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Orthogonal Semiderivations and Symmetric Bi-semiderivations in Semiprime Rings

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ABSTRACT

In this paper, orthogonality for symmetric bi-semiderivations is defined and some results are obtained when two symmetric bi-semiderivations are orthogonal. Also, this paper gives the notion of orthogonality between semiderivations and symmetric bi-semiderivations of a 2-torsion free semiprime ring and offers some results of orthogonality.

Keywords: Semiderivation, Bi-semiderivation, Semiprime ring, Orthogonal derivation.

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Introduction

The concept of derivation in rings was firstly given by E. C. Posner [1]. An additive mapping $d: R \to R$ is said to be a derivation if $d(rs) = d(r)s + rd(s)$ for all $r, s \in R$. In the above paper, E. C. Posner examined the commutativity conditions for a prime ring by associating them with derivation. In the following years, different derivations have been defined and the properties of these derivations in prime and semiprime rings have been the subject of many researchers. In these studies, on different derivations, the conditions for the ring to be commutative are examined.

In 1980, the definition of symmetric bi-derivation on a ring was given by Gy. Maksa [2]. A mapping $D(.,.): R \times R \to R$ is called symmetric if $D(r, s) = D(s, r)$ holds for all $r, s \in R$. A mapping $d: R \to R$, $d(r) = D(r, r)$ is called trace of D, where $D(.,.): R \times R \to R$ is a symmetric mapping. It is clear that d the trace of D satisfies the relation $d(r + s) = d(r) + d(s) + 2D(r, s)$ for all $r, s \in R$. A symmetric bi-additive mapping $D(.,.): R \times R \to R$ is called a symmetric bi-derivation if $D(rs, t) = D(r, t)s + rD(s, t)$ holds for all $r, s, t \in R$. Then the relation $D(r, st) = D(r, s)t + sD(r, t)$ holds for all $r, s, t \in R$. J. Vukman has achieved some conclusions regarding symmetric bi-derivations on prime and semiprime rings [3, 4].

The notion of semiderivation in rings was given by J. Bergen in [5]. An additive mapping f of a ring R into R is called a semiderivation if there exists a function $g: R \to R$ such that $f(rs) = f(r)g(s) + rf(s) = f(r)s + g(r)f(s)$ and $f(g(r)) = g(f(r))$ for all $r, s \in R$. J. C. Chang generalized some well-known properties to semiderivations in [6]. In the above study, it has been shown that if R is a prime ring and f is a semiderivation associated with function g (not necessarily surjective), then g is a homomorphism.

The definition of orthogonal derivation in rings was given in 1989 by M. Bresar and J. Vukman [7]. Let R be a ring and d, g be nonzero derivations. If for all $r, s \in R$, $d(r)Rg(s) = g(s)Rd(r)$ holds, then d and g are called orthogonal derivations. In the above study, the following theorem has been proved. Let R be a semiprime ring with $char\ R \neq 2$, d and g be nonzero derivations. Then, d and g are orthogonal derivations if and only if one of the following conditions holds:

(i) $dg = 0$,
(ii) $dg + gd = 0$,
(iii) For all $r \in R$, $d(r)g(r) = 0$,
(iv) For all $r \in R$, $d(r)g(r) + g(r)d(r) = 0$,
(v) dg is a derivation.

Similar situations have been proved by many researchers for different derivations. In 2016, C. J. S. Reddy and B. R. Reddy obtained similar results for orthogonal symmetric bi-derivations in semiprime rings [8].

D. Yılmaz and H. Yazarlı, based on the concepts of symmetric bi-derivation and semiderivation, defined a symmetric bi-semiderivation in a prime ring [9]. Moreover, in [9], symmetric Jordan bi-semiderivations are defined, examples are given and when these two concepts are related is examined. Let R be a ring. A symmetric bi-additive function $D: R \times R \to R$ is called a symmetric bi-semiderivation associated with a function $f: R \to R$ (or simply a symmetric bi-semiderivation of a ring R) if

$$D(rs, t) = D(r, t)f(s) + rD(s, t) = D(r, t)s + f(r)D(s, t) \text{ and } d(f(r)) = f(d(r)) \text{ for all } r, s, t \in R,$$

where $d: R \to R$ is the trace of D.

