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Quasi ideals of nearness semirings

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Abstract

This article introduces quasi-ideals in semirings on weak nearness approximation spaces. Concepts and definitions are given to clarify the subject of quasi ideals in semirings on weak nearness approximation spaces. Some basic properties of quasi ideals are also given. Furthermore, it is given that the definition of upper-near quasi ideals. And, it is examined that the relationship between quasi ideals and upper near quasi ideals. Therefore, the features described in this study will contribute greatly to the theoretical development of the nearness semirings theory.

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1. Introduction

Peters defined near sets theory that is a generalization of rough sets [1]. Peters introduced new form of indiscernibility relation by using the characteristics of the objects to find the nearness of the objects [2]. Afterward, he generalized approaching theory in the study of the nearness of non-empty sets which are similar to each other [3], [4]. İnan and Öztürk introduced the notion of nearness groups and nearness semigroups [5], [6], [7]. Also, other approaches have been studied in [8], [9], [10], [11], [12], [13].

Vandier introduced the concept of semiring theory in 1934 [14] and many mathematicians proved important properties for semiring theory. Especially, semirings are very important for determinants and matrices. One of the most important notions for semirings is ideals. Shabir et al. [15] defined ideals for semirings. The subject of quasi ideals for semigroups and rings was formally defined by Steinfeld in 1956 [16]. Iseki defined the concept of quasi- ideal for a semiring [17]. Rao introduced other types of ideals and their properties for semirings [18], [19].
In this article, quasi ideals in semirings are defined and some of the concepts and definitions on weak nearness approximation spaces are explained. Then, we study some basic properties of quasi ideals.

2. Preliminaries

An object characterization is specified by means a tuple of function values $\Phi(x)$ deal with an $x \in X$. $B \subseteq \mathcal{F}$ is a set of probe functions and these functions stand for characteristics of sampling objects $X \subseteq \mathcal{O}$. Let $\varphi_i \in B$, that is $\varphi_i : \mathcal{O} \to \mathbb{R}$. The functions showing object characteristics supply a basis for $\Phi: \mathcal{O} \to \mathbb{R}^L, \Phi(x) = (\varphi_1(x), \varphi_2(x), \dots, \varphi_L(x))$ a vector consisting of measurements deal with every functional value $\varphi_i(x)$ with the description length $|\Phi| = L$ ([2]).
The selection of functions $\varphi_i \in B$ is very fundamental by using to determine sampling objects. $X \subseteq \mathcal{O}$ are near each other if and only if the sample items have alike characterizations. Every φ shows a descriptive pattern of an object. Hence, $\triangle_{\varphi_i}$ means $\triangle_{\varphi_i} = |\varphi_i(x) - \varphi_i(x')|$ such that $x, x' \in \mathcal{O}$. $\triangle_{\varphi_i}$ means to a description of the indiscernibility relation "$\sim_B$" defined by Peters in [2]. B_r is probe functions in B for $r \leq |B|$.

Definition 2.1 [2]

$$\sim_B = \{(x, x') \in \mathcal{O} \times \mathcal{O} \mid \triangle_{\varphi_i} = 0, \forall \varphi_i \in B, B \subseteq \mathcal{F}\}\}$$

means an indiscernibility relation on $\mathcal{O}$ with $i \leq |\Phi|$. $\sim_{B_r}$ is also indiscernibility relation determined by utilizing B_r.

Near equivalence class is stated as $[x]_{B_r} = \{x' \in \mathcal{O} \mid x \sim_{B_r} x'\}$. After getting near equivalence classes, quotient set $\mathcal{O}/\sim_{B_r} = \{[x]_{B_r} \mid x \in \mathcal{O}\} = \xi_{\mathcal{O},B_r}$ and set of partitions $N_r(B) = \{\xi_{\mathcal{O},B_r} \mid B_r \subseteq B\}$ can be found. By using near equivalence classes, $N_r(B)^*X = \bigcup_{[x]_{B_r} \cap X \neq \emptyset} [x]_{B_r}$ upper approximation set can be attained.

Definition 2.1 [9] Let $\mathcal{O}$ be a set of sample objects, F a set of the probe functions, $\sim_{B_r}$ an indiscernibility relation, and $N_r(B)$ a collection of partitions. Then,

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$(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$ is called a weak nearness approximation space.

