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Proposed nonparametric tests for the ordered alternative in a completely randomized and randomized block mixed design

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Abstract

Two nonparametric tests are proposed for the mixed design consisting of a randomized complete block and a completely randomized design to test for k nondecreasing treatment effects. The Hollander test and the Page test are used in randomized complete block design and completely randomized design, respectively. We compared the performance of the proposed tests against the Z_{CombI}, Z_{CombII} Page and Hollander tests. A Monte Carlo simulation study was conducted comparing the estimated powers of the tests 3, 4 and 5 treatments under various treatment effects and three different underlying distributions. In conclusion, the two proposed tests have higher powers than the Page, Hollander, Z_{CombI} and Z_{CombII} tests.

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1. Introduction

When researchers may wish to compare the effects of treatments, they need to plan an experimental design. Design structures can occur in many ways. Magel and Ndungu [1] gave an example in which a business may have several thousand employees and wants to reduce the annual cost of health care for the employees. For example, a business may have several thousand employees and wants to reduce the annual cost of health care for the employees. For this purpose, the business wants to educate workers to change food habits, as to good nutrition and fitness and make a more appropriate exercise program. The business believes that some factors, including cholesterol level, blood pressure, body mass index, the amount of sleep a person gets, and the amount and types of exercises, are related to the health of workers, represented by a health number. This number is based on the values of the observed factors for the employee with a higher health number indicating better health. A voluntary worker may likely to skip a period time or periods when these factors are measured. For this reason, there could be missing observations within a block; hence, in testing whether this program is efficient, the business may decide to collect some observations using a completely randomized design in which additional random samples of the employees involved only during a period time. This mixed design consists of a randomized complete block portion and a completely randomized design portion. It is possible to have a mixed design which is a combination of a randomized complete block, paired data, a balanced incomplete block and a completely randomized design.

As can be seen from the above example, in real life problems, researchers may have to change the design structure in order to avoid loss of information. For this reason, these block designs that form the mixed design structure should be analyzed with non-parametric test combinations. However, there are very few studies on mixed designs in the literature. Therefore, in this article, we proposed nonparametric tests which is combined Hollander [2] test statistic (randomized complete block) and Jonckheere-Terpstra (*JT*) test statistic [3-4] (completely randomized design) for ordered alternatives. We would like to test the following set of hypotheses:

$$H_0: \tau_1 = \tau_2 = \cdots = \tau_k$$
$$H_1: \tau_1 \leq \tau_2 \leq \cdots \leq \tau_k \tag{1}$$

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with τ_i indicating the treatment effect for the i^{th} time period and k indicating the number of treatments.

Several nonparametric tests available to test the differences for more than two treatments in a completely randomized design. Kruskal-Wallis test statistic [5] is a nonparametric test which is an extension of the Mann-Whitney test statistic [6] comparing two treatments in a completely randomized design. The *JT* test is designed to test the non-decreasing treatment effects for this type of design. Akdur et al. [7] modified generalized Jonckheere test for repeated measures in randomized blocks with circular bootstrap method. To compute the *JT* test statistic, *JT*, we calculate the $k(k-1)/2$ Mann-Whitney counts u_{uv} given by

$$U_{uv} = \sum_{i=1}^{n_u}\sum_{j=1}^{n_v} \varphi(X_{iu}, X_{jv}), 1 \le u < v \le k,$$

where $\varphi(a,b) = 1$ if $a < b$, 0 otherwise. (Thus, U_{uv} is the number of sample *u* before sample *v* precedences). The *JT* test statistic is the sum of these $k(k-1)/2$ Mann-Whitney counts:

$$JT = \sum_{u=1}^{v-1}\sum_{v=2}^{k} U_{uv}. \quad (2)$$

The expected value and variance of *JT* are:

$$E(JT) = \frac{N^2 - \sum_{j=1}^{k} n_j^2}{4},$$

and

$$Var(JT) = \frac{N^2(2N+3) - \sum_{j=1}^{k} n_j^2(2n_j+3)}{72}.$$

The standardized version of the test statistic, Z_{JT}, is given as:

$$Z_{JT} = \frac{JT - E(JT)}{\sqrt{Var(JT)}} \quad (3)$$

where n_j is j^{th} treatment sample and *N* is the total sample size of all treatments. Under H_0, the test statistic, Z_{JT}, has an asymptotic normal distribution [8]. It is rejected when standardized version is greater than or equal to Z_α at the α level of significance.

