



An efficient method for the plane vibration analysis of composite sandwich beam with an orthotropic core

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Abstract

The free vibration characteristics of composite sandwich beam is examined using an efficient numerical solution scheme. The simply supported beam is assumed to be composed of two isotropic face sheets and an orthotropic core. The plane elasticity formulations are used to derive the equations of motion and the reduced governing differential equation is solved by Complementary Functions Method. The dimensionless analysis of natural frequencies is done to acquire the high precision along with few divisions. The influences of material and geometric parameters on the natural frequency are also illustrated. The solutions are validated with the results obtained from finite element software and it is shown that presented method is an efficient solution technique for the vibration problems of composite beams with a core.

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1. Introduction

Sandwich composites have been extensively using as a structural member in engineering areas such as automotive, aerospace, computer, biomedical and so on. Sandwich composites have been found in structural systems like beams, plates, annular members. Accordingly, understanding the dynamic behavior of such systems becomes necessary to improve their performance and the mathematical analysis and developing effective numerical methods are important to achieve this aim.

The papers related with the free vibration analysis of cored composited beams are introduced. Arikoglu and Ozkol[1] have studied the vibration behavior of sandwich beam using Differential Transform Method. The experimental and numerical natural frequency analysis of sandwich beam have been conducted by Baba[2]. In his work, the beam made of fiber-glass laminate skins wrapped over polyurethane foam core is subjected to clamped-clamped boundary conditions. The vibration characteristics of layered beam are examined by Lou et. al.[3] where the core material is pyramidal lattice truss. The viscoelastic-core composite beam is investigated by Sadeghnejad et. al.[4] using extended high-order sandwich panel theory. They have considered the effects of transverse shear and core compressibility. The beam with lattice truss core has been studied by Xu and Qui[5] using the

interval optimization method. In their study, the combining theory of Euler–Bernoulli beam and Timoshenko beam is used. Wang and Wang[6] have investigated the free vibration behavior of soft-core layered beams using the extended high-order sandwich panel theory and weak form quadrature element method. Cheng et. al.[7] have examined the vibration analysis of fiber-reinforced polymer honeycomb sandwich beam using refined sandwich beam theory. The shear-deformable Timoshenko porous beam is studied by Chen et. al.[8] where Ritz method in combination with a direct iterative algorithm is employed. The zig-zag beam theory is applied by Khdeir and Aldraihem[9] to the free vibration of soft-core laminated beams. Zhang et. al.[10] have studied the vibration of honeycomb-corrugation hybrid core beam using finite element method. Demir et. al.[11] have investigated natural frequency of curved beam with laminated face sheets and a viscoelastic core using general differential quadrature method. Chanthanumataporn and Watanabe[12] have presented a finite element solution on the free vibration of a layered beam coupling with ambient air where the shear deformation of the sandwich core is considered. The free and forced vibration behavior of sandwich beam having carbon/epoxy face sheets and a magnetorheological elastomer honeycomb core are examined numerically and experimentally by Eloy et. al.[13] where the magnetic field is assumed to be applied on the free end and on the center of the beam.

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Rahmani et. al.[14] have investigated the vibration characteristics of a flexible functionally-graded core beam where the classical beam theory for the face sheets and elasticity theory for core are employed. The natural frequency study has been conducted by Asgari et. al.[15] on the free vibration of simply supported soft-core beam resting on a nonlinear foundation. Xu et. al.[16] have presented a continuous homogeneous theory to derive the governing equations of free vibration problems of sandwich beam with graded corrugated lattice core.

The problem addressed in the present study is to analyze the free vibration of three-layered sandwich beam having an orthotropic core. The plane elasticity formulations are employed and the coupled second-order governing differential equations are reduced to a fourth-order differential equation. The face sheets and core material are assumed to be interconnected through the equilibrium and compatibility. The layers are considered to be in a perfect bond with each other. CFM has been proven to be an accurate and efficient numerical method [17-19] for the present type problem is infused into the analysis.