Let R be a 2-torsion free semiprime ring, $D_1, D_2 : R \times R \to R$ be two nonzero symmetric bi-semiderivations associated with a surjective homomorphism f. In this paper, some cases are investigated when D_1 and D_2 orthogonal. Also, the notion of orthogonality between semiderivations and symmetric bi-semiderivations of a 2-torsion free semiprime ring is introduced and some features of this concept are examined.

Orthogonal Symmetric Bi-semiderivations in Semiprime Rings

Definition 2.1: Let R be a semiprime ring. Two symmetric bi-semiderivations D_1 and D_2 associated with a surjective function f are called orthogonal if

$$D_1(r,s)RD_2(s,t) = (0) = D_2(s,t)RD_1(r,s)$$

for all $r,s,t \in R$.

Example 2.2: Assume that S is a commutative additively idempotent semiprime ring. Then $R = \left\{\begin{pmatrix} r & 0 \\ 0 & s\end{pmatrix} : r,s \in S\right\}$ is a semiprime ring with matrix addition and multiplication. We define

$D_1: R\times R \to R$, $D_1\left(\begin{pmatrix} r & 0 \\ 0 & s\end{pmatrix}, \begin{pmatrix} t & 0 \\ 0 & w\end{pmatrix}\right) = \begin{pmatrix} rt & 0 \\ 0 & 0\end{pmatrix}$ and $D_2: R\times R \to R$, $D_2\left(\begin{pmatrix} r & 0 \\ 0 & s\end{pmatrix}, \begin{pmatrix} t & 0 \\ 0 & w\end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ 0 & sw\end{pmatrix}$. Let $f: R \to R$, $f\left(\begin{pmatrix} r & 0 \\ 0 & s\end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ 0 & s\end{pmatrix}$. Then D_1 and D_2 are orthogonal symmetric bi-semiderivations associated with function f.

Lemma 2.3: ([7]) Suppose that R is a 2-torsion free semiprime ring and $r,s \in R$. Then, the following conditions are equivalent:

(i) $rxs = 0$,
(ii) $sxr = 0$,
(iii) $rxs + sxr = 0$ for all $x \in R$.

If one of the above conditions holds, then $rs = sr = 0$.

Lemma 2.4: ([8], Lemma 2) Let R be a semiprime ring. Assume that $B, D: R \times R \to R$ are two bi-additive mappings satisfy $B(r,s)RD(r,s) = 0$ for all $r,s \in R$. Then,

$$B(r,s)RD(s,t) = (0) \text{ for all } r,s,t \in R.$$

Remark 2.5: ([8], Proposition 1) Let B and D be two bi-derivations of a ring R. The following identity holds
$(BD)(rs,t) = B(D(rs,t),w) = r(BD)(s,t) + B(r,w)D(s,t) + D(r,t)B(s,w) + (BD)(r,t)s$
for all $r,s,t,w \in R$.

Remark 2.6: Let D_1, $D_2: R\times R \to R$ be two symmetric bi-semiderivations associated with a surjective function f. Then, for all $r,s,t,w \in R$

$$\begin{aligned}(D_1D_2)(rs,t) = D_1(D_2(rs,t),w) = {} & (D_1D_2)(r,t)f(s) + f\big(D_2(r,t)\big)D_1(f(s),w) \\ & + D_1(r,w)f\big(D_2(s,t)\big) + r(D_1D_2)(s,t).\end{aligned} \tag{1}$$