There are several nonparametric tests to compare more than two treatments in a randomized complete block design. One of the most important tests is the Page test statistic [9], which compares the treatment effects for non-decreasing alternatives. Gokpinar et al. [10] compared performances of permutation version of several non parametric tests such as Page, Hollander tests for ordered alternative hypotheses in RCBD. Recently, Akdur et al. [11] proposed a nonparametric test for ordered alternatives in randomized complete block designs (RCBD). Hollander test statistic, *H*, [2] based on Wilcoxon signed-rank test statistic [12] is used for testing non-decreasing alternatives. For each of pair (u, v) treatment and each of $1 < u < v < t$, T_{uv} is defined as a signed-rank statistic and written as:

$$T_{uv} = \sum_{i=1}^{b} R_{uv}^i, \psi_{uv}^i,$$

where

$$\psi_{uv}^i = \begin{cases} 1 & X_{iu} < X_{iv} \\ \frac{1}{2} & X_{iu} = X_{iv} \\ 0 & X_{iu} > X_{iv} \end{cases}.$$

H test statistic depending on the statistic of T_{uv} is

$$H = \sum_{u=1}^{t-1}\sum_{v=u+1}^{t} T_{uv}. \quad (4)$$

The expected value and variance of the *H* test statistic are

$$E(H) = \frac{bt(t-1)(b+1)}{8}$$

and

$$Var(H) = \frac{bt(b+1)(2b+1)(t-1)\{3 + 2(t-2)p_U^b\}}{144}$$

where p_U^b, depend on *b*, is the value of the null correlation between two overlapping signed rank statistics based on *n* observations. Wilcoxon signed-rank test statistic. The standardized version of the test statistic, Z_H, is given as:

$$Z_H = \frac{H - E(H)}{\sqrt{Var(H)}}. \quad (5)$$

Under H_0, the test statistic, Z_H, has an asymptotic normal distribution. It is rejected when standardized version is greater than or equal to Z_α at the α level of significance.

Dubnicka et al. [13] proposed a rank-based test for the mixed two-sample design which is a combination of paired data and independent observations. Their proposed test statistic is sum of the Wilcoxon signed rank statistic (paired data) and the Mann-Whitney statistic (independent samples).

Magel et al. [14] proposed tests for testing the equality of k medians when the data are mixture of a randomized complete block, a completely randomized and incomplete block design. They developed two tests for the umbrella alternatives. The two proposed tests are compared to each other and give suggestions.

Magel and Fu [15] proposed a nonparametric test for a mixed design which is a combination of a paired sample portion and a two-independent-sample portion to test for a difference in treatment effects.

Olet ve Magel [16] proposed six nonparametric tests to test for a difference between the control and any of k-1 treatments in a completely randomized and randomized complete block mixed design.

Magel et al. [17] introduced two tests for the non-decreasing alternative for mixed designs consisting of a randomized complete block portion and a completely randomized design portion. Their proposed test statistics are linear combinations of Page's test statistic and the *JT* test statistic.

The first version of the test can be written as

$$Z_{CombI} = \frac{Z_p + Z_{JT}}{\sqrt{2}}, \tag{6}$$

where Z_p and Z_{JT} are the standardized version of Page's test and standardized version of *JT* test, respectively. Under H_0, Z_{CombI} has an asymptotic standard normal distribution since the asymptotic distributions of Z_p and Z_{JT} under H_0 are standard normal. H_0 is rejected if $Z_{CombI} > Z_\alpha$, where Z_α is the $(1-\alpha)100$ percentile of a standard normal distribution.

The second version of the test can be formulated as

$$Z_{CombII} = \frac{L + JT - E(L+JT)}{\sqrt{Var(L+JT)}}, \tag{7}$$

where

$$E(L+JT) = \frac{bk(k+1)^2}{4} + \frac{(N^2 - \sum_{i=1}^{k} n_i^2)}{4},$$

$$Var(L+JT) = \frac{b(k^3-k)^2}{144(k-1)} + \frac{N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i+3)}{72}.$$

Here, L and *JT* are Page's test statistic and the *JT* test statistic, respectively. The test statistic has an asymptotic standard normal distribution when H_0 is true. It is rejected when $Z_{CombII} > Z_\alpha$, where Z_α is the $(1-\alpha)$ percentile of a standard normal distribution.