Theorem 2.1 [9] Let $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$ be a weak nearness approximation space and $X, Y \subset \mathcal{O}$. The followings are held:

1) $X \subseteq N_r(B)^*X$,

2) $N_r(B)^*(X \cup Y) = (N_r(B)^*X) \cup (N_r(B)^*Y)$,

3) $X \subseteq Y$ implies $N_r(B)^*X \subseteq N_r(B)^*Y$,

4) $N_r(B)^*(X \cap Y) \subseteq (N_r(B)^*X) \cap (N_r(B)^*Y)$.

After this, $\mathcal{O}$ means a $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$ is weak near approximation spaces unless otherwise stated.

Definition 2.3 [13] If S is a nearness semigroup and there exists an $e \in N_r(B)^*S$ satisfying $x \cdot e = x = e \cdot x$ for all $x \in S$, then $(S,\cdot)$ is called a nearness monoid. If $x \cdot y = y \cdot x$ ($x + y = y + x$) for all $x, y \in S$, then $(S,\cdot)$ ($(S,+)$) is called a commutative (abelian).

Definition 2.4 [13] Let $S \in \mathcal{O}$. Then, S is called a semiring on weak near approximation spaces $\mathcal{O}$ if the following properties hold:
SR_1) $(S,+)$ is an abelian monoid on $\mathcal{O}$ with identity element 0,
NSR_2) $(S,\cdot)$ is a monoid on $\mathcal{O}$ with identity element 1,
NSR_3) for all $x, y, z \in S$,
$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$ and $(x + y) = (x \cdot z) + (y \cdot z)$
properties hold in $N_r(B)^*S$,
NSR_4) for all $x \in S$, $0 \cdot x = 0 = x \cdot 0$
properties hold in $N_r(B)^*$,
NSR_5) $1 \neq 0$.

Theorem 2.2 [13] Let S be a nearness semiring, $\sim_{B_r}$ a complete congruence indiscernibility relation on S, and X, Y two non-empty subsets of S. The following properties hold:

1) $(N_r(B)^*X) + (N_r(B)^*Y) = N_r(B)^*(X + Y)$,
2) $(N_r(B)^*X) \cdot (N_r(B)^*Y) = N_r(B)^*(X \cdot Y)$.

Definition 2.5 [13] Let S be a nearness semiring, and A is a non-empty subset of S.

1) A is called a subnearness semiring of S, if $A + A \subseteq N_r(B)^*A$ and $A \cdot A \subseteq N_r(B)^*A$.
2) A is called a upper-near subnearness semiring of S, if $(N_r(B)^*A) + (N_r(B)^*A) \subseteq N_r(B)^*A$ and $(N_r(B)^*A) \cdot (N_r(B)^*A) \subseteq N_r(B)^*A$.

Definition 2.6 [13] Let S be a nearness semiring, and A be a subnearness semigroup of S, where $A \neq S$.

1) A is called a right (left) ideal of S, if $A \cdot S \subseteq N_r(B)^*A$ ($S \cdot A \subseteq N_r(B)^*A$).
2) A is called a upper-near right (left) ideal of S, if $(N_r(B)^*A) \cdot S \subseteq N_r(B)^*A$ ($S \cdot (N_r(B)^*A) \subseteq N_r(B)^*A$).

Definition 2.7 [20] Let S be a semiring and A be a non-empty subset of semiring S, where $A \neq S$.
1) If A is a subsemigroup of S, $AS \subseteq A$ and $SA \subseteq A$, then A is called an ideal of S.
2) If A is a subsemigroup of S and $AS \cap SA \subseteq A$, then A is called a quasi-ideal of S.

3. Quasi Ideals of Nearness Semirings

Definition 3.1 Let S be a nearness semiring and Q be a non-empty subset of S, where $Q \neq S$.

1) Q is called quasi-ideal of S if Q is a subnearness semigroup of S and $QS \cap SQ \subseteq N_r(B)^*Q$.
2) Q is called a upper-near quasi ideal of S if Q is a subnearness semigroup of S and $(N_r(B)^*Q)S \cap S(N_r(B)^*Q) \subseteq N_r(B)^*Q$.