Proof: Assume that D_1 and D_2 are two symmetric bi-semiderivations associated with a surjective function f. For all $r,s,t,w \in R$, we get

$$\begin{aligned}(D_1D_2)(rs,t) &= D_1(D_2(rs,t),w) = D_1(D_2(r,t)f(s) + rD_2(s,t),w) \\ &= D_1(D_2(r,t)f(s),w) + D_1(rD_2(s,t),w) \\ &= D_1(D_2(r,t),w)f(s) + f(D_2(r,t))D_1(f(s),w) + D_1(r,w)f(D_2(s,t)) + rD_1(D_2(s,t),w) \\ &= (D_1D_2)(r,t)f(s) + f\big(D_2(r,t)\big)D_1(f(s),w) + D_1(r,w)f\big(D_2(s,t)\big) + r(D_1D_2)(s,t).\end{aligned}$$

Lemma 2.7: Let R be a 2-torsion free semiprime ring and D_1, $D_2: R\times R \to R$ be two symmetric bi-semiderivations associated with a surjective function f.
Then, D_1 and D_2 are orthogonal $\Leftrightarrow$ For all $r,s,t \in R$, $D_1(r,s)D_2(s,t) + D_2(r,s)D_1(s,t) = 0$.

Proof: Firstly, we suppose that for all $r,s,t \in R$,

$$D_1(r,s)D_2(s,t) + D_2(r,s)D_1(s,t) = 0. \tag{2}$$

Replacing *t* by *tr* in (2), we have
$0 = D_1(r,s)D_2(s,tr) + D_2(r,s)D_1(s,tr)$

$$= D_1(r,s)D_2(s,t)f(r) + D_1(r,s)tD_2(s,r) + D_2(r,s)D_1(s,t)f(r) + D_2(r,s)tD_1(s,r)$$
$$= D_1(r,s)tD_2(s,r) + D_2(r,s)tD_1(s,r) + \big(D_1(r,s)D_2(s,t) + D_2(r,s)D_1(s,t)\big)f(r).$$

By using (2) in the last equation, we get

$$D_1(r,s)tD_2(s,r) + D_2(r,s)tD_1(s,r) = 0 \text{ for all } r,s,t \in R. \quad (3)$$

From Lemma 2.3, we obtain $D_1(r,s)RD_2(s,r) = (0)$ for all $r,s \in R$. Since D_1 and D_2 are bi-additive mappings, $D_1(r,s)RD_2(s,t) = (0)$ for all $r,s,t \in R$, by Lemma 2.4.

Using Lemma 2.3, we get $D_2(s,t)RD_1(r,s) = (0)$ for all $r,s,t \in R$. Thus, for all $r,s,t \in R$

$D_1(r,s)RD_2(s,t) = (0) = D_2(s,t)RD_1(r,s)$.

Therefore, D_1 and D_2 are orthogonal.

Now, we suppose that D_1 and D_2 are orthogonal symmetric bi-semiderivations.

Then, for all $r,s,t,w \in R$

$D_1(r,s)wD_2(s,t) = 0 = D_2(s,t)wD_1(r,s)$.

By Lemma 2.3, we have $D_1(r,s)D_2(s,t) = 0 = D_2(s,t)D_1(r,s)$. Therefore, we arrive $D_1(r,s)D_2(s,t) + D_2(r,s)D_1(s,t) = 0$ for all $r,s,t \in R$. This completes the proof.

Theorem 2.8: Let R be a 2-torsion free semiprime ring and $D_1,\ D_2: R \times R \to R$ be two symmetric bi-semiderivations associated with surjective homomorphism f.

(i) For all $r,s,t \in R$, $D_1(r,s)D_2(s,t) = 0$ or $D_2(r,s)D_1(s,t) = 0$ if and only if D_1 and D_2 are orthogonal.

(ii) If D_1 and D_2 are orthogonal, then D_1D_2 is a symmetric bi-semiderivation associated with function f^2.

(iii) If D_1 and D_2 are orthogonal, then $D_1D_2 = 0$.

