} = \{0.7/p_2, 0.85/p_4\}$$

and

$$\underline{Y} = \{0.2/p_2, 0.3/p_3\}, \ \overline{Y} = \{0.5/p_2, 0.65/p_3\},$$

then the virtual fuzzy parameterized soft sets Ψ_X and Ψ_Y are written by

$$\Psi_X = \begin{Bmatrix} (0.42/p_2, \{u_1, u_2, u_3, u_7, u_8\}), (0.45/p_4, \{u_1, u_3, u_5, u_6, u_8\}) \\ (0.64/p_2, \{u_2, u_3, u_7, u_8\}), (0.55/p_4, \{u_3, u_5, u_6, u_8\}) \\ (0.7/p_2, \{u_2, u_3, u_8\}), (0.85/p_4, \{u_3, u_6, u_8\}) \end{Bmatrix}$$

and

$$\Psi_Y = \begin{Bmatrix} (0.2/p_2, \{u_1, u_2, u_4, u_5, u_7\}), (0.3/p_3, \{u_1, u_3, u_4, u_5, u_6\}) \\ (0.3/p_2, \{u_2, u_4, u_5, u_7\}), (0.48/p_3, \{u_3, u_4, u_5, u_6\}) \\ (0.5/p_2, \{u_2, u_4, u_5\}), (0.65/p_3, \{u_4, u_5, u_6\}) \end{Bmatrix}.$$

Then,

$$\Psi_X \boxplus \Psi_Y = \begin{Bmatrix} (0.536/p_2, \{u_1, u_2, u_3, u_4, u_5, u_7, u_8\}), (0.3/p_3, \{u_1, u_3, u_4, u_5, u_6\}), \\ (0.45/p_4, \{u_1, u_3, u_5, u_6, u_8\}), (0.748/p_2, \{u_2, u_3, u_4, u_5, u_7, u_8\}), \\ (0.48/p_3, \{u_3, u_4, u_5, u_6\}), (0.55/p_4, \{u_3, u_5, u_6, u_8\}), \\ (0.85/p_2, \{u_2, u_3, u_4, u_5, u_8\}), (0.65/p_3, \{u_4, u_5, u_6\}), (0.85/p_4, \{u_3, u_6, u_8\}) \end{Bmatrix}$$

Proposition 3.1. Let $\Psi_X, \Psi_\emptyset, \Psi_{\tilde{P}} \in VFPS(U)$. Then,

i. $\Psi_X \boxplus \Psi_{\tilde{P}} = \Psi_{\tilde{P}}$,
ii. $\Psi_X \breve{\boxtimes} \Psi_{\tilde{P}} = \Psi_X$,
iii. $\Psi_X \boxplus \Psi_\emptyset = \Psi_X$,
iv. $\Psi_X \breve{\boxtimes} \Psi_\emptyset = \Psi_\emptyset$.

Proof. Straightforward.

Proposition 3.2. Let $\Psi_X, \Psi_Y \in VFPS(U)$. Then,

i. $(\Psi_X \boxplus \Psi_Y)^c = \Psi_X^c \breve{\boxtimes} \Psi_Y^c$,

ii. $\left(\Psi_X \overset{\smile}{\boxtimes} \Psi_Y\right)^c = \Psi_X^c \boxplus \Psi_Y^c$,
iii. $(\Psi_X \boxtimes \Psi_Y)^c = \Psi_X^c \overset{\smile}{\boxplus} \Psi_Y^c$,
iv. $\left(\Psi_X \overset{\smile}{\boxplus} \Psi_Y\right)^c = \Psi_X^c \boxtimes \Psi_Y^c$.

Proof. i. For all $p^{\underline{\gamma}} \in \underline{P}$,

$$\begin{aligned}
&\underline{\mu_{(X\boxplus Y)^c}}(p^{\underline{\gamma}}) = 1 - \underline{\mu_{X\boxplus Y}}(p^{\underline{\gamma}})\\
&= 1 - \left(\left[\left(\mu_X(p) - \underline{\alpha}\right) + \left(\mu_Y(p) - \underline{\beta}\right)\right] - \left(\mu_X(p) - \underline{\alpha}\right)\cdot\left(\mu_Y(p) - \underline{\beta}\right)\right)\\
&= 1 - \mu_X(p) + \underline{\alpha} - \mu_Y(p) + \underline{\beta} + \left(\mu_X(p) - \underline{\alpha}\right)\cdot\left(\mu_Y(p) - \underline{\beta}\right)\\
&= \left(1 - \mu_X(p) + \underline{\alpha}\right)\left(1 - \mu_Y(p) + \underline{\beta}\right)\\
&= \underline{\mu_{X^c \overset{\smile}{\boxtimes} Y^c}}(p^{\underline{\gamma}})
\end{aligned}$$