We propose two forms of a test statistic for a mixed design consisting of a completely randomized portion and a randomized block portion. We may wish to compare the effects of k treatments. The underlying distributions considered include the normal distribution, exponential distribution and t distribution with 3 degrees of freedom. To use parametric tests, random sample should be drawn from normal distribution or large enough sample size, and hence, we are considering nonparametric tests.

The paper is organized as follows. Section 2 introduces the proposed nonparametric tests for a mixed design consisting of randomized complete block design and completely randomized design. The simulation results and discussions are provided in Section 3-4. Finally, we give some concluding remarks in Section 5.

2. Proposed Tests

In this article, we propose and compare nonparametric tests for a mixed design consisting of randomized complete block design and completely randomized design based on hypothesis given in Eq. (1). These tests are linear combinations of Hollander test statistic and *JT* test statistic. The tests we propose are similar to the idea of Magel et al. [17].

The first version of the test statistic considered adds the standardized versions of the Hollander test statistic, denoted by Z_H, and the *JT* test statistic, denoted by Z_{JT}, and then divided by $\sqrt{2}$. The first proposed test statistic, denoted by Z_1, is given in Eq. (8),

$$Z_I = \frac{Z_H + Z_{JT}}{\sqrt{2}}. \tag{8}$$

Under H_0, Z_I has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_I > Z_\alpha$.

The second version of the test statistic considered adds the nonstandardized versions of Hollander test statistic, denoted by *H*, and the *JT* test statistic, denoted by *JT*, together and then restandardized. The second proposed test statistic, denoted by Z_{II}, is given in Eq. (9):

$$Z_{II} = \frac{H + JT - [E(H) + E(JT)]}{\sqrt{[V(H) + V(JT)]}}. \tag{9}$$

Under H_0, Z_{II} has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_{II} > Z_\alpha$.

3. Simulation Study

A simulation study was conducted using MATLAB (R2017b) to compare the powers of the proposed test versions with the powers of the tests constructed by [17] and the powers of Page's test and Hollander test discarding the additional observations from the completely randomized design portion. The underlying population distributions considered were the normal, exponential and student's *t* with 3 degrees of freedom. All powers were estimated based on 5000 iterations for each combination of the distributions, several different equal and unequal arrangements of sample sizes and different location parameter arrangements. Since the convergence required for the power values and the estimated alpha is at 5000 iterations, so it is taken as the number of iterations. Estimated alpha and power values were found in each case. A relative difference percentage between the two test versions developed by [17] was calculated by:

$$D = \frac{100*(Power_{Z_I} - Power_{Z_{II}})}{Power_{Z_I}}. \quad (10)$$

When this difference is positive, the first test version, Z_I, has a higher estimated power. When this difference is negative, the second version, Z_{II}, has a higher estimated power. In the simulation study, the equal and unequal sample sizes denoted by *n* (recall, the *JT* test is used on this portion) were used for the completely randomized design portion. The cases were considered so that the completely randomized portion was 1/8, 1/4 and 1/2 that of the randomized complete block portion (recall, the Page's test and Holander test were used on the block portion). The following is a list of all of the sample sizes considered where block is randomized block portion and n_i is completely randomized block portion:

1. Block=40, $n_i = 5$.
2. Block=40, $n_i = 10$.
3. Block=40, $n_i = 20$.
4. Block=16, $n_1 = 8, n_2 = n_3 = n_4 = 4$.
5. Block=32, $n_1 = 8, n_2 = n_3 = n_4 = 4$.
6. Block=40, $n_1 = 10, n_2 = n_3 = n_4 = 5$.

In the simulation study, we considered increasing ordered alternatives. Various location parameters were added when estimating the powers of the test statistics. The number of treatments (denoted by *k*) were taken 3,4 and 5. The u_i value in tables is the location parameter arrangement for i^{th} treatment, *i*=1, 2, 3, 4 and 5. The value n_i is the sample size of i^{th} treatment for the completely randomized portion.

4. Results

Estimated rejection percentages are given for each two proposed test, the two proposed test are given by [17], Page's test and Hollander test (discarding observations from the completely randomized design for the Page's and Hollander test). The percentage rejection difference is defined in Eq. (10). Percentage of rejections for Page's test, Hollander test, the proposed test, denoted by Z_{CombI} and Z_{CombII}, are given by [17], the proposed test version one and proposed test version two are shown in columns Page (%), Hollander (%), Z_{CombI} (%), Z_{CombII} (%), Z_I (%) and Z_{II} (%). *D* is the percentage rejection difference between the two proposed tests.