Proof: (i) Assume that for all $r,s,t \in R$, $D_1(r,s)D_2(s,t) = 0$. In this equation replacing r by rw and using hypothesis, we get $D_1(r,s)wD_2(s,t) = 0$ for all $r,s,t,w \in R$. Thus, we obtain $D_1(r,s)RD_2(s,t) = (0)$ for all $r,s,t \in R$. Hence, D_1 and D_2 are orthogonal.

Now, assume that D_1 and D_2 are orthogonal. Then, $D_1(r,s)RD_2(s,t) = (0)$ for all $r,s,t \in R$. By Lemma 2.3, $D_1(r,s)D_2(s,t) = 0$ for all $r,s,t \in R$.

Similarly, it is proved that D_1 and D_2 are orthogonal $\Leftrightarrow$ $D_2(r,s)D_1(s,t) = 0$ for all $r,s,t \in R$.

(ii) Assume that D_1 and D_2 are orthogonal. In view of (i), $D_2(r,s)D_1(s,w) = 0$ for all $r,s,w \in R$. Taking $s = r$ yields that

$$d_2(r)D_1(r,w) = 0 \text{ for all } r,w \in R. \quad (4)$$

Replacing r by $r + v$ in (4), we obtain

$$0 = d_2(r)D_1(r,w) + d_2(r)D_1(v,w) + d_2(v)D_1(r,w) + d_2(v)D_1(v,w)$$
$$+ 2\, D_2(r,v)D_1(r,w) + 2\, D_2(r,v)D_1(v,w).$$

Since D_1 and D_2 are orthogonal and the equation (2.4), we get

$$d_2(r)D_1(v,w) + d_2(v)D_1(r,w) = 0 \text{ for all } r,v,w \in R.$$

Replacing v by $-v$ in the last equation, we get

$$d_2(r)D_1(v,w) = 0 \text{ for all } r,v,w \in R. \quad (5)$$

Replacing r by $r + t$ in (5), we get $2D_2(r,t)D_1(v,w) = 0$. Since R is a 2-torion free ring, we have

$$D_2(r,t)D_1(v,w) = 0 \text{ for all } r,t,v,w \in R. \quad (6)$$

Again using (i), we have

$D_1(r,s)D_2(s,t) = 0$ for all $r,s,t \in R$.

Taking s for r, we get

$$d_1(s)D_2(s,t) = 0 \text{ for all } s,t \in R. \quad (7)$$

A linearization of (7) gives

$$0 = d_1(s)D_2(s,t) + d_1(s)D_2(u,t) + d_1(u)D_2(s,t) + d_1(u)D_2(u,t)$$

$$+ 2D_1(s,u)D_2(s,t) + 2D_1(s,u)D_2(u,t).$$

Since D_1 and D_2 are orthogonal, we get

$$d_1(s)D_2(u,t) + d_1(u)D_2(s,t) = 0 \text{ for all } s,t,u \in R.$$

Letting $s = -s$ in the last equation, we obtain

$$d_1(s)D_2(u,t) = 0 \text{ for all } s,t,u \in R.$$

Since f is a surjective homomorphism, we get $d_1(f(s))f(D_2(u,t)) = 0$. Putting $s + w$ instead of s in the last equation and using it, we find

$$2D_1\big(f(s),f(w)\big)f\big(D_2(u,t)\big) = 0 \text{ for all } s,t,u,w \in R.$$

Since R is a 2-torion free ring and f is a surjective homomorphism, we get

$$D_1(r,v)f\big(D_2(u,t)\big) = 0 \text{ for all r}, t,u,v \in R. \quad (8)$$

On the other hand, for all $r,s,t,w \in R$

$$\begin{aligned}(D_1D_2)(rs,t) = D_1(D_2(rs,t),w) &= D_1(D_2(r,t)f(s) + rD_2(s,t),w)\\ &= D_1(D_2(r,t),w)f^2(s) + D_2(r,t)D_1(f(s),w)\\ &\quad + D_1(r,w)f\big(D_2(s,t)\big) + rD_1(D_2(s,t),w).\end{aligned} \quad (9)$$

