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Description of even-even Ti isotopes within IBM-1 model

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Abstract

In this work, the some collective properties of even-even Ti isotopes in the A~50 mass region were studied by using the interacting boson model-1 (IBM-1). This study includes calculations of the energy levels and the electromagnetic transition rates of $^{44\text{-}48}$Ti and $^{52\text{-}60}$Ti isotopes. The neutron number ^{50}Ti isotope is 28, the magic number, and this isotope was excluded from the IBM-1 calculations. First, the energy ratios were analyzed by comparing the typical values of U(5), SU(3), O(6) dynamical symmetries and E(5), X(5) critical point symmetries. Later model Hamiltonian was constructed according to the behavior of given isotopes. The low lying energy levels of each Ti isotopes were calculated by using the fitted parameters of Hamiltonian. B(E2) values were also calculated by using the corresponding electromagnetic transition operator in the IBM-1. The calculation results were compared with experimental data and they are in good agreement with each other. Finally, $\mathrm{R}_{4/2}=\mathrm{E}(4_1^+)/\mathrm{E}(2_1^+)$, $\mathrm{R}_{0/2}=\mathrm{E}(0_2^+)/\mathrm{E}(2_1^+)$ energy ratios and $\mathrm{B}(\mathrm{E2}: 4_1^+ \to 2_1^+)/\mathrm{B}(\mathrm{E2}: 2_1^+ \to 0_1^+)$, $\mathrm{B}(\mathrm{E2}: 0_2^+ \to 2_1^+)/\mathrm{B}(\mathrm{E2}: 2_1^+ \to 0_1^+)$ ratios were analized to see the behavior of given isotopes. According to obtained results, Ti isotopes show E(5) behavior along the transition path from spherical to deformed γ-unstable region. Overall analysis indicates that these isotopes can be good example for the quantum shape phase studies along the isotopic chain.

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1. Introduction

In nuclear physics studies, nuclear models [1] were developed to understand the complex structure of nucleus consisting of protons and neutrons. One of them is the interacting boson model-1 (IBM-1) [2] introduced to explain the nuclear collective structure of even-even nuclei. The IBM-1 is a group theoretical model and quite successful for the phenomenologically description of the collective properties of nuclei, especially for the medium and heavy ones. This model is still actively used along the isotopic/isotonic chains and different mass regions of the nuclear valley.

During the last decades, the nuclei in A~50 mass region were investigated as reported in Refs. [3-9]. The shape phase transition was studied for the light nuclei with number of proton Z=20–28 including also Ti isotopes [3]. The fully-microscopic shell-model (SM) were used to calculate the first 2^+ states of Ti isotopes and their B(E2) transition rates along the isotopic chain [4]. Excitation 2^+, 4^+ energy states, B(E2) values, and Q(2^+) of even-even Ca, Ti, Cr isotopes were calculated in the generalized seniority with realistic interactions and compared with results of full SM calculations performed by using FPD6 and GXPF1 interactions [5]. The shape phase transition was studied for $^{38\text{-}66}$Ti isotopes within the Hartree-Fock-Bogoliubov (HFB) approach and ^{46,52,60}Ti isotopes show E(5) behavior [6]. Another SM calculations were performed by using the same interactions for N=Z isotopes [7]. SM was used for the systematic calculations of first 2^+ levels and B(E2) transitions of the pf-shell nuclei including also even-even Ti isotopes [8]. Excitation energies of $J^{\pi}=2^+$ states, two-neutron separation energies (S_{2n}), and B(E2) values along the isotopic chains of the fp shell nuclei were analyzed within the SM [9].