Example 3.1

Let $\mathcal{O} = \{a, b, c, d, e, f, g, h, j, k, l, m, n\}$ be a set of perceptual objects where

$$a = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, c = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, d = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},$$
$$e = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, f = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, g = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, h = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix},$$
$$j = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, k = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, l = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, m = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$
$$n = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$$

for $\mathcal{O} = \{[a_{ij}]_{2x2} | a_{ij} \in \mathbb{Z}_3\}, r = 1,\ B, \{\varphi_1, \varphi_2, \varphi_3\} \subseteq \mathcal{F}$ is a set of probe functions. Let $S = \{c, d, e, f\} \subset \mathcal{O}$. Probe functions' values

$$\varphi_1 : \mathcal{O} \to V_1 = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\},$$
$$\varphi_2 : \mathcal{O} \to V_2 = \{\alpha_2, \alpha_3, \alpha_5, \alpha_6, \alpha_7\},$$
$$\varphi_3 : \mathcal{O} \to V_3 = \{\alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$$

are presented in Table 1.

Table 1: Features' Table

	a	b	c	d	e	f	g	h	j	k	l	m	n
φ_1	α_1	α_2	α_4	α_5	α_4	α_5	α_3	α_4	α_5	α_1	α_2	α_1	α_3
φ_2	α_3	α_5	α_6	α_3	α_3	α_2	α_6	α_6	α_6	α_5	α_5	α_7	α_7
φ_3	α_2	α_3	α_2	α_4	α_3	α_2	α_4	α_5	α_6	α_5	α_5	α_6	α_6

Now, we find the nearness equivalence classes as follows;

$[a]_{\varphi_1} = \{x' \in \mathcal{O} | \varphi_1(x') = \varphi_1(a) = \alpha_1\} = \{a, k, m\} = [k]_{\varphi_1} = [m]_{\varphi_1}$,

$[b]_{\varphi_1} = \{x' \in \mathcal{O} | \varphi_1(x') = \varphi_1(b) = \alpha_2\} = \{b, l\} = [l]_{\varphi_1}$,

$[c]_{\varphi_1} = \{x' \in \mathcal{O} | \varphi_1(x') = \varphi_1(c) = \alpha_4\} = \{c, e, h\} = [e]_{\varphi_1} = [h]_{\varphi_1}$,

$[d]_{\varphi_1} = \{x' \in \mathcal{O} | \varphi_1(x') = \varphi_1(d) = \alpha_5\} = \{d, f, j\}, = [f]_{\varphi_1} = [j]_{\varphi_1}$,

$[g]_{\varphi_1} = \{x' \in \mathcal{O} | \varphi_1(x') = \varphi_1(g) = \alpha_3\} = \{g, n\}, = [n]_{\varphi_1}$.

Then, we have that $\xi_{\varphi_1} = \{[a]_{\varphi_1}, [b]_{\varphi_1}, [c]_{\varphi_1}, [d]_{\varphi_1}, [g]_{\varphi_1}\}$.

$[a]_{\varphi_2} = \{x' \in \mathcal{O} | \varphi_2(x') = \varphi_2(a) = \alpha_3\} = \{a, d, e\} = [d]_{\varphi_2} = [e]_{\varphi_2}$,

$[b]_{\varphi_2} = \{x' \in \mathcal{O} | \varphi_2(x') = \varphi_2(b) = \alpha_5\} = \{b, k, l\} = [k]_{\varphi_2} = [l]_{\varphi_2}$,

$[c]_{\varphi_2} = \{x' \in \mathcal{O} | \varphi_2(x') = \varphi_2(c) = \alpha_6\} = \{c, g, h, j\} = [g]_{\varphi_2} = [h]_{\varphi_2} = [j]_{\varphi_2}$,

$[f]_{\varphi_2} = \{x' \in \mathcal{O} | \varphi_2(x') = \varphi_2(f) = \alpha_2\} = \{f\}$,

$[m]_{\varphi_2} = \{x' \in \mathcal{O} | \varphi_2(x') = \varphi_2(m) = \alpha_7\} = \{m, n\} = [n]_{\varphi_2}$.

We attain that $\xi_{\varphi_2} = \{[a]_{\varphi_2}, [b]_{\varphi_2}, [c]_{\varphi_2}, [f]_{\varphi_2}, [m]_{\varphi_2}\}$.

$[a]_{\varphi_3} = \{x' \in \mathcal{O} | \varphi_3(x') = \varphi_3(a) = \alpha_2\} = \{a, c, f\} = [c]_{\varphi_3} = [f]_{\varphi_3}$,

$[b]_{\varphi_3} = \{x' \in \mathcal{O} | \varphi_3(x') = \varphi_3(b) = \alpha_3\} = \{b, e\} = [e]_{\varphi_3}$,

$[d]_{\varphi_3} = \{x' \in \mathcal{O} | \varphi_3(x') = \varphi_3(d) = \alpha_4\} = \{d, g\} = [g]_{\varphi_3}$,

$[h]_{\varphi_3} = \{x' \in \mathcal{O} | \varphi_3(x') = \varphi_3(h) = \alpha_5\} = \{h, k, l\} = [k]_{\varphi_3} = [l]_{\varphi_3}$,

$[j]_{\varphi_3} = \{x' \in \mathcal{O} | \varphi_3(x') = \varphi_3(j) = \alpha_6\} = \{j, m, n\} = [m]_{\varphi_3} = [n]_{\varphi_3}$.