2. Theoretical Analysis

Consider a rectangular cross-section sandwich beam of length L and depth h . The beam is assumed to be under the conditions of plane stress in the x - z plane hence it has a unit width. The governing equation of motion including faces and core is given as follows:

$$\frac{\partial \sigma_{xxi}}{\partial x} + \frac{\partial \tau_{xzi}}{\partial z} = \rho_i \frac{\partial^2 u_i}{\partial t^2} \tag{1}$$

$$\frac{\partial \tau_{xzi}}{\partial x} + \frac{\partial \sigma_{zzi}}{\partial z} = \rho_i \frac{\partial^2 w_i}{\partial t^2} \quad i = b, c, t$$

where σ and τ are normal and shear stresses, ρ is material density, $u(x, z, t)$ and $w(x, z, t)$ are the axial and transverse displacements, respectively. Here subscripts b , c and t are, respectively, denote bottom face, core and top face.

The constitutive equations are written as follows:

$$\begin{Bmatrix} \sigma_{xxi} \\ \sigma_{zzi} \\ \tau_{xzi} \end{Bmatrix} = \begin{bmatrix} C_{11}^i & C_{13}^i & 0 \\ C_{13}^i & C_{33}^i & 0 \\ 0 & 0 & C_{55}^i \end{bmatrix} \begin{Bmatrix} \epsilon_{xxi} \\ \epsilon_{zzi} \\ \gamma_{xzi} \end{Bmatrix} \tag{2}$$

The strain components in the terms of horizontal and vertical displacements:

$$\epsilon_{xxi} = \frac{\partial u_i}{\partial x}, \quad \epsilon_{zzi} = \frac{\partial w_i}{\partial z}, \quad \gamma_{xzi} = \frac{\partial u_i}{\partial z} + \frac{\partial w_i}{\partial x} \tag{3}$$

Substituting Eqs. (2, 3) and into Eq. (1) give the following two equations in the axial and transverse displacements:

$$\frac{\partial}{\partial x} \left(C_{11}^i \frac{\partial u_i}{\partial x} + C_{13}^i \frac{\partial w_i}{\partial z} \right) + \frac{\partial}{\partial z} \left(C_{55}^i \frac{\partial u_i}{\partial z} + C_{55}^i \frac{\partial w_i}{\partial x} \right) = \rho_i \frac{\partial^2 u_i}{\partial t^2} \tag{4}$$

$$\frac{\partial}{\partial x} \left(C_{55}^i \frac{\partial u_i}{\partial z} + C_{55}^i \frac{\partial w_i}{\partial x} \right) + \frac{\partial}{\partial z} \left(C_{11}^i \frac{\partial u_i}{\partial x} + C_{13}^i \frac{\partial w_i}{\partial z} \right) = \rho_i \frac{\partial^2 w_i}{\partial t^2}$$

The typical boundary conditions for the simply supported beam are:

$$w_i(0, z, t) = w_i(L, z, t) = 0 \tag{5}$$

$$\sigma_{xxi}(0, z, t) = \sigma_{xxi}(L, z, t) = 0$$

The displacements satisfying the boundary conditions above are assumed as follows:

$$u_i(x, z, t) = U_i(z) \cos[\xi x] \sin[\omega x] \tag{6}$$

$$w_i(x, z, t) = W_i(z) \sin[\xi x] \sin[\omega x]$$

where $\xi = m\pi/L$ and ω is the natural frequency. Substituting Eq. (6) into Eq. (4) leads to a pair of coupled differential equations for $U(z)$ and $W(z)$:

$$A_1^i U_i'' + A_3^i U_i + A_4^i W_i' = 0 \tag{7}$$

$$B_1^i U_i' + B_3^i W_i'' + B_5^i W_i = 0 \tag{8}$$

where ()' denotes the derivative with respect to transverse coordinate z and

$$A_1^i = C_{55}^i, \quad A_3^i = [\rho_i \omega^2 - C_{11}^i \xi^2], \quad A_4^i = [C_{13}^i \xi + C_{55}^i \xi]$$

$$B_1^i = [-C_{13}^i \xi - C_{55}^i \xi], \quad B_3^i = C_{33}^i, \quad B_5^i = [\rho_i \omega^2 - C_{55}^i \xi^2]$$

These coupled second order equations must be reduced to a simple fourth-order differential equation. In order to obtain an uncoupled governing equation, Eq. is differentiated with respect to z and rearranged as:

$$U_i''' + C_2^i U_i' + C_3^i W_i'' = 0 \tag{9}$$

where $C_2^i = \frac{A_3^i}{A_1^i}$ and $C_3^i = \frac{A_4^i}{A_1^i}$.