Using (6) and (8) in the equation (9), we arrive that

$$(D_1D_2)(rs,t) = (D_1D_2)(r,t)f^2(s) + r(D_1D_2)(s,t) \text{ for all } r,s,t \in R.$$

Hence, D_1D_2 is a symmetric bi-semiderivation associated with function f^2 and we conclude that desired result.
(iii) Suppose that D_1 and D_2 are orthogonal. Then, we have

$$D_1(r,s)D_2(s,t) = 0 \text{ for all } r,s,t \in R.$$

This implies that $D_1(D_1(r,s)D_2(s,t),\ w) = 0$ for all $r,s,t,w \in R$. If this statement is regulated, we get

$$D_1(D_1(r,s),w)f\big(D_2(s,t)\big) + D_1(r,s)(D_1D_2)(s,t) = 0.$$

Using the equation (8) in the last expression, we obtain $D_1(r,s)(D_1D_2)(s,t) = 0$ for all $r,s,t \in R$. Replacing r by ru, we get

$$D_1(r,s)u(D_1D_2)(s,t) = 0 \text{ for all } r,s,t,u \in R.$$

In particular, putting $r = D_2(s,t)$ gives $(D_1D_2)(s,t)u(D_1D_2)(s,t) = 0$. Since R is a semiprime ring, we get $D_1D_2 = 0$, the conclusion is obtained.

Orthogonality of Semiderivations and Symmetric Bi-semiderivations

Definition 3.1: Let R be a semiprime ring, g be a semiderivation of R and D be a symmetric bi-semiderivation of R associated with a function f. If
$D(r,s)Rg(t) = (0) = g(t)RD(r,s)$ for all $r,s,t \in R$,
Then g and D are called orthogonal.

Example 3.2: Let S be a commutative ring and let $M_3(S) = \left\{\begin{pmatrix} 0 & s & t\\ 0 & 0 & 0\\ 0 & 0 & w\end{pmatrix} : s,t,w \in S\right\}$.

Define $g\colon M_3(S) \to M_3(S)$ by $g\left(\begin{pmatrix} 0 & s & t\\ 0 & 0 & 0\\ 0 & 0 & w\end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & t\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$,

$D: M_3(S) \times M_3(S) \rightarrow M_3(S)$ by $D\left(\begin{pmatrix} 0 & s & t \\ 0 & 0 & 0 \\ 0 & 0 & w \end{pmatrix}, \begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & c \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & tc \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

and $f: M_3(S) \rightarrow M_3(S)$, $\begin{pmatrix} 0 & s & t \\ 0 & 0 & 0 \\ 0 & 0 & w \end{pmatrix} \mapsto \begin{pmatrix} 0 & s & 0 \\ 0 & 0 & 0 \\ 0 & 0 & w \end{pmatrix}$.

It is obvious that g is a semiderivation of $M_3(S)$ associated with f and D is a symmetric bi-semiderivation of $M_3(S)$ associated with f. Also, g and D are orthogonal.

Lemma 3.3: Let R be a semiprime ring. Suppose that an additive mapping h on R and a bi-additive mapping $f: R \times R \rightarrow R$ satisfy $h(r)Rf(r,s) = (0)$ for all $r, s \in R$. Then, $h(r)Rf(t,s) = (0)$ for all $r, s, t \in R$.

Proof: We have

$$h(r)wf(r,s) = 0 \text{ for all } r, s, w \in R.$$

Linearising the above equation on r gives

$$h(r)wf(r,s) + h(t)wf(r,s) + h(r)wf(t,s) + h(t)wf(t,s) = 0\,.$$

Then, we get

$$h(r)wf(t,s) + h(t)wf(r,s) = 0 \text{ for all } r, s, t, w \in R.$$

Hence, we obtain $h(r)wf(t,s) = -h(t)wf(r,s)$. Replacing w by $wf(t,s)vh(r)w$ and using hypothesis, we get

$$h(r)wf(t,s)vh(r)wf(t,s) = -h(t)wf(r,s)vh(r)wf(t,s) = 0.$$

Since R is semiprime ring, we get $h(r)wf(t,s) = (0)$ for all $r, s, t, w \in R$.