for all $p \in P$,

$$\begin{aligned}
&\mu_{(X\boxplus Y)^c}(p) = 1 - \mu_{X\boxplus Y}(p)\\
&= 1 - \left(\mu_X(p) + \mu_Y(p) - \mu_X(p)\cdot\mu_Y(p)\right)\\
&= 1 - \mu_X(p) - \mu_Y(p) + \mu_X(p)\cdot\mu_Y(p)\\
&= \left(1 - \mu_X(p)\right)\left(1 - \mu_Y(p)\right)\\
&= \mu_{X^c \overset{\smile}{\boxtimes} Y^c}(p)
\end{aligned}$$

for all $p^{\overline{\gamma}} \in \overline{P}$,

$$\begin{aligned}
&\overline{\mu_{(X\boxplus Y)^c}}\left(p^{\overline{\gamma}}\right) = 1 - \overline{\mu_{X\boxplus Y}}\left(p^{\overline{\gamma}}\right)\\
&= 1 - \left(\left[(\mu_X(p) + \overline{\alpha}) + \left(\mu_Y(p) + \overline{\beta}\right)\right] - (\mu_X(p) + \overline{\alpha})\cdot\left(\mu_Y(p) + \overline{\beta}\right)\right)\\
&= 1 - \mu_X(p) - \overline{\alpha} - \mu_Y(p) - \overline{\beta} + (\mu_X(p) + \overline{\alpha})\cdot\left(\mu_Y(p) + \overline{\beta}\right)\\
&= (1 - \mu_X(p) - \overline{\alpha})\left(1 - \mu_Y(p) - \overline{\beta}\right)\\
&= \overline{\mu_{X^c \overset{\smile}{\boxtimes} Y^c}}\left(p^{\overline{\gamma}}\right)
\end{aligned}$$

and for all $p^{\underline{\alpha}}, p^{\underline{\beta}}, p^{\underline{\gamma}} \in \underline{P}$, $p \in P$ and $p^{\overline{\alpha}}, p^{\overline{\beta}}, p^{\overline{\gamma}} \in \overline{P}$;

$$\underline{\psi_{(X\boxplus Y)^c}}(p^{\underline{\gamma}}) = U \setminus \left(\underline{\psi_X}(p^{\underline{\alpha}}) \cup \underline{\psi_Y}\left(p^{\underline{\beta}}\right)\right) = \left(U \setminus \underline{\psi_X}(p^{\underline{\alpha}})\right) \cup \left(U \setminus \underline{\psi_Y}\left(p^{\underline{\beta}}\right)\right) = \underline{\psi_{X^c}}(p^{\underline{\alpha}}) \cap \underline{\psi_{Y^c}}(p^{\underline{\beta}}),$$

$$\psi_{(X\boxplus Y)^c}(p) = U \setminus \left(\psi_X(p) \cup \psi_Y(p)\right) = \left(U \setminus \psi_X(p)\right) \cup \left(U \setminus \psi_Y(p)\right) = \psi_{X^c}(p) \cap \psi_{Y^c}(p),$$

$$\overline{\psi_{(X\boxplus Y)^c}}\left(p^{\overline{\gamma}}\right) = U \setminus \left(\overline{\psi_X}\left(p^{\overline{\alpha}}\right) \cup \overline{\psi_Y}\left(p^{\overline{\beta}}\right)\right) = \left(U \setminus \overline{\psi_X}\left(p^{\overline{\alpha}}\right)\right) \cup \left(U \setminus \overline{\psi_Y}\left(p^{\overline{\beta}}\right)\right) = \overline{\psi_{X^c}}\left(p^{\overline{\alpha}}\right) \cap \overline{\psi_{Y^c}}\left(p^{\overline{\beta}}\right).$$

The remaining parts can also be proved in a similar way.

Proposition 3.3. Let $\Psi_X, \Psi_Y, \Psi_Z \in VFPS(U)$. Then, for $\star = \left\{\overset{\smile}{\boxtimes}, \boxtimes, \overset{\smile}{\boxplus}, \boxplus\right\}$ and $* = \{\cap, \cup\}$,

i. $\Psi_X \star \Psi_Y = \Psi_Y \star \Psi_X$
ii. $\Psi_X \star (\Psi_Y \star \Psi_Z) = (\Psi_X \star \Psi_Y) \star \Psi_Z$,
iii. $\Psi_X \star (\Psi_Y * \Psi_Z) = (\Psi_X \star \Psi_Y) * (\Psi_X \star \Psi_Z)$,

Proof. Straightforward.

4. The Proposed Decision-Making Model

In this section, we propose a different decision-making method in expressing uncertainty problems using virtual fuzzy parameterized soft sets. For this, we first define the fuzzy decision sets of a given universe set using the OR-t-norm, AND-t-norm, OR-t-conorm and AND-t-conorm. Then, we propose an algorithm by using the defined fuzzy decision sets.