The differentiation Eq. (9) gives a fourth order equation as follows:

$$U_i^{iv} + C_2^i U_i'' + C_3^i W_i'' = 0 \tag{10}$$

The first derivative of transverse displacement w can be obtained from Eq. (7). Taking the derivative of obtained equation and substituting it into Eq. (8) gives the followings for transverse displacement function:

$$W_i = \frac{1}{J_4^i} U_i''' + \frac{J_2^i}{J_4^i} U_i' \tag{11}$$

where $J_2^i = C_3^i D_3^i$ and $J_4^i = C_3^i D_2^i$.

Substituting Eq. (9) and its derivatives, the simple forth-order uncoupled governing differential equation may be obtained as:

$$U_i^{iv} + H_2^i U_i'' + H_4^i U_i = 0 \tag{12}$$

where $H_2^i = [D_2^i + C_3^i D_3^i]$ and $H_4^i = [C_2^i D_2^i]$.

The beam is assumed to be composed of three layers, namely bottom, core and top, 12 constants (4 for each layer) are to be determined. The traction-free boundary conditions at the bottom and top surfaces are:

$$\tau_{xzb}(x, 0, t) = \left[C_{55}^b \frac{\partial u_b}{\partial z} + C_{55}^b \xi \frac{\partial w_b}{\partial x} \right] = 0 \tag{13}$$

$$\sigma_{zxb}(x, 0, t) = \left[C_{13}^b \frac{\partial u_b}{\partial x} + C_{33}^b \frac{\partial w_b}{\partial z} \right] = 0 \tag{14}$$

$$\tau_{xzt}(x, h, t) = \left[C_{55}^t \frac{\partial u_t}{\partial z} + C_{55}^t \xi \frac{\partial w_t}{\partial x} \right] = 0 \tag{15}$$

$$\sigma_{zxt}(x, h, t) = \left[C_{13}^t \frac{\partial u_t}{\partial x} + C_{33}^t \frac{\partial w_t}{\partial z} \right] = 0 \tag{16}$$

Displacement continuity conditions through lamina interfaces:

$$u_i = u_{i+1} \tag{17}$$

$$w_i = w_{i+1} \quad i = 1, 2, 3 \tag{18}$$

Axial and shear stress continuity conditions through lamina interfaces:

$$\sigma_{zzi} = \sigma_{zzi+1} \tag{19}$$

$$\tau_{xzi} = \tau_{xzi+1} \quad i = 1, 2, 3 \tag{20}$$

Upon substitution of displacements given by Eq. (6), the necessary conditions above may be obtained in the terms of U_i and W_i :

$$\left[C_{55}^b \frac{\partial U_b}{\partial z} + C_{55}^b \xi W_b \right]_{z=0} = 0 \tag{21}$$

$$\left[-C_{13}^b \xi U_b + C_{33}^b \frac{\partial W_b}{\partial z} \right]_{z=0} = 0 \tag{22}$$

$$\left[C_{55}^t \frac{\partial U_t}{\partial z} + C_{55}^t \xi W_t \right]_{z=h} = 0 \tag{23}$$

$$\left[-C_{13}^t \xi U_t + C_{33}^t \frac{\partial W_t}{\partial z} \right]_{z=h} = 0 \tag{24}$$

at the lamina interfaces:

$$U_i = U_{i+1} \tag{25}$$

$$W_i = W_{i+1} \tag{26}$$

$$\left[C_{55}^i \frac{\partial U_i}{\partial z} + C_{55}^i \xi W_i \right] = \left[C_{55}^{i+1} \frac{\partial U_{i+1}}{\partial z} + C_{55}^{i+1} \xi W_{i+1} \right] \tag{27}$$

$$\left[-C_{13}^i \xi U_i + C_{33}^i \frac{\partial W_i}{\partial z} \right] = \left[-C_{13}^{i+1} \xi U_{i+1} + C_{33}^{i+1} \frac{\partial W_{i+1}}{\partial z} \right] \quad i = 1, 2, 3 \tag{28}$$