From here, we get that $\xi_{\varphi_3} = \{[a]_{\varphi_3}, [b]_{\varphi_3}, [d]_{\varphi_3}, [h]_{\varphi_3}, [j]_{\varphi_3}\}$. Consequently, a set of partitions of $\mathcal{O}$ is $N_r(B) = \{\xi_{\varphi_1}, \xi_{\varphi_2}, \xi_{\varphi_3}\}$ for $r = 1$.

Hence,

$N_1(B)^* S = \bigcup_{[x]_{\varphi_i} \cap S \neq \emptyset} [x]_{\varphi_i}$

$= [c]_{\varphi_1} \cup [d]_{\varphi_1} \cup [a]_{\varphi_2} \cup [c]_{\varphi_2} \cup [f]_{\varphi_2} \cup [a]_{\varphi_3} \cup [b]_{\varphi_3} \cup [d]_{\varphi_3}$
$= \{a, b, c, d, e, f, g, h, j\}$.

Taking operation tables for S in Table 2 and Table 3.

Table 2: "+" operation table for S.

$+$	c	d	e	f
c	b	j	f	h
d	j	e	a	c
e	f	a	d	j
f	h	c	j	g

Table 3: "·" operation table for S.

$\cdot$	c	d	e	f
c	b	e	d	g
d	e	b	c	f
e	d	c	b	g
f	g	f	g	f

In this case, $(S, +, \cdot)$ is a nearness semiring. Let take $Q = \{d, e, f\}$ is subset of S. Considering operation tables for Q in Table 4 and Table 5.

Table 4: "+" operation table for .

$+$	d	e	f
d	e	a	c
e	a	d	j
f	c	j	g

Table 5: "+" operation table for Q.

$\cdot$	d	e	f
d	b	c	f
e	c	b	g
f	f	g	f

$N_1(B)^*Q = \bigcup_{[x]_{\varphi_i} \cap Q \neq \emptyset} [x]_{\varphi_i}$

$= [c]_{\varphi_1} \cup [d]_{\varphi_1} \cup [a]_{\varphi_2} \cup [f]_{\varphi_2} \cup [a]_{\varphi_3} \cup [b]_{\varphi_3} \cup [d]_{\varphi_3}$

$= \{a, b, c, d, e, f, g, h, j\}$.

Since Q is a subnearness semiring of S and $QS \cap SQ \subseteq N_r(B)^*Q$, Q is quasi-ideal of S.

Lemma 3.1 Let S be a nearness semiring. If S is commutative, then each quasi-ideal of S is two-sided ideal of S.

Proof Let S be a commutative nearness semiring and Q be a quasi-ideal of S. Then $QS \cap SQ \subseteq N_r(B)^*Q$. Moreover, S is commutative and $Q \subseteq S$, then $SQ = QS$, and so $SQ \subseteq N_r(B)^*Q$. Therefore, Q is a left ideal of semiring S. Similarly, Q is right ideal of S. Hence, each quasi-ideal of S is a two-sided ideal of S.

Example 3.2 Let $\mathcal{O} = \{a, b, c, d, e, f, g, h, m\}$ be a set of perceptual objects where for $r = 1$, $B = \{\varphi_1, \varphi_2\} \subseteq \mathcal{F}$ be a set of probe functions. Let $S = \{a, b, c, d\} \subset \mathcal{O}$. Here are some sample probe functions values

$\varphi_1: \mathcal{O} \to V_1 = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$,
$\varphi_2: \mathcal{O} \to V_2 = \{\alpha_1, \alpha_3, \alpha_4, \alpha_5, \alpha_7\}$

are presented in Table 6.