More recently, the IBM-1 calculations were applied to neighbours of Ti nuclei, e.g. even-even Cr [10, 11] and Fe nuclei [12] along the isotopic chain. In this study, the structural properties of even-even $^{44\text{-}48}$Ti and $^{52\text{-}60}$Ti isotopes in A~50 mass region were investigated by using the IBM-1 model [2]. Our aim is to calculate their low-lying energy levels by fitting the parameters of the model Hamiltonian and B(E2) transition rate, and also to understand their structural behavior by applying to the analysis of the energy ratios; $\mathrm{R}_{4/2}=$

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$E(4_1^+)/E(2_1^+)$, $R_{0/2} = E(0_2^+)/E(2_1^+)$ and the B(E2) ratios; $B(E2: 4_1^+ \to 2_1^+)/B(E2: 2_1^+ \to 0_1^+)$, $B(E2: 0_2^+ \to 2_1^+)/B(E2: 2_1^+ \to 0_1^+)$. The results of the model calculations are compared with experimental data [13] and the mentioned ratios are analyzed by comparing typical values of dynamical symmetries and that of the critical point symmetries.

This paper is structured as follows; in first section, the introduction is presented to give brief information about this work and the some recent studies. In the second section, the IBM-1 model is explained with its Hamiltonian and other formulations related the calculations. Later, the obtained results are discussed in third section. Finally, the conclusion are summarized in the last section.

2. Materials and Methods

For the present application, we used the IBM-1 model making no distinguish between neutron and proton, they are taken into account as nucleons. This basic version is also called sd-IBM model since using an interacting system of s and d bosons. In the following subsections, the IBM-1 model is explained with its Hamiltonian used for the calculations of energy levels and other formulations related the calculations of B(E2) values.

2.1. Interacting boson model-1 (IBM-1)

The IBM-1 model was first proposed by Feshbach and Iachello in the 1970s and later developed by Arima and Iachello [2]. This model is still widely used and very successful for the investigation of even-even nuclei, especially in deformed regions. The basic idea of this model tries to describe the low-lying collective states of the given nucleus with a system of s- and d- bosons of angular momenta J=0 and J=2, respectively. This algebraic model is established on the U(6) unitary group since s and d bosons have six magnetic moments in total, one is μ=0 of the angular momentum J=0 and other five magnetic moments are μ=0,±1, ±2 of J=2. The U(6) group has three possible subgroups U(5), SU(3), O(6) called as dynamical symmetries and these symmetries correspond to the spherical, the axially deformed (prolate/oblate) and deformed γ-unstable nuclei, respectively [2]. The other concept is the critical point symmetries [14] in relation to the structural behavior of nuclei, introduced by Iachello [15, 16]. The X(5) and E(5) symmetries exist in between U(5)–SU(3) and U(5)–O(6) symmetries, respectively. The X(5) critical point symmetry [15] describe the first-order phase transition appearing along the path from spherical to axially deformed (prolate/oblate) shapes. The E(5) symmetry [16,17] corresponds to a system undergoing a second-order phase transition appear in between the spherical and deformed γ-unstable shapes.

2.2. Model Hamiltonian

To calculate the nuclear collective properties of given nucleus with a system formed by the interaction of N bosons, one can constructs the model Hamiltonian in terms of the boson operators. The general Hamiltonian of the IBM-1 can also be reconstructed as a combination of the linear and quadratic Casimir invariants of corresponding subgroups of U(6) [2]. For the present application, we used another Hamiltonian in multipole expansion form given as following [18];

$$\hat{H} = \varepsilon'' \hat{n}_d + a_0 \hat{P}^\dagger \hat{P} + a_1 \hat{L}^2 + a_2 \hat{Q}^2 + a_3 \hat{T}_3^2 + a_4 \hat{T}_4^2 \quad (1)$$

where constants given by ε'', a_0, a_1, a_2, a_3, and a_4 are free parameters fitted from experimental data. The $\hat{n}_d$, $\hat{P}$, $\hat{L}$, $\hat{Q}$, $\hat{T}_3$, and $\hat{T}_4$ operators are called the boson-number, the pairing, the angular momentum, the quadrupole, the octupole, and the hexadecapole operators, respectively. These operators can be defined in terms of the creation and annihilation operators of the s and d bosons as [18];

$$\hat{T}_l = (d^\dagger \tilde{d})^{(l)} \;;\; (l = 0,1,2,3,4)\,,\; \hat{n}_d = \sqrt{5}\hat{T}_0\,,\;\; \hat{L} = \sqrt{10}\hat{T}_1$$

$$\hat{P} = \frac{1}{2}(\tilde{d}^2 - s^2)\,,\; \hat{Q} = (d^\dagger s + s^\dagger \tilde{d}) + \chi (d^\dagger \tilde{d})^{(2)} \quad (2)$$

As seen, these operators acts in boson states, not in fermion spaces. The constant χ given in quadrupole operator ($\hat{Q}$) is another free parameter. This parameter changes in between $-\sqrt{7}/2$ and $+\sqrt{7}/2$ to move from prolate side to oblate side, and $\chi = 0$ corresponds to γ-unstable behavior [18].