Selected results are given in Tables 1-7. Selected results for the normal distribution and four treatments, *k*=4, are given in Tables 1 and 6. In Tables 1-3, the results are shown for 40 blocks with various equal sample sizes for the completely randomized portion. In Tables 4-6, the results are shown for 16, 32 and 40 blocks with various unequal sample sizes for the completely randomized portion.

5. Conclusion

In this article, we proposed two nonparametric test statistics for a mixed design consisting of randomized complete block design and completely randomized design. These test statistics are weighted versions of the standardized Hollander test statistic and the standardized *JT* test statistic. When $\sigma_H^2 = \sigma_{JT}^2$, Z_I is a special case of Z_{II}.

$$Z_I = \frac{Z_H + Z_{JT}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{H - U_H}{\sqrt{\sigma_H^2}} + \frac{JT - U_{JT}}{\sqrt{\sigma_{JT}^2}}\right),$$

$$Z_{II} = \frac{H + JT - [E(H) + E(JT)]}{\sqrt{[V(H) + V(JT)]}}$$

$$= \frac{(H + JT) - (u_H + U_{JT})}{\sqrt{\sigma_H^2 + \sigma_{JT}^2}}$$

$$= \frac{\sqrt{\sigma_H^2}}{\sqrt{\sigma_H^2 + \sigma_{JT}^2}} \frac{H - u_H}{\sqrt{\sigma_H^2}} + \frac{\sqrt{\sigma_{JT}^2}}{\sqrt{\sigma_H^2 + \sigma_{JT}^2}} \frac{JT - u_{JT}}{\sqrt{\sigma_{JT}^2}}.$$

In the first version of proposed nonparametric tests, denoted by Z_I, the weights of variances of Hollander test and *JT* test are $1/\sqrt{2}$. In the second version of proposed nonparametric tests, denoted by Z_{II}, if the variance of Hollander test is larger, then Hollander test will get more weight; otherwise, *JT* test will get more weight.

The results showed that the empirical type I error rate of all tests are close to nominal level within acceptable values ranging between 4.38 and 5.48 over all of the cases considered.

In Tables 1 and 2, one can see that the power of the Z_{II} test statistic is superior to other tests. In Table 3, the Z_I test statistic has higher powers than the Z_{II} test statistic in some cases. We noticed as in comparing Tables 1,2 and 3, when the sample size, *n*, is equal, the rejection percentage of Z_{II} gradually decreases, which can be explained by weights. Since the sample size of Hollander test is more than *JT* test, the weight of the Hollander test will be higher and the power of the Hollander test is higher than the *JT* test. When the sample size of each treatment in the completely randomized portion starts increasing, the weight of the *JT* test will increase with the power.

In Tables 4, 5 and 6, unequal sample sizes, n_i, are considered. The proposed two nonparametric test versions, Z_I and Z_{II}, again have higher powers of any of the tests. When Tables 4 and 5 are compared, the rejection percentage of Z_{II} gradually increases because of weights. When the sample size of each treatment in the randomized complete block portion starts increasing, the weight of the Hollander test will increase as the power increases. When Tables 5 and 6 are compared, there is no noticeable difference in terms of the rejection percentage of Z_{II} because both cases are considered, so that the completely randomized portion is 1/8 that of the randomized complete block portion.

The overall recommendation is to use either Z_I or Z_{II} tests. If the completely randomized portion is 1/8 or 1/4 that of the randomized complete block portion, Z_{II} test is recommended. If the completely randomized portion is 1/2 that of the randomized complete block portion, Z_I and Z_{II} tests are recommended together.

Table 7 give the weights of the Hollander's test and *JT* test for the first and second proposed tests versions, Z_I and Z_{II}. The weights were compared with the rejection percentage results. Moreover, in this table, number of treatments is considered *k*=3 for exponential distribution and *k*=5 for student's *t* distribution respectively.