Definition 4.1. Let $\Psi_X, \Psi_Y \in VFPS(U)$. Then, the OR-fuzzy decision set of $\Psi_X \boxplus \Psi_Y$, denoted $\Xi_{\Psi_X \boxplus \Psi_Y}$, is defined by

$$\Xi_{\Psi_X \boxplus \Psi_Y} = \left\{ \frac{\mu_{\boxplus}(u_j)}{u_j} : u_j \in U \right\}$$

which is a fuzzy set over U, its membership function $\mu_{\boxplus}$ is defined by $\mu_{\boxplus}: U \to [0,1]$,

$$\mu_{\boxplus}(u_j) = \frac{1}{3n} \sum_{\substack{1 \le i \le m \\ 1 \le j \le n}} \begin{bmatrix} \underline{\mu_{X \boxplus Y}}\left(p_i^{\underline{\gamma}}\right) \cdot \chi_{\underline{\psi_{X \boxplus Y}}\left(p_i^{\underline{\gamma}}\right)}(u_j) + \\ \mu_{X \boxplus Y}(p_i) \cdot \chi_{\psi_{X \boxplus Y}(p_i)}(u_j) + \\ \overline{\mu_{X \boxplus Y}}\left(p_i^{\overline{\gamma}}\right) \cdot \chi_{\overline{\psi_{X \boxplus Y}}\left(p_i^{\overline{\gamma}}\right)}(u_j) \end{bmatrix} \tag{37}$$

where

$$\chi_{\underline{\psi_{X \boxplus Y}}\left(p_i^{\underline{\gamma}}\right)}(u_j) = \begin{cases} 1, & u_j \in \underline{\psi_{X \boxplus Y}}\left(p_i^{\underline{\gamma}}\right) \\ 0, & u_j \notin \underline{\psi_{X \boxplus Y}}\left(p_i^{\underline{\gamma}}\right) \end{cases}, \tag{38}$$

$$\chi_{\psi_{X \boxplus Y}(p_i)}(u_j) = \begin{cases} 1, & u_j \in \psi_{X \boxplus Y}(p_i) \\ 0, & u_j \notin \psi_{X \boxplus Y}(p_i) \end{cases} \tag{39}$$

and

$$\chi_{\overline{\psi_{X \boxplus Y}}\left(p_i^{\overline{\gamma}}\right)}(u_j) = \begin{cases} 1, & u_j \in \overline{\psi_{X \boxplus Y}}\left(p_i^{\overline{\gamma}}\right) \\ 0, & u_j \notin \overline{\psi_{X \boxplus Y}}\left(p_i^{\overline{\gamma}}\right) \end{cases}. \tag{40}$$

Here, the fuzzy decision sets $\Xi_{\Psi_X \breve{\boxtimes} \Psi_Y}$, $\Xi_{\Psi_X \boxtimes \Psi_Y}$ and $\Xi_{\Psi_X \breve{\boxplus} \Psi_Y}$ are defined in a similar way.

Next, we construct the algorithm given below for decision-making (i.e., the application of a virtual fuzzy parameterized soft set):

Algorithm

Step 1: Choose fuzzy subsets X and Y over P, (also "fuzzy subsets $\underline{X}$ and $\underline{Y}$ over $\underline{P}$" and "fuzzy subsets $\overline{X}$ and $\overline{Y}$ over $\overline{P}$")

Step 2: Construct the virtual fuzzy parameterized soft sets Ψ_X and Ψ_Y over U,

Step 3: Find the OR-t-conorm $\Psi_X \boxplus \Psi_Y$,

Step 4: Compute the OR-fuzzy decision set $\Xi_{\Psi_X \boxplus \Psi_Y}$.

Step 5: Find r, for which $\mu_{\Xi_{\Psi_X \boxplus \Psi_Y}}(u_r) = max\left\{\mu_{\Xi_{\Psi_X \boxplus \Psi_Y}}(u) : u \in U\right\}$.

Note that, for other defined products, an algorithm can be constructed similar to the algorithm given above.

Now, we show the steps and principle of the above algorithm by using the following example. It should also be noted that the examples to be given for algorithms created by using other products can be expressed in a similar way.

Example 4.1. Suppose a school wants to choose the students that best suit its parameters. For this, the school has posted an announcement. According to the announcement, a three-stage exam will be held for candidate students. Then; the set of candidate students applying for admission to the school is $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9\}$ and the set of parameters the school requires from students is $P = \{p_1, p_2, p_3, p_4\} = \{self-confident,\ successful,\ willing\ to\ learn,\ talented\}$.