2.3. B(E2) transition probabilities

General form of electromagnetic transition operator in the IBM-1 model is written as follows [2],

$$\hat{T}^{(L)} = \gamma_0 \left[\hat{s}^\dagger \times \tilde{\hat{s}}\right]^{(0)} + \alpha_2 \left[\hat{d} \times \tilde{\hat{s}} + \hat{s}^\dagger \times \tilde{\hat{d}}\right]^{(2)} + \beta_L \left[\hat{d}^\dagger \times \tilde{\hat{d}}\right]^{(L)} \quad (3)$$

where γ_0, α_2 and β_L (L=0,1,2,3,4) are parameters specifying various terms in the corresponding operators. In addition to the wave functions of the initial and final states, we need the $\hat{T}^{(E2)}$ operator to calculate B(E2) values. This operator is given as;

$$\hat{T}^{(E2)} = \alpha_2\left[\hat{d}^\dagger \times \tilde{\hat{s}} + \hat{s}^\dagger \times \tilde{\hat{d}}\right]^{(2)} + \beta_2\left[\hat{d}^\dagger \times \tilde{\hat{d}}\right]^{(2)} \quad (4)$$

where, the α_2 and β_2 constants are free parameters for E2 transition polarities. The B(E2) values are defined in terms of reduced matrix elements as [2];

$$B(E2; L_i \rightarrow L_f) = \frac{1}{2L_i+1}\left|(< L_f \left\| \hat{T}^{(E2)} \right\| L_i >)^2 (eb)^2\right|. \quad (5)$$

3. Results and Discussion

The energy ratio $R_{4/2}$ of the first 4^+ and 2^+ levels in the ground state (g.s.) band of the given $^{44-60}$Ti isotopes in the A~50 mass region were analyzed to have idea about their behavior. Dynamical symmetries of the IBM-1 are U(5), SU(3), and O(6) limits corresponding to the geometric shapes of nuclei [2] as discussed in subsection 2.1 including also the concept of the X(5) and E(5) critical point symmetries [16,17] appeared in between U(5)–SU(3) and U(5)–O(6) symmetries, respectively. Typical values [14] of the energy ratio ($R_{4/2}$) are listed in Table 1 for dynamical and critical point symmetries.

Table 1. The energy ratios of the dynamical and critical points symmetries, and experimental [13] ones of the Ti isotopes.

$R_{4/2}$	U(5)	O(6)	SU(3)	E(5)	X(5)	^{44}Ti	^{46}Ti	^{48}Ti	^{52}Ti	^{54}Ti	^{56}Ti	^{58}Ti	^{60}Ti
	2.00	2.50	3.33	2.19	2.91	2.27	2.26	2.33	2.21	1.70	2.03	1.95	2.02

The experimental [13] energy ratios of even-even $^{44-48}$Ti and $^{52-60}$Ti isotopes are given in Table 1 including also typical energy ratios of dynamical symmetries and critical point symmetries. According to the experimental energy ratios, $^{44-52}$Ti and ^{52}Ti isotopes are close to E(5) region and other even-even $^{54-60}$Ti isotopes are closer to U(5). ^{50}Ti isotope is excluded from the following IBM-1 calculations since its neutron number (N=28) is the magic number.

For the calculations of energy levels, the following model Hamiltonian given equation (1) can be formed as follows;

$$H = (EPS)\hat{n}_d + (PAIR)\hat{P}^\dagger \cdot \hat{P} + (ELL)\hat{L}\cdot\hat{L} + (QQ)\hat{Q}\cdot\hat{Q}, \quad (6)$$

where the constants are free parameters written in the form of the PHINT code [19] used for the IBM-1 calculations. The fitted parameters are listed in Table 2.