In Table 7, for unequal cases, if the weight of *JT* test is higher in Z_{II}, the first proposed version is better. For example, the first case: *k*=3, block=16, $n_1 = 8, n_2 = n_3 = n_4 = 4$, the second case: *k*=4, block=16, $n_1 = 8, n_2 = n_3 = n_4 = 4$ and the third case: *k*=5, block=16, $n_1 = 8, n_2 = n_3 = n_4 = n_5 = 4$. Therefore, it is better to use Z_I, in these cases. If the weight of Hollander test is higher in Z_{II}, the second proposed version is better.

For equal cases, the weight of Hollander test is higher in Z_{II} in all situations. Therefore, it is better to use Z_{II}, in these cases.

The range of average *D* is (-10.94, -0.04). The case for exponential distributions, the number of treatment is 3, block = 16, n = 4 has the lowest average *D* which equals -0.04. The case for *t* distributions, the number of treatment is 5, block = 16, *n* = 8 has the highest average *D* which equals -10.94. Since *D* < 0, the second proposed test, Z_{II}, is better.

The range of average *D* is (0.08, 4.45). The case for exponential distributions, the number of treatment is 3, block = 16, $n_1 = 8, n_2 = n_3 = 4$ has the lowest average *D* which equals 0.08. The case for *t* distributions, the number of treatment is 5, block = 16, $n_1 = 8, n_2 = n_3 = n_4 = n_5 = 4$ has the highest average *D* which equals 4.45. Since *D* > 0, the first proposed test, Z_I, is better.

Table 1. Percentage of rejection for *k*=4; Normal distributions: block = 40 and *n* = 5

$u1$	$u2$	$u3$	$u4$	P(%)	H(%)	Z_{CombI}(%)	Z_{CombII} (%)	Z_I (%)	Z_{II} (%)	D
0.000	0.000	0.000	0.000	4.60	4.60	4.48	4.38	4.90	4.74	3.26
0.000	0.100	0.200	0.300	33.22	38.76	32.84	35.30	36.58	**39.80**	-8.80
0.000	0.000	0.250	0.250	32.96	38.28	32.64	35.12	36.44	**39.22**	-7.63
0.000	0.125	0.250	0.250	24.42	32.72	27.62	29.10	30.62	**33.44**	-9.21
0.000	0.000	0.000	0.500	56.34	65.06	55.98	59.06	61.26	**66.82**	-9.08
0.050	0.100	0.300	0.500	59.08	68.28	59.00	61.68	65.56	**69.88**	-6.59
0.000	0.000	0.500	0.500	78.30	85.98	77.48	80.38	83.04	**87.26**	-5.08
0.000	0.250	0.500	0.500	68.78	76.82	67.92	70.88	73.92	**78.08**	-5.63
0.000	0.500	0.500	1.000	97.40	99.18	97.16	98.18	98.64	**99.40**	-0.77
0.000	0.250	0.250	0.500	57.10	65.90	57.00	59.96	62.62	**67.46**	-7.73
0.000	0.250	0.250	0.250	22.34	27.42	22.58	23.86	26.08	**28.08**	-7.67
0.100	0.200	0.600	1.000	98.18	99.64	98.24	98.80	99.24	**99.72**	-0.48
0.250	0.250	0.500	0.500	32.96	38.50	32.64	35.12	36.30	**39.80**	-9.64
0.000	0.100	0.300	0.700	87.18	93.22	86.16	89.12	90.38	**94.14**	-4.16
0.000	0.050	0.150	0.350	39.88	45.94	39.86	42.56	43.56	**47.20**	-8.36
0.000	0.150	0.200	0.500	59.26	67.49	59.06	61.90	64.14	**69.04**	-7.64
0.000	0.000	0.100	0.600	74.20	81.88	73.28	76.40	77.76	**83.32**	-7.15
0.000	0.000	0.050	0.300	30.94	37.18	30.52	32.86	34.50	**38.08**	-10.38

Table 2. Percentage of rejection for *k*=4; Normal distributions: block = 40 and *n* = 10