Lemma 3.4: Let R be a semiprime ring. Suppose that an additive mapping h on R and a bi-additive mapping $f: R \times R \rightarrow R$ satisfy $h(r)Rf(r,s) = (0)$ for all $r, s \in R$. Then, $h(t)Rf(r,s) = (0)$ for all $r, s, t \in R$.

Proof: We have

$$h(r)wf(r,s) = 0 \text{ for all } r, s, w \in R.$$

Linearising the above equation on r gives

$$h(r)wf(r,s) + h(t)wf(r,s) + h(r)wf(t,s) + h(t)wf(t,s) = 0\,.$$

Then, we get

$$h(r)wf(t,s) + h(t)wf(r,s) = 0 \text{ for all } r, s, t, w \in R.$$

From hypothesis and the last equation, we obtain

$$h(t)wf(r,s)Rh(t)wf(r,s) = -h(t)wf(r,s)Rh(r)wf(t,s) = (0).$$

Since R is semiprime ring, we get $h(t)wf(r,s) = (0)$ for all $r, s, t, w \in R$.

Lemma 3.5: Let R be a 2-torsion free semiprime ring, g be a semiderivation of R and D be a symmetric bi-semiderivation of R associated with a function f. Then, g and D are orthogonal if and only if

$$D(r,s)g(t) + g(r)D(t,s) = 0 \text{ for all } r, s, t \in R.$$

Proof: Suppose g and D satisfy

$$D(r,s)g(t) + g(r)D(t,s) = 0 \text{ for all } r, s, t \in R. \tag{10}$$

Replacing t by tr in (10) and using (10), we get

$$D(r,s)tg(r) + g(r)tD(r,s) = 0 \text{ for all } r,s,t \in R.$$

From Lemma 2.3, we have $g(r)RD(r,s) = (0)$ for all $r,s \in R$.
Then, Lemma 3.4 gives $g(t)RD(r,s) = (0)$ for all $r,s,t \in R$. Using Lemma 2.3 again, we obtain

$$\mathrm{g(t)RD}(r,s) = (0) = D(r,s)\mathrm{Rg(t)} \text{ for all } r,s,t \in R.$$

Thus, g and D are orthogonal.
Now, suppose that g and D are orthogonal. Then, $D(r,s)g(t) = g(r)D(t,s) = 0$, from Lemma 2.3. This gives the desired result.

Remark 3.6: Let R be a ring. Suppose that g is a semiderivation of R and D is a symmetric bi-semiderivation of R associated with a surjective function f. Then, the following equation holds:

$$(gD)(rs,t) = (gD)(r,t)f^2(s) + D(r,t)g\big(f(s)\big) + g(r)f\big(D(s,t)\big) + r(gD)(s,t) \text{ for all } r,s,t \in R.$$

Proof: For all $r,s,t \in R$, we have

$$\begin{aligned}(gD)(rs,t) &= g\big(D(rs,t)\big) = g\big(D(r,t)f(s) + rD(s,t)\big)\\ &= g\big(D(r,t)f(s)\big) + g\big(rD(s,t)\big)\\ &= (gD)(r,t)f^2(s) + D(r,t)g\big(f(s)\big) + g(r)f\big(D(s,t)\big) + r(gD)(s,t).\end{aligned}$$

Theorem 3.7: Let R be a 2-torsion free semiprime ring, g be a semiderivation of R and D be a symmetric bi-semiderivation of R associated with a surjective homomorphism f. Then, g and D are orthogonal if and only if $g(r)D(r,s) = 0$ for all $r,s \in R$.