Table 2. The PHINT parameters (in units MeV) of Hamiltonian given in equation (6), CHQ is dimensionless parameter.

	^{44}Ti	^{46}Ti	^{48}Ti	^{52}Ti	^{54}Ti	^{56}Ti	^{58}Ti	^{60}Ti
EPS	0.85	0.6	1.1	1.3	1.351	1.0399	0.85	0.858
$PAIR$	-0.1	-0.17	0.14	0.3	--	--	--	--
QQ	-0.181	-0.46	-0.04	0.03	0.29	0.142	0.115	-0.0006
ELL	--	-0.10	--	--	--	--	--	--
CHQ	-1.5	-0.84	-0.6	-2	-1.3	-1.39	-2.2	-2

The energy levels of $^{44-60}$Ti isotopes were the calculated using the parameters given in Table 2. ^{44}Ti, ^{48}Ti and ^{52}Ti isotopes have two bosons, and the IBM-1 model can calculate the energy levels up to $2N$ (N is boson number). Thus, only 2_1^+, 4_1^+ levels in the g.s. band and also 0_2^+, 2_2^+ levels were calculated of these two isotopes. ^{46}Ti and ^{54}Ti have three bosons and their energy levels were calculated up to 6_1^+ in the g.s. band. An extra parameter (ELL) was added to the model Hamiltonian (6) to obtain the better results for ^{46}Ti isotope. Another fitted parameter of PHINT is CHQ in related to the parameter χ (CHI) with a relation by $CHQ \cong 2.24 \cdot CHI$. χ (CHI) is the constant in the $\hat{Q}$ operator given by equation (2). The rest isotopes have more bosons but limited levels are experimentally known. Overall calculated levels are exhibited in Figure 1 and clearly seen that these calculations are very close experimental ones.

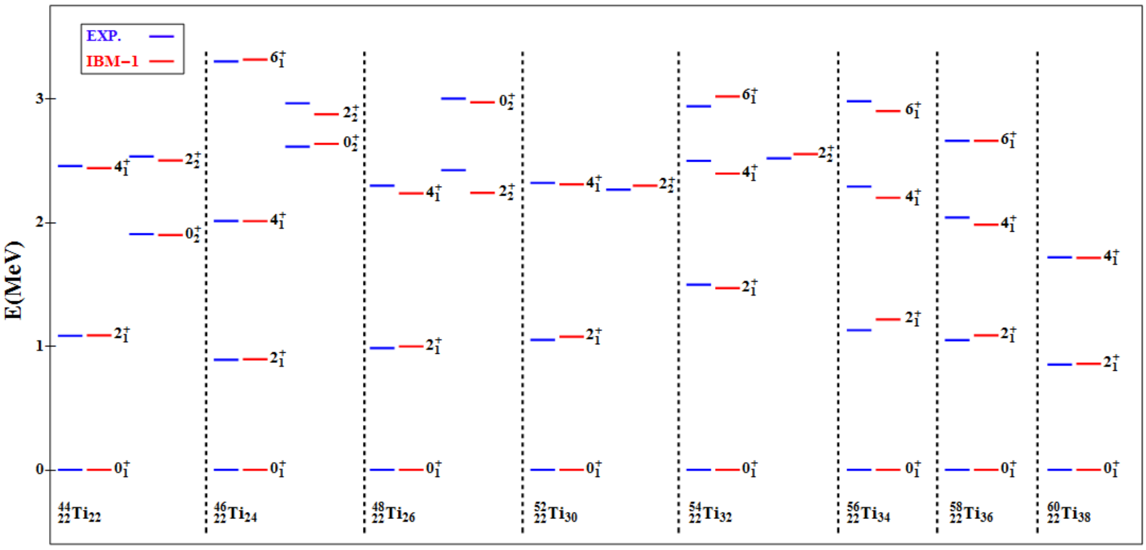

Figure 1. The experimental [13] and calculated energy spectra of given even-even Ti isotopes.