$u1$	$u2$	$u3$	$u4$	P(%)	H(%)	Z_{CombI}(%)	Z_{CombII} (%)	Z_I (%)	Z_{II} (%)	D
0.000	0.000	0.000	0.000	4.70	4.60	5.16	5.48	4.78	4.62	3.34
0.000	0.100	0.200	0.300	39.40	38.76	39.52	29.78	43.02	**43.10**	-0.19
0.000	0.000	0.250	0.250	38.76	38.28	39.02	30.34	41.56	**42.50**	-2.26
0.000	0.125	0.250	0.250	33.00	32.72	32.54	25.50	34.86	**36.44**	-4.53
0.000	0.000	0.000	0.500	65.84	65.06	64.96	51.64	68.92	**70.39**	-2.13
0.050	0.100	0.300	0.500	68.78	68.28	68.00	54.62	73.34	**74.22**	-1.20
0.000	0.000	0.500	0.500	86.28	85.98	85.80	72.74	87.92	**89.86**	-2.21
0.000	0.250	0.500	0.500	77.50	76.82	77.38	62.90	81.26	**82.06**	-0.98
0.000	0.500	0.500	1.000	99.36	99.18	99.12	95.92	99.68	**99.72**	-0.04
0.000	0.250	0.250	0.500	65.96	65.90	66.16	52.52	70.94	**71.30**	-0.51
0.000	0.250	0.250	0.250	26.08	27.42	26.58	21.34	28.22	**28.32**	-0.35
0.100	0.200	0.600	1.000	99.44	99.64	99.28	96.68	99.83	**99.90**	-0.07
0.250	0.250	0.500	0.500	37.82	38.50	39.02	30.34	40.88	**41.52**	-1.57
0.000	0.100	0.300	0.700	93.22	93.22	93.08	82.54	94.88	**95.67**	-0.83
0.000	0.050	0.150	0.350	46.90	45.94	47.28	36.60	50.63	**51.42**	-1.56
0.000	0.150	0.200	0.500	68.12	67.49	68.38	54.64	71.58	**73.08**	-2.10
0.000	0.000	0.100	0.600	81.94	81.88	82.34	68.84	85.54	**86.76**	-1.43
0.000	0.000	0.050	0.300	36.55	37.18	36.42	28.10	39.30	**40.33**	-2.62

Table 3. Percentage of rejection for k=4; Normal distributions: block = 40 and n = 20

$u1$	$u2$	$u3$	$u4$	P(%)	H(%)	Z_{CombI}(%)	Z_{CombII} (%)	Z_I (%)	Z_{II} (%)	D
0.000	0.000	0.000	0.000	4.80	4.90	5.22	4.96	4.58	4.60	-0.43
0.000	0.100	0.200	0.300	32.90	38.46	48.24	32.40	50.10	**50.20**	-0.20
0.000	0.000	0.250	0.250	33.32	39.66	47.60	32.40	49.70	**50.66**	-1.93
0.000	0.125	0.250	0.250	27.20	33.00	40.16	27.32	41.82	**42.22**	-0.96
0.000	0.000	0.000	0.500	56.24	65.28	75.98	53.88	78.52	**79.20**	-0.87
0.050	0.100	0.300	0.500	59.58	67.17	78.70	57.34	81.64	**81.74**	-0.12
0.000	0.000	0.500	0.500	78.78	86.78	92.78	75.74	95.08	**95.54**	-0.48
0.000	0.250	0.500	0.500	68.38	76.75	86.52	65.60	88.94	**89.16**	-0.25
0.000	0.500	0.500	1.000	97.50	99.24	99.82	96.22	**99.96**	99.92	0.04
0.000	0.250	0.250	0.500	56.18	64.75	76.08	54.84	78.42	**79.00**	-0.74
0.000	0.250	0.250	0.250	22.52	26.97	32.62	22.50	**34.68**	34.26	1.21
0.100	0.200	0.600	1.000	98.44	99.52	99.88	97.12	**99.99**	99.96	0.03
0.250	0.250	0.500	0.500	33.32	39.76	47.60	32.40	49.70	**50.56**	-1.73
0.000	0.100	0.300	0.700	87.14	93.40	97.28	84.92	98.52	**98.66**	-0.14
0.000	0.050	0.150	0.350	39.76	46.33	57.22	38.32	59.56	**60.00**	-0.74
0.000	0.150	0.200	0.500	59.52	67.20	78.72	56.94	**82.42**	81.38	1.26
0.000	0.000	0.100	0.600	73.88	82.36	90.62	71.70	93.12	**93.16**	-0.04
0.000	0.000	0.050	0.300	30.28	35.90	44.20	29.96	**46.58**	46.54	0.09

Table 4. Percentage of rejection for k=4; Normal distributions: block = 16 and $n_1 = 8$ and $n_2 = n_3 = n_4 = 4$