Proof: Suppose g and D such that

$$g(r)D(r,s) = 0 \text{ for all } r,s \in R. \tag{11}$$

A linearization of (11) gives

$$g(r)D(r,s) + g(r)D(t,s) + g(t)D(r,s) + g(t)D(t,s) = 0 \text{ for all } r,s,t \in R.$$

Using (11), we have

$$g(r)D(t,s) + g(t)D(r,s) = 0 \text{ for all } r,s,t \in R. \tag{12}$$

Replacing t by tw in (12), we get

$$g(r)D(t,s)f(w) + g(r)tD(w,s) + g(t)f(w)D(r,s) + tg(w)D(r,s) = 0 \text{ for all } r,s,t,w \in R. \tag{13}$$

By (12), we have $g(r)D(t,s) = -g(t)D(r,s)$ and $g(w)D(r,s) = -g(r)D(w,s)$. Thus, for all $r,s,t,w \in R$, the equation (13) becomes

$$-g(t)D(r,s)f(w) + g(r)tD(w,s) + g(t)f(w)D(r,s) - tg(r)D(w,s) = 0. \tag{14}$$

Taking t by $g(r)$ in (14), we have

$$-g^2(r)D(r,s)f(w) + g(r)^2D(w,s) + g^2(r)f(w)D(r,s) - g(r)^2D(w,s) = 0$$
which implies that

$$g^2(r)[f(w),D(r,s)] = 0 \text{ for all } r,s,w \in R.$$

Since f is a surjective homomorphism, we obtain

$$g^2(r)[u,D(r,s)] = 0 \text{ for all } r,s,u \in R. \tag{15}$$

Letting $u = ut$ in (15), we get $g^2(r)u[t, D(r,s)] + g^2(r)[u, D(r,s)]t = 0$. Using (15) in the last equation, we get $g^2(r)u[t, D(r,s)] = 0$. From Lemma 3.3, we obtain

$$g^2(r)R[t, D(v,s)] = (0) \text{ for all } r,s,t,v \in R. \tag{16}$$

Taking $r = ru$ in (16), we get

$$\left(g^2(r)f^2(u) + g(r)g\big(f(u)\big) + g(r)f\big(g(u)\big) + rg^2(u)\right)R[t, D(v,s)] = (0) \text{ for all } r,s,t,u,v \in R.$$

By (16), we have $\left(g(r)g\big(f(u)\big) + g(r)f\big(g(u)\big)\right)R[t, D(v,s)] = (0)$ for all $r,s,t,u,v \in R$. Since g is a semiderivation of R associated with f and R is a 2-torsion free ring, we get $g(r)g\big(f(u)\big)R[t, D(v,s)] = (0)$ for all $r,s,t,u,v \in R$. Since f is a surjective homomorphism, we have

$$g(r)g(w)R[t, D(v,s)] = (0) \text{ for all } r,s,t,v,w \in R. \tag{17}$$

Replacing r by rz in (17), we obtain $g(r)Rg(w)R[t, D(v,s)] = (0)$ for all $r,s,t,v,w \in R$.
In particular,

$$g(r)R[t, D(v,s)]Rg(r)R[t, D(v,s)] = (0) \text{ for all } r,s,t,v \in R.$$

Since R is a semiprime ring, we get $g(r)R[t, D(v,s)] = (0)$ for all $r,s,t,v \in R$. Then, the equation $[g(r), D(v,s)]R[g(r), D(v,s)] = (0)$ holds for all $r,s,v \in R$. Then, we have

$g(r)D(v,s) = D(v,s)g(r)$ for all $r,s,v \in R$. Hence, the equation (12) can be written as

$$D(t,s)g(r) + g(t)D(r,s) = 0 \text{ for all } r,s,t \in R.$$

Therefore, Lemma 3.5 gives the required result.
Now, suppose that g and D are orthogonal. Then we have $g(r)RD(r,s) = (0)$ for all $r,s \in R$. By Lemma 2.3, we get $g(r)D(r,s) = 0$ for all $r,s \in R$.