The experimental and calculated $R_{4^+/2^+}$ energy ratios (given in Table 1) are plotted in Figure 2(a) along the isotopic chain. According to the experimental values, ^{44}Ti, ^{46}Ti, ^{52}Ti isotopes are located around E(5) region while other even-even $^{54-60}$Ti isotopes are closer to U(5) symmetry. As seen this figure, ^{48}Ti isotope is between E(5)-O(6) symmetries but closer to E(5) point and $^{56-60}$Ti isotopes are close to U(5) region. However, the energy ratio of ^{54}Ti is smaller than that of U(5) and it is close to ^{50}Ti isotope having magic neutron number. The calculated $R_{4^+/2^+}$ ratios are mostly same with the experimental ones except for ^{56}Ti and ^{58}Ti isotopes but their calculated ratios are also close U(5). Changing of $R_{0^+/2^+}$ energy ratios are also exhibited in Figure 2(b) along the isotopic chain of Ti nuclei. This figure only includes experimentally known $^{44-48}$Ti isotopes. According to the $R_{0^+/2^+}$ ratios, the ^{44}Ti isotope is close to U(5) symmetry, ^{46}Ti and ^{48}Ti isotopes are very close to E(5) point.

In addition to energy levels, the IBM-1 model can also calculate the B(E2) values of given Ti isotopes by fitting the α_2 and β_2 parameters in the the $\hat{T}^{(E2)}$ operator given by equation (3). The parameters α_2 and β_2 are given by E2SD and E2DD in the PBEM code [19]. For the calculation, E2DD $= 0$ and E2SD is fitted as 0.075, 0.068, 0.083, 0.0714, 0.0657, 0.0412, 0.0416, 0.03 for given ^{44}Ti, ^{46}Ti, ^{48}Ti, ^{52}Ti, ^{54}Ti, ^{56}Ti, ^{58}Ti, ^{60}Ti isotopes, respectively. Overall calculated $B(E2: L_i^\pi \rightarrow L_f^\pi)$ transitions values are listed in Table 3.

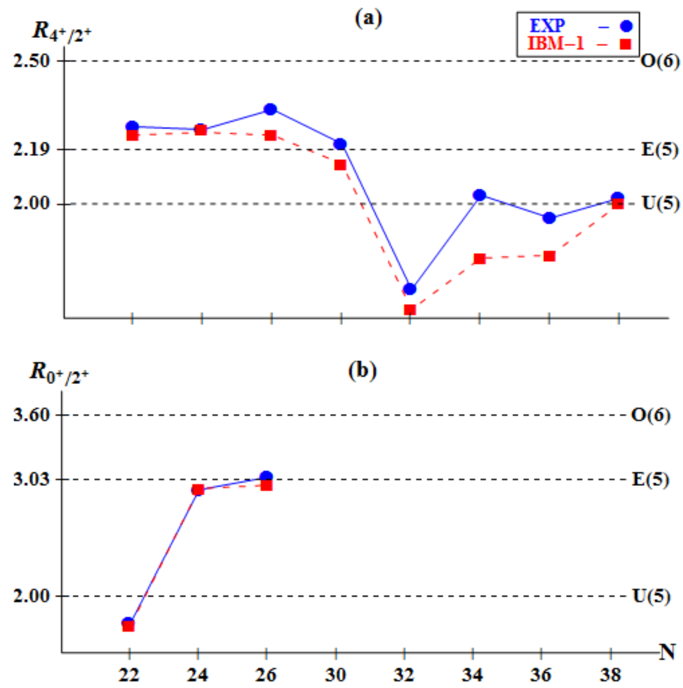

Figure 2. Energy ratios of given Ti nuclei along the isotopic chain. The experimental [13] ratios are denoted by blue and the calculated ones by red. (a) $R_{4^+/2^+}$ ratios (b) $R_{0^+/2^+}$ ratios.

Table 3. Experimental [13] and calculated $B(E2: L_i^{\pi} \rightarrow L_f^{\pi})$ transitions (in units of 10^{-2} e^2b^2) in given Ti isotopes.