Step 1: Suppose that the school administration has selected two experts, X and Y, who are authorized on this subject. The fuzzy sets in which each expert express his opinion on this subject are as follows:

$$\underline{X} = \{0.5/p_1, 0.65/p_3\}, \qquad X = \{0.56/p_1, 0.72/p_3\}, \qquad \overline{X} = \{0.76/p_1, 0.84/p_3\}$$

and

$$\underline{Y} = \{0.45/p_3, 0.36/p_4\}, \qquad Y = \{0.62/p_3, 0.5/p_4\}, \qquad \overline{Y} = \{0.75/p_3, 0.67/p_4\}$$

over $\underline{P}$, P, $\overline{P}$, respectively.

Step 2: The results obtained based on the opinions of the experts are expressed as virtual fuzzy parameterized soft sets Ψ_X and Ψ_Y over U as follows:

$$\Psi_X = \begin{Bmatrix} (0.5/p_1, \{u_2, u_4, u_5, u_7, u_9\}), (0.65/p_3, \{u_1, u_4, u_6, u_7, u_8\}) \\ (0.56/p_1, \{u_2, u_5, u_7, u_9\}), (0.72/p_3, \{u_1, u_4, u_6, u_8\}) \\ (0.76/p_1, \{u_2, u_7, u_9\}), (0.84/p_3, \{u_1, u_4, u_8\}) \end{Bmatrix}$$

$$\Psi_Y = \begin{Bmatrix} (0.45/p_3, \{u_1, u_2, u_4, u_5, u_7\}), (0.36/p_4, \{u_1, u_3, u_4, u_5, u_6\}) \\ (0.62/p_3, \{u_2, u_4, u_5, u_7\}), (0.5/p_4, \{u_3, u_4, u_5, u_6\}) \\ (0.75/p_3, \{u_2, u_4, u_5\}), (0.67/p_4, \{u_4, u_5, u_6\}) \end{Bmatrix}.$$

Step 3: The OR-t-conorm of Ψ_X and Ψ_Y is formed as,

$$\Psi_X \boxplus \Psi_Y = \begin{Bmatrix} (0.5/p_1, \{u_2, u_4, u_5, u_7, u_9\}), (0.8075/p_3, \{u_1, u_2, u_4, u_5, u_6, u_7, u_8\}), \\ (0.36/p_4, \{u_1, u_3, u_4, u_5, u_6\}), (0.56/p_1, \{u_2, u_5, u_7, u_9\}), \\ (0.8936/p_3, \{u_1, u_2, u_4, u_5, u_6, u_7, u_8\}), (0.5/p_4, \{u_3, u_4, u_5, u_6\}), \\ (0.76/p_1, \{u_2, u_7, u_9\}), (0.96/p_3, \{u_1, u_2, u_4, u_5, u_8\}), (0.67/p_4, \{u_4, u_5, u_6\}) \end{Bmatrix}$$

Step 4: The OR-fuzzy decision set of $\Psi_X \boxplus \Psi_Y$ is computed as

$$\Xi_{\Psi_X \boxplus \Psi_Y} = \begin{Bmatrix} \frac{0.112}{u_1}, \frac{0.166}{u_2}, \frac{0.032}{u_3}, \frac{0.174}{u_4}, \frac{0.194}{u_5} \\ \frac{0.120}{u_6}, \frac{0.130}{u_7}, \frac{0.099}{u_8}, \frac{0.067}{u_9} \end{Bmatrix}$$

For example, the value $\mu_{\boxplus}(u_7)$ for u_7 is calculated as follows:

$$\mu_{\boxplus}(u_7) = \frac{1}{3 \cdot 9} \sum_{\substack{1 \le i \le 4 \\ 1 \le j \le 9}} [0.5 + 0.8075 + 0 + 0.56 + 0.8936 + 0 + 0.76 + 0 + 0] = \frac{3.5211}{27} \cong 0.130$$

Step 5: We conclude from the values of u that $\mu_{\Xi_{\Psi_X \boxplus \Psi_Y}}(u_5) = max\left\{\mu_{\Xi_{\Psi_X \boxplus \Psi_Y}}(u): u \in U\right\} = 0.194$ and hence $r = 5$. Thus u_5 is the optimal choice candidate and so u_5 is the most suitable student candidate for the desired parameters.

5. Conclusion

To date, many hybrid sets have been constructed by considering fuzzy sets and soft sets. However, these sets have not questioned the margin of error of the decision-maker. For this reason, it is usual to encounter certain problems in solving uncertainty problems. This paper has focused on virtual fuzzy parameterized soft sets, which is the first mathematical model that can detect a possible margin of error in the data expressed by decision-makers. Moreover, the present study has studied the algebraic operations of VFPSSs. Afterward, a decision-making method has been proposed. We think that the approach given in this study can be instrumental in expressing with virtual fuzzy parameterized soft sets, which is a new mathematical model, the uncertainty problems encountered in fields such as current life state, computer science, decision making, etc.

Conflicts of interest

The author state that did not have conflict of interests.

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