Table 6: Features' Table

	a	b	c	d	e	f	g	h	m
φ_1	α_1	α_2	α_2	α_1	α_3	α_4	α_3	α_3	α_4
φ_2	α_3	α_7	α_7	α_1	α_3	α_1	α_4	α_5	α_5

Now, we find the nearness equivalence classes according to the indiscernibility relation $\sim_{B_r}$ of elements in $\mathcal{O}$:

$[a]_{\varphi_1} = \{x' \in \mathcal{O} | \varphi_1(x') = \varphi_1(a) = \alpha_1\} = \{a, d\}$

$= [d]_{\varphi_1}$,

$[b]_{\varphi_1} = \{x' \in \mathcal{O} | \varphi_1(x') = \varphi_1(b) = \alpha_2\} = \{b, c\}$

$= [c]_{\varphi_1}$,

$[e]_{\varphi_1} = \{x' \in \mathcal{O} | \varphi_1(x') = \varphi_1(e) = \alpha_3\} = \{e, g, h\}$

$= [g]_{\varphi_1} = [h]_{\varphi_1}$,

$[f]_{\varphi_1} = \{x' \in \mathcal{O} | \varphi_1(x') = \varphi_1(f) = \alpha_5\} = \{f, m\}$,

$= [m]_{\varphi_1}$.

Then, we have that $\xi_{\varphi_1} = \{[a]_{\varphi_1}, [b]_{\varphi_1}, [e]_{\varphi_1}, [f]_{\varphi_1}\}$.

$[a]_{\varphi_2} = \{x' \in \mathcal{O} | \varphi_2(x') = \varphi_2(a) = \alpha_3\} = \{a, e\}$

$= [e]_{\varphi_2}$,

$[b]_{\varphi_2} = \{x' \in \mathcal{O} | \varphi_2(x') = \varphi_2(b) = \alpha_7\} = \{b, c\}$

$= [c]_{\varphi_2}$,

$[d]_{\varphi_2} = \{x' \in \mathcal{O} | \varphi_2(x') = \varphi_2(d) = \alpha_1\} = \{d, f\}$

$= [f]_{\varphi_2}$,

$[g]_{\varphi_2} = \{x' \in \mathcal{O} | \varphi_2(x') = \varphi_2(g) = \alpha_4\} = \{g\}$,

$[h]_{\varphi_2} = \{x' \in \mathcal{O} | \varphi_2(x') = \varphi_2(h) = \alpha_5\} = \{h, m\}$

$= [m]_{\varphi_2}$.

We attain that

$\xi_{\varphi_2} = \{[a]_{\varphi_2}, [b]_{\varphi_2}, [d]_{\varphi_2}, [g]_{\varphi_2}, [h]_{\varphi_2}\}$.

Consequently, we obtain that a set of partitions of $\mathcal{O}$ is

$N_r(B) = \{\xi_{\varphi_1}, \xi_{\varphi_2}\}$ for $r = 1$. Hence,

$N_1(B)^*S = \bigcup_{[x]_{\varphi_i} \cap S \neq \emptyset} [x]_{\varphi_i}$

$= [a]_{\varphi_1} \cup [b]_{\varphi_1} \cup [a]_{\varphi_2} \cup [b]_{\varphi_2} \cup [d]_{\varphi_2}$

$= \{a, b, c, d, e, f\}$.

Taking operation tables for S in Table 7 and Table 8.

Table 7: "+" operation table for S.

+	***a***	***b***	***c***	***d***
a	a	b	c	d
b	b	c	d	e
c	c	d	e	f
d	d	e	f	a

Table 8: "$\cdot$" operation table for S.

$\cdot$	**a**	**b**	**c**	**d**
a	b	d	f	a
b	d	a	e	b
c	f	e	d	c
d	a	b	c	d

In this case, $(S,+,\cdot)$ is a nearness semiring. Let take $Q=\{b,c,d\}$ is subset of S. Let's find $N_1(B)^*Q$:

$$N_1(B)^*Q = \bigcup_{[x]_{\varphi_i}\cap Q\neq\emptyset}[x]_{\varphi_i}$$
$$= [a]_{\varphi_1}\cup[b]_{\varphi_1}\cup[b]_{\varphi_2}\cup[d]_{\varphi_2}$$
$$=\{a,b,c,d,f\}.$$

Hence, it can be easily seen that Q is not quasi-ideal of nearness semiring S. Since $e\in QS\cap SQ$, but $e\notin N_r(B)^*Q$.

Proposition 3.1 Let S be a nearness semiring. Each one or two-sided ideal of S is a quasi-ideal of S.