Nucleus	$L_i^{\pi} \rightarrow L_f^{\pi}$	Experimental	Calculation
^{44}Ti	$2_1^+ \rightarrow 0_1^+$	1,200 (0,369)	1,195
	$4_1^+ \rightarrow 2_1^+$	2,768 (0,461)	1,108
	$0_2^+ \rightarrow 2_1^+$	<0,313	0,676
^{46}Ti	$2_1^+ \rightarrow 0_1^+$	1,909 (0,59)	1,825
	$4_1^+ \rightarrow 2_1^+$	1,978 (0,127)	2,032
	$6_1^+ \rightarrow 4_1^+$	1,606 (0,147)	1,347
	$0_2^+ \rightarrow 2_1^+$	0,245 (0,069)	0,182
^{48}Ti	$2_1^+ \rightarrow 0_1^+$	1,523 (0,420)	1,556
	$4_1^+ \rightarrow 2_1^+$	1,906 (0,176)	1,377
	$0_2^+ \rightarrow 2_1^+$	0,110 (0,013)	0,481
^{52}Ti	$2_1^+ \rightarrow 0_1^+$	1,141 (0,127)	1,139
	$4_1^+ \rightarrow 2_1^+$	--	1,019
^{54}Ti	$2_1^+ \rightarrow 0_1^+$	0,715 (0,133)	0,724
	$4_1^+ \rightarrow 2_1^+$	--	0,745
^{56}Ti	$2_1^+ \rightarrow 0_1^+$	--	0,432
	$4_1^+ \rightarrow 2_1^+$	--	0,665
	$6_1^+ \rightarrow 4_1^+$	--	0,683
^{58}Ti	$2_1^+ \rightarrow 0_1^+$	--	0,502
	$4_1^+ \rightarrow 2_1^+$	--	0,818
	$6_1^+ \rightarrow 4_1^+$	--	0,909
^{60}Ti	$2_1^+ \rightarrow 0_1^+$	--	0,542
	$4_1^+ \rightarrow 2_1^+$	--	0,903

Table 3 includes experimentally known B(E2) values of ^{44,46,48,52,54}Ti isotopes and their calculations and also some predictions for unknown B(E2) values of ^{52,54,56,58,60}Ti isotopes. As seen this table, the calculated $B(E2: 2_1^+ \rightarrow 0_1^+)$ transition values are in good agreement with the experimental data. Other $B(E2: L_i^+ \rightarrow L_f^+)$ transitions are close to experimental data.

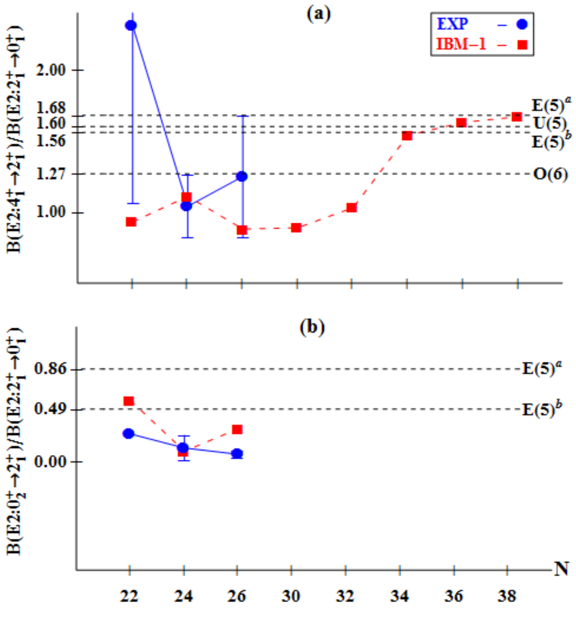

Figure 3. The comparison of the experimental (blue) [13] and the calculated (red) B(E2) ratios of Ti isotopes. (a) $B(E2: 4_1^+ \rightarrow 2_1^+)/B(E2: 2_1^+ \rightarrow 0_1^+)$ (b) $B(E2: 0_2^+ \rightarrow 2_1^+)/B(E2: 2_1^+ \rightarrow 0_1^+)$. The different values of B(E2) ratios are taken from Refs. [16, 17] marked by a and b for E(5) critical point symmetries.