The stiffness matrix for the isotropic face sheets and orthotropic core are, respectively:

$$C_{jk}^{b,t} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E(1-\nu)}{2(1-\nu^2)} \end{bmatrix} \tag{29}$$

$$C_{jk}^c = \begin{bmatrix} \frac{E_1}{1-\nu_{13}\nu_{31}} & \frac{E_3\nu_{13}}{1-\nu_{13}\nu_{31}} & 0 \\ \frac{E_3\nu_{13}}{1-\nu_{13}\nu_{31}} & \frac{E_3}{1-\nu_{13}\nu_{31}} & 0 \\ 0 & 0 & G_{13} \end{bmatrix} \tag{30}$$

CFM is applied to the present problem as an efficient solution method. The laborious mathematical manipulations and moderate tasks such as integral transformation, finite element model or series solution are not required by solutions steps of CFM[20]. CFM transforms the two-point boundary value problem to a

system of initial-value problems which can be solved by a numerical method. In this case, the fifth-order Runge-Kutta(RK-5) is chosen. A-six-digit accuracy is obtained at 4 intervals through the transverse coordinate for each lamina. The theoretical background and detailed solution steps for the present type of problems with CFM are available in the literature[18, 19, 21-23]. Upon the application of CFM to the problem on hand, the complete solution of Eq. is obtained in following form:

$$U_i = \sum_{j=1}^n e^j U_i^j, \quad i = b, c, t \quad n = 4 \quad (31)$$

Applying the boundary conditions prescribed in Eqs. (21-28) for the particular problem on hand results in the following system of algebraic equations for the coefficients e_i^j :

$$[T_{kl}] \{e_i^j\} = 0 \quad k, l = 1, 2, \dots, 12 \quad (32)$$

where $[T_{kl}]$ including the variables z, ω, ζ is the coefficient matrix. The frequencies which make the determinant of coefficients matrix $[T_{kl}]$ zero are the natural frequencies of the composite beam.

3. Results and Discussion

Two different materials for isotropic face sheets and orthotropic core are considered and the material properties are given in Table 1. The thickness of bottom and top face is equal to each other which is taken as 25mm and the thickness of core is 75mm.

The natural frequencies ω are determined from the coefficient matrix given in Eq. (32). It is mainly concentrated on the effectiveness of CFM for the present type of problems. The efficiency and accuracy of the method are compared to the finite element software and results are tabulated in Table 2. As it can be seen that CFM results match quite well with those of finite element software ones. Four divisions through the transverse coordinate for each laminate are sufficient to obtain a-six-digit accuracy. In the finite element model, the structural solid element having quadratic displacement behavior is used. The element has eight nodes with two degrees of freedom at each node which are translations in the nodal x and z axes. The beam is divided into 200 equal elements for each layer. The element has three nodes and six degrees of freedoms based on the first-order shear deformation theory may also be used in the finite element model[24, 25]. The effects of material properties of isotropic face sheets and orthotropic core, wave number and length-to-thickness ratio (L/h) are also examined. Table 3 and 4 show the variation of non-dimensional natural

frequencies of Al face sheets beam with property of core material, wave number and length-to-thickness ratio. As an inspection of these tables, using Glass/Epoxy instead of Graphite/Epoxy as a core material decreases the natural frequency. Also, increasing the wave number and decreasing the length-to-thickness ratio increase the natural frequency. The free vibration results of ceramic face sheet (ZrO_2) composite beam are also obtained and illustrated in Figure 1 and 2 for the first three wave numbers. The results are given in figure form preventing the redundancy to properly compare the differences between Al and ZrO_2 face sheets beams. It is seen that zirconia as face sheets decreases the natural frequency.

Table 1. Elastic properties of the material used in the analysis.

	E_1 (GPa)	E_3 (GPa)	G_{13} (GPa)	ν_{13}	ρ (kg / m ³)
Aluminum (Al)	70	70	-	0.3	2707
Zirconia (ZrO ₂)	200	200	-	0.22	5700
Graphite/Epoxy	181	10.3	7.17	0.28	1600
Glass/Epoxy	38.6	8.27	4.14	0.26	1900

Table 2. Comparison of CFM with ANSYS for the first 5 natural frequencies (Hz)

m	CFM	Finite Element
1	1127.67836	1127.68083
2	3015.91541	3015.90980
3	5046.92617	5046.93427
4	7197.83090	7197.83472
5	9462.65677	9462.89252