$u1$	$u2$	$u3$	$u4$	P(%)	H(%)	Z_{CombI}(%)	Z_{CombII} (%)	Z_I (%)	Z_{II} (%)	D
0.000	0.000	0.000	0.000	5.10	4.90	5.00	5.20	5.12	5.06	1.17
0.000	0.100	0.200	0.300	20.14	20.84	23.04	22.34	**24.18**	24.00	0.74
0.000	0.000	0.250	0.250	20.20	21.88	22.24	21.64	22.68	**24.09**	-6.22
0.000	0.125	0.250	0.250	17.02	18.54	20.54	19.94	**20.66**	20.18	2.32
0.000	0.000	0.000	0.500	32.22	33.98	33.98	32.64	36.44	**38.70**	-6.20
0.050	0.100	0.300	0.500	33.76	38.66	37.90	36.64	42.46	**43.98**	-3.58
0.000	0.000	0.500	0.500	47.20	52.38	53.16	51.14	56.66	**58.58**	-3.39
0.000	0.250	0.500	0.500	39.98	45.51	48.22	46.92	**53.33**	52.83	0.94
0.000	0.500	0.500	1.000	76.00	81.76	86.40	84.74	**89.34**	88.68	0.74
0.000	0.250	0.250	0.500	32.82	34.30	38.40	37.18	**41.44**	39.94	3.62
0.000	0.250	0.250	0.250	15.04	14.84	18.56	18.42	**17.29**	16.66	3.64
0.100	0.200	0.600	1.000	79.32	83.78	85.84	83.52	88.62	**89.75**	-1.28
0.250	0.250	0.500	0.500	20.20	21.54	22.24	21.64	23.20	**23.64**	-1.90
0.000	0.100	0.300	0.700	56.52	61.46	63.30	60.20	67.02	**68.42**	-2.09
0.000	0.050	0.150	0.350	23.66	25.20	26.16	25.16	**28.32**	28.24	0.28
0.000	0.150	0.200	0.500	34.38	36.74	38.62	37.38	42.24	**42.62**	-0.90
0.000	0.000	0.100	0.600	43.60	48.10	46.70	44.54	51.92	**54.32**	-4.62
0.000	0.000	0.050	0.300	18.94	20.36	20.60	19.66	21.84	**22.48**	-2.93

Table 5. Percentage of rejection for k=4; Normal distributions: block = 32 and $n_1 = 8$ and and $n_2 = n_3 = n_4 = 4$

$u1$	$u2$	$u3$	$u4$	P(%)	H(%)	Z_{CombI}(%)	Z_{CombII} (%)	Z_I (%)	Z_{II} (%)	D
0.000	0.000	0.000	0.000	5.28	4.68	4.78	4.62	4.93	4.68	5.07
0.000	0.100	0.200	0.300	30.24	33.62	31.06	30.84	33.96	**35.54**	-4.65
0.000	0.000	0.250	0.250	29.66	33.70	29.84	30.06	31.66	**34.82**	-9.98
0.000	0.125	0.250	0.250	25.22	27.82	26.12	26.20	27.43	**28.92**	-5.43
0.000	0.000	0.000	0.500	50.86	56.36	48.08	48.78	52.18	**58.02**	-11.19
0.050	0.100	0.300	0.500	54.04	57.98	52.72	53.62	56.14	**60.81**	-8.32
0.000	0.000	0.500	0.500	72.50	79.30	71.14	72.02	75.64	**81.36**	-7.56
0.000	0.250	0.500	0.500	62.48	68.46	64.48	65.12	70.48	**71.41**	-1.32
0.000	0.500	0.500	1.000	95.22	97.88	95.76	96.18	98.02	**98.56**	-0.55
0.000	0.250	0.250	0.500	52.08	56.87	52.90	53.34	57.84	**59.84**	-3.46
0.000	0.250	0.250	0.250	20.56	23.88	22.84	22.56	25.84	**25.40**	1.70
0.100	0.200	0.600	1.000	96.26	98.32	95.88	96.44	97.70	**98.82**	-1.15
0.250	0.250	0.500	0.500	29.66	32.66	29.84	30.06	31.18	**34.08**	-9.30
0.000	0.100	0.300	0.700	82.56	88.24	81.14	82.16	85.86	**89.58**	-4.33
0.000	0.050	0.150	0.350	36.46	39.24	35.92	36.22	38.36	**40.54**	-5.68
0.000	0.150	0.200	0.500	54.30	59.68	53.70	54.16	58.32	**61.70**	-5.80
0.000	0.000	0.100	0.600	67.96	73.66	65.02	65.92	68.89	**75.44**	-9.51
0.000	0.000	0.050	0.300	27.84	31.50	26.92	27.00	28.99	**32.82**	-13.21