Proof. Assume that Q is left ideal of S. From Definition 3.1. (1), $SQ\subseteq N_r(B)^*Q$. Then, $QS\cap SQ\subseteq SQ\subseteq N_r(B)^*Q$. We get $QS\cap SQ\subseteq N_r(B)^*Q$, and so Q is a quasi-ideal of S. Similarly, we can show that if Q is a right ideal of S, then Q is a quasi-ideal of S. $QS\subseteq N_r(B)^*Q$. Thus, $QS\cap SQ\subseteq QS\subseteq N_r(B)^*Q$. Hence, Q is a quasi-ideal of S.

Theorem 3.1 Let S be a nearness semiring and $\{Q_i|i\in I\}$ be set of quasi-ideals of the nearness semiring S where I is index set. If $N_r(B)^*\left(\bigcap_{i\in I}Q_i\right)=\bigcap_{i\in I}N_r(B)^*Q_i$, then $\bigcap_{i\in I}Q_i=\emptyset$ or $\bigcap_{i\in I}Q_i$ is a quasi-ideal of S.

Proof. Let $\bigcap_{i\in I}Q_i=Q$. Let show that Q is either empty or a quasi-ideal of S. Assume that Q is non-empty. Since Q_i is quasi-ideals of S for $i\in I$. We get that $Q_iS\cap SQ_i\subseteq N_r(B)^*Q_i$ for all $i\in I$.

$$SQ=S(\bigcap_{i\in I}Q_i)=\bigcap_{i\in I}(SQ_i)\subseteq SQ_i$$
and
$$QS=(\bigcap_{i\in I}Q_i)S=\bigcap_{i\in I}(Q_iS)\subseteq Q_iS$$

Then, we obtain

$$QS\cap SQ\subseteq Q_iS\cap SQ_i\subseteq N_r(B)^*Q_i,\qquad \forall i\in I.$$

Therefore, we have that $QS\cap SQ\subseteq N_r(B)^*Q$. Hence, Q is a quasi-ideal of S.

Lemma 3.2 Let S be a nearness semiring, L be a left ideal and R be a right ideal of S. If $(N_r(B)^*R)\cap(N_r(B)^*L)\subseteq N_r(B)^*(R\cap L)$, then

1) $RL\subseteq N_r(B)^*(L\cap R)$.
2) $Q=L\cap R$ is a quasi-ideal of S.

Proof. 1) Let L be left ideal and R be right ideal of S. Since $R\subseteq S$ and L is left ideal of S, $RL\subseteq SL\subseteq N_r(B)^*L$.

Similarly, since $L\subseteq S$ and R is right ideal of S, $RL\subseteq RS\subseteq N_r(B)^*R$. Then, considering these $RL\subseteq(N_r(B)^*R)\cap(N_r(B)^*L)$. Hence, we get $RL\subseteq N_r(B)^*(L\cap R)$ by the hypothesis.

2) Let show that $QS\cap SQ\subseteq N_r(B)^*Q$. Since L is a left ideal and R is a right ideal of S,
$SQ=S(L\cap R)=SL\cap SR\subseteq SL\subseteq N_r(B)^*L$
and
$QS=(L\cap R)S=LS\cap RS\subseteq RS\subseteq N_r(B)^*R.$
Thus,
$QS\cap SQ\subseteq(N_r(B)^*L)\cap(N_r(B)^*R)\subseteq N_r(B)^*(L\cap R)=N_r(B)^*Q$ by the hypothesis.

Theorem 3.2 Let Q be quasi ideal of nearness semiring S such that $N_r(B)^*(N_r(B)^*Q)=N_r(B)^*Q$. If S is commutative, then Q is an upper-near quasi ideal of S.

Proof. $(N_r(B)^*Q)S\cap S(N_r(B)^*Q)\subseteq(N_r(B)^*Q)(N_r(B)^*S)\cap(N_r(B)^*S)(N_r(B)^*Q)$ by Theorem 2.1.(1) . From Theorem 2.2. (2) we get that $(N_r(B)^*Q)S\cap S(N_r(B)^*Q)\subseteq N_r(B)^*(QS)\cap N_r(B)^*(SQ)$. Since $QS\subseteq N_r(B)^*Q$ and $SQ\subseteq N_r(B)^*Q$ by Lemma 3.1, we have $(N_r(B)^*Q)S\cap S(N_r(B)^*Q)\subseteq(N_r(B)^*(N_r(B)^*Q))\cap(N_r(B)^*(N_r(B)^*Q))$.
Thus, we obtain $(N_r(B)^*Q)S\cap S(N_r(B)^*Q)\subseteq N_r(B)^*Q$ by the hypothesis.

Conflicts of interest

The authors state that did not have a conflict of interests

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