Table 3. Non-dimensional natural frequencies ($\bar{\omega} = \omega \sqrt{\rho_{Al} / E_{Al}}$) of Al-Graphite/Epoxy-Al composite beam for different wave number

L/h	$\bar{\omega}$				
	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$
4	1.39335	3.72644	6.23594	8.89358	11.69198
	2.61538	5.23176	7.84614	10.46252	13.07890
	9.76088	11.03734	11.83498	13.22415	15.12607
10	0.28431	0.97403	1.83720	2.76555	3.72644
	1.04675	2.09250	3.13826	4.18401	5.23176
	4.01882	7.90453	10.82299	10.88692	11.03734
15	0.13098	0.48284	0.97403	1.53916	2.14191
	0.69750	1.39400	2.09250	2.78901	3.48751
	2.68761	5.33509	7.90453	10.36411	10.83562

Table 4. Non-dimensional natural frequencies $\bar{\omega} = \omega\sqrt{\rho_{Al}/E_{Al}}$ of Al-Glass/Epoxy-Al composite beam for different wave number

L/h	ω				
	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$
4	1.11007	2.97860	4.97436	7.09937	9.11956
	1.82311	3.64722	5.47133	7.29544	9.36666
	5.89605	9.36641	9.98482	11.07141	12.64135
10	0.22439	0.77394	1.46641	2.21098	2.97860
	0.72944	1.45989	2.18833	2.91878	3.64722
	2.36969	4.72858	7.04112	8.91523	9.36641
15	0.10317	0.38195	0.77394	1.22710	1.71103
	0.48663	0.97226	1.45989	1.94552	2.43115
	1.58033	3.15797	4.72858	6.28135	7.77255

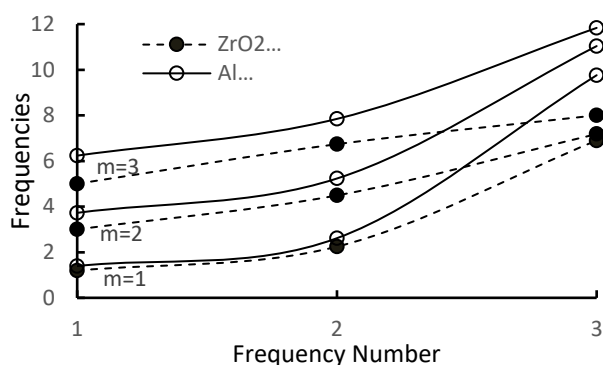


Figure 1. Non-dimensional natural frequencies($\bar{\omega} = \omega\sqrt{\rho_{Al}/E_{Al}}$) of Graphite/Epoxy core beam for $L/h=4$

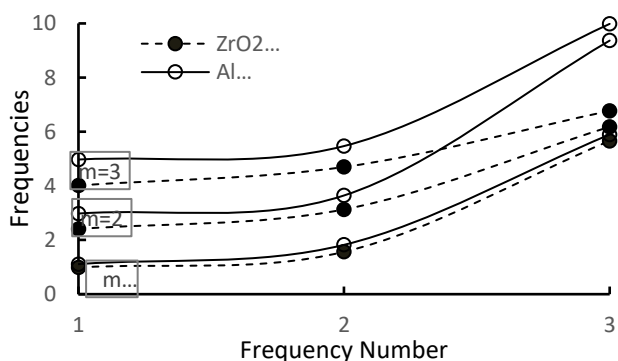


Figure 2. Non-dimensional natural frequencies($\bar{\omega} = \omega\sqrt{\rho_{Al}/E_{Al}}$) of Glass/Epoxy core beam for $L/h=4$

4. Conclusions

Plane vibration analysis of composite beam with an orthotropic core has been conducted. The governing equation of motion is obtained by plane elasticity theory and readily solved by CFM using very coarse collocation points. The results are obtained in the terms of a non-dimensional frequency parameter in order to accelerate the convergence of study. It is observed that

CFM is very convenient numerical technique for the vibration problems of composite core beams. The influences of laminate properties, geometric parameter and wave number on the natural frequency are also examined. The results show that the core material with low elastic property ratio decreases the natural frequency and increasing the wave number and decreasing the geometric parameter also increase the natural frequency.

Conflicts of interest

The authors stated that did not have conflict of interests.

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