Table 6. Percentage of rejection for k=4; Normal distributions: block = 40 and and $n_1 = 10$ and $n_2 = n_3 = n_4 = 5$

$u1$	$u2$	$u3$	$u4$	P(%)	H(%)	Z_{CombI}(%)	Z_{CombII} (%)	Z_I (%)	Z_{II} (%)	D
0.000	0.000	0.000	0.000	5.34	5.28	5.24	5.28	4.88	5.22	-6.96
0.000	0.100	0.200	0.300	33.98	39.42	35.32	34.44	39.26	**41.33**	-5.27
0.000	0.000	0.250	0.250	33.90	39.28	34.58	33.50	37.12	**41.09**	-10.70
0.000	0.125	0.250	0.250	28.48	32.26	30.58	29.84	33.18	**34.08**	-2.71
0.000	0.000	0.000	0.500	56.80	64.24	55.12	53.18	59.78	**66.40**	-11.07
0.050	0.100	0.300	0.500	59.42	69.48	60.00	58.56	66.44	**71.48**	-7.59
0.000	0.000	0.500	0.500	78.04	86.32	79.40	77.64	84.44	**87.94**	-4.14
0.000	0.250	0.500	0.500	67.88	78.44	72.40	70.86	78.92	**80.46**	-1.95
0.000	0.500	0.500	1.000	97.78	99.36	98.64	98.40	99.32	**99.46**	-0.14
0.000	0.250	0.250	0.500	56.66	66.62	60.14	58.82	66.14	**69.10**	-4.48
0.000	0.250	0.250	0.250	23.70	26.91	27.08	26.48	27.86	**27.96**	-0.36
0.100	0.200	0.600	1.000	98.34	99.56	98.70	98.26	99.16	**99.70**	-0.54
0.250	0.250	0.500	0.500	33.90	38.58	34.58	33.50	36.96	**39.98**	-8.17
0.000	0.100	0.300	0.700	87.48	94.14	88.20	86.84	92.00	**95.00**	-3.26
0.000	0.050	0.150	0.350	40.26	47.74	41.40	39.96	45.83	**49.50**	-8.01
0.000	0.150	0.200	0.500	59.58	68.58	60.94	59.48	66.38	**70.86**	-6.75
0.000	0.000	0.100	0.600	74.20	81.66	73.28	71.36	77.40	**83.42**	-7.78
0.000	0.000	0.050	0.300	31.78	37.12	31.22	30.16	33.56	**38.22**	-13.89

Table 7. Comparing the Highest Percentages of Rejection with Weights of Hollander's Test and *JT* Test (equal cases) for Z_{II}

Case		Z_I	Z_{II}	$\frac{\sigma_H^2}{\sigma_H^2+\sigma_{JT}^2}$	$\frac{\sigma_{JT}^2}{\sigma_H^2+\sigma_{JT}^2}$
Block	n_i				
K=3					
16	$n_1 = 8. n_2 = n_3 = 4$	X		0.3200	0.6800
32	$n_1 = 8. n_2 = n_3 = 4$		X	0.7834	0.2166
40	$n_1 = 10. n_2 = n_3 = 5$		X	0.7830	0.2170
K=4					
16	$n_1 = 8. n_2 = n_3 = n_4 = 4$	X		0.2028	0.7972
32	$n_1 = 8. n_2 = n_3 = n_4 = 4$		X	0.6623	0.3377
40	$n_1 = 10. n_2 = n_3 = n_4 = 5$		X	0.6614	0.3386
K=5					
16	$n_1 = 8. n_2 = n_3 = n_4 = n_5 = 4$	X		0.1311	0.8689
32	$n_1 = 8. n_2 = n_3 = n_4 = n_5 = 4$		X	0.5383	0.4617
40	$n_1 = 10. n_2 = n_3 = n_4 = n_5 = 5$		X	0.8026	0.1974

Concflicts of interest

There is no conflict of interest.

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