Theorem 3.8: Let R be a 2-torsion free semiprime ring. Suppose that a semiderivation g of R and a symmetric bi-semiderivation D of R associated with a surjective homomorphism f are orthogonal. Then

$$D(r,t)g\big(f(s)\big) + g(r)f\big(D(s,t)\big) = 0 \text{ for all } r,s,t \in R.$$

Proof: Let g and D are orthogonal. Hence, we have

$$D(r,t)g(s) = 0 = g(r)D(s,t) \text{ for all } r,s,t \in R. \tag{18}$$

In the equation $D(r,t)g(s) = 0$, replacing s by $f(s)$ yield

$$D(r,t)g\big(f(s)\big) = 0 \text{ for all } r,s,t \in R. \tag{19}$$

On the other hand, we have $g(r)D(s,t) = 0$ by (18). Taking $s = t$ in the last equation we get

$$g(r)d(t) = 0 \text{ for all } r,t \in R. \tag{20}$$

Replacing t by $f(t)$ in (20), we get

$$g(r)f\big(d(t)\big) = 0 \text{ for all } r,t \in R. \tag{21}$$

Since f is a homomorphism taking $t = t + s$ in (21),

$$g(r)f\big(d(t)\big) + g(r)f\big(d(s)\big) + 2g(r)f\big(D(s,t)\big) = 0 \text{ for all } r,s,t \in R.$$

Using (21), we get $2g(r)f\big(D(s,t)\big) = 0$. Since R is 2-torsion free ring, we get

$$g(r)f\big(D(s,t)\big) = 0 \text{ for all } r,s,t \in R. \tag{22}$$

The equations (19) and (22) give the required result.

Theorem 3.9: Let R be a 2-torsion free semiprime ring, g be a semiderivation of R and D be a symmetric bi-semiderivation of R associated with a surjective homomorphism f. If g and $D$are orthogonal, then $gD = 0$.

Proof: We assume that g and D are orthogonal.
Then, we have $g(r)wD(s,t) = 0$ for all $r,s,t,w \in R$. Then,

$$0 = g\big(g(r)\big)wD(s,t) + g\big(f(r)\big)g(w)D(s,t) + g\big(f(r)\big)f(w)(gD)(s,t).$$

Since g and D are orthogonal, we get $g\big(f(r)\big)f(w)(gD)(s,t) = 0$ for all $r,s,t,w \in R$. Since f is a surjective homomorphism, we get

$$g(u)R(gD)(s,t) = (0) \text{ for all } s,t,u \in R. \tag{23}$$

Let $u = D(s,t)$ in (23). Hence, $(gD)(s,t)R(gD)(s,t) = (0)$ for all $s,t \in R$.
By semiprimeness of R, $(gD)(s,t) = 0$ for all $s,t \in R$. It implies that $gD = 0$.

Theorem 3.10: Suppose R be a 2-torsion free semiprime ring, g be a semiderivation of R and D be a symmetric bi-semiderivation of R associated with a surjective homomorphism f. If g and D are orthogonal, then gD is a symmetric bi-semiderivation associated with f^2 function.

Proof: Let g and D be orthogonal. By Remark 3.6, we have for all $r,s,t \in R$,

$$(gD)(rs,t) = (gD)(r,t)f^2(s) + D(r,t)g\big(f(s)\big) + g(r)f\big(D(s,t)\big) + r(gD)(s,t). \tag{24}$$

Also, by Theorem 3.8, we have $D(r,t)g\big(f(s)\big) + g(r)f\big(D(s,t)\big) = 0$ for all $r,s,t \in R$. From the last two expression, we get $(gD)(rs,t) = (gD)(r,t)f^2(s) + r(gD)(s,t)$ for all $r,s,t \in R$. It implies that gD is a symmetric bi-semiderivation of R associated with f^2 function.

Conflicts of interest

The author declares no conflict of interest

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