The ratios of B(E2) values are also analyzed along the isotopic chain as shown in Figure 3.

The $B(E2: 4_1^+ \rightarrow 2_1^+)/B(E2: 2_1^+ \rightarrow 0_1^+)$ ratios are plotted as a function of the neutron number for given Ti isotopes in Figure 3 (a) including the typical ratios of U(5), O(6), and E(5) symmetries. There are two different ratios for E(5) point symmetry taken from Refs. [16, 17]. The experimental data of ^{44}Ti isotope is close to E(5)^a, while the calculation value is close to O(6). The experimental ratios and the calculated ones of ^{46}Ti and ^{48}Ti isotopes are very close to O(6) symmetry. As seen from Figure 3(a), ^{46}Ti and ^{48}Ti isotopes are close to O(6) symmetry and the ratio of ^{44}Ti isotope is bigger but the error bars of ^{44}Ti and ^{48}Ti isotopes touch to the ratios of O(6) and E(5). The calculated B(E2) ratios of ^{44,46,48}Ti isotopes are close to O(6) symmetry. Figure 3(a) also includes the prediction of $B(E2: 4_1^+ \rightarrow 2_1^+)/B(E2: 2_1^+ \rightarrow 0_1^+)$ ratios for ^{52,54,56,58,50}Ti isotopes, and clearly seen that ^{52,54}Ti isotopes are very close to O(6) symmetry, and ^{56,58,60}Ti isotopes are around the E(5) region. For more detailed analysis, the $B(E2: 0_2^+ \rightarrow 2_1^+)/B(E2: 2_1^+ \rightarrow 0_1^+)$ ratios are plotted in Figure 3(b) along the isotopic chain. This figure includes only experimentally known ratio, and seen that ^{44,46,48}Ti isotopes are close E(5) critical point taken from Refs. [16, 17].

4. Conclusion

In this study, the collective properties of even-even Ti isotopes in the A~50 mass region were investigated within the IBM-1 model. First, the parameters of the model Hamiltonian were fitted to calculate their low-lying energy levels and B(E2) values by using these parameters in the PHINT and PBEM codes [19]. The IBM-1 calculations of the energy levels and also B(E2) values are in good agreement with experimental data for given isotopes. Their energy ratios ($R_{4^+/2^+}$, $R_{0^+/2^+}$) and B(E2) ratios were also analyzed along the isotopic chain to understand their structural behavior. According to obtained results, given even-even Ti isotopes show E(5) behavior along the transition path from spherical to deformed γ-unstable region. The $R_{4^+/2^+}$ values along the isotopic chain show that ^{44}Ti, ^{46}Ti, ^{48}Ti, and ^{52}Ti isotopes are close to E(5) critical point while ^{56}Ti, ^{58}Ti, ^{60}Ti isotopes are close to U(5) region, the energy ratio of ^{54}Ti isotope is close to that of ^{50}Ti having magic neutron number. However, the $R_{0^+/2^+}$ ratios show that ^{44}Ti isotope is close to U(5) symmetry and ^{46}Ti, ^{48}Ti isotopes are located around the E(5) point. The B(E2) ratios also suggest that even-even Ti isotopes have signatures for E(5) behavior. Overall analysis indicates that these isotopes can be good example for the quantum shape phase transition from spherical to γ-unstable shapes along the isotopic chain. We again remind that the neutron number ^{50}Ti isotope is the magic number (N=28) and we excluded this isotope from the IBM-1 calculations. For the shape phase transition description of even-even Ti isotopes was performed within HFB approach in Ref. [6] and E(5) behavior was determined for ^{46,52,60}Ti isotopes. Systematic studies can be applied to A~50 mass region to see the behavior of even-even Ti, Cr, and Fe nuclei. For example, it can be interesting to apply the analysis of the energy ratios $\mathrm{R(L)} = \mathrm{E}(\mathrm{L}_1^+)/\mathrm{E}(2_1^+)$ in the g.s. band of these nuclei to see the E(5) behavior along the shape phase transition from U(5) to O(6) symmetry.

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Conflicts of interest

The authors declare no conflict of interest.

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