Cumhuriyet Science Journal

e-ISSN: 2587-246X
ISSN: 2587-2680

Cumhuriyet Sci. J., 41(2) (2020) 360-368
http://dx.doi.org/10.17776/csj.644158

Common properties and approximations of local function and set operator ψ

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Abstract

Through this paper, we shall obtain common properties of local function and set operator ψ and introduce the approximations of local function and set operator ψ. We also determined expansion of local function and set operator ψ.

Article info

History:
Received: 08.11.2019
Accepted: 01.04.2020
Keywords: Local function, ψ-operator, $A = B[mod \mathcal{I}]$, compatible ideal, codense ideal.

1. Introduction

The study of ideals on topological spaces was first introduced by K. Kuratowski [1]. The authors, Al-Omari and Noiri [2,3], Modak [4,5,6], Modak and Islam [7,8,9,10], Ekici and Elmali [11], Modak and Mistry [12], Khan and Noiri [13], Csaszar [14], Özbakir and Yildirim [15] have introduced the study of ideals on the generalized topological space. We know from [1] that, a subcollection $\mathcal{I}$ of $\wp(X)$, the powerset of X, is called an ideal on X if (i) for $A, B \subseteq X$ and $A \subseteq B \in \mathcal{I}$, $A \in \mathcal{I}$ (hereditary) and (ii) for $A, B \subseteq X$ and $A, B \in \mathcal{I}$, $A \cup B \in \mathcal{I}$ (finite additivity). Hayashi [16] introduced localization properties of ideal topological space (an ideal $\mathcal{I}$ on a topological space (X, τ) is called an ideal topological space and it is denoted as $(X, \tau, \mathcal{I})$). For a subset A of X, the local function of A in the ideal topological space $(X, \tau, \mathcal{I})$, is denoted as A^* and defined as:

$A^* = \{x \in X : U_x \cap A \notin \mathcal{I}, U_x \in \tau(x)\}$, where $\tau(x)$ is collection of all open sets of (X, τ) containing x.

In regards of local function, Natkaniec [17] had defined the ψ operator. For an ideal topological space $(X, \tau, \mathcal{I})$, the ψ operator is defined as follows:

$\psi(A) = X \setminus (X \setminus A)^*$, for every $A \subseteq X$.

Again for its equivalent definition, see [18] and [19].

In this paper, our intension is to study the properties of local function and ψ operator in which topological space, generalized topological space [20], m-space [2], minimal space [21] and supra-topological space [22] etc. are not an essential part. That means we study the properties which hold in any subcollecton of $\wp(X)$. We also characterize the Newcomb's idea $A = B[mod \mathcal{I}]$, τ-boundary [23] and Njastad's compatibility [24]. Secondly, we introduce and study the approximations of local function and the operator ψ. Finally we have

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considered the expansion of above two operators. However A. Pavlovic[25] have studied “local function versus local closure function in ideal topological spaces”.

2. Common Properties

In this section, we shall consider the properties of local function and operator ψ which are not dependent on topology, generalized topology, minimal structure etc.

Let X be a set, $\mathcal{A} \subseteq \wp(X)$ and $\mathcal{I}$ be an ideal on X, then we call $(X,\mathcal{A},\mathcal{I})$ a space.

Definition 2.1. Let $(X,\mathcal{A},\mathcal{I})$ be a space. A set operator $()^{c^*}:\wp(X)\to\wp(X)$, called the common-local function of $\mathcal{I}$ on X with respect to $\mathcal{A}$, is defined as: $(A)^{c^*}(\mathcal{A},\mathcal{I})=\{x\in X : U\cap A\notin\mathcal{I}, \text{for every } U\in\mathcal{A}(x)\}$, where $\mathcal{A}(x)=\{U\in\mathcal{A}: x\in U\}$.

This is simply called $c*$-local function and denoted as A^{c^*}, for $A\subseteq X$.

Following is the example of a $c*$-local function of a set:

Example 2.2. Let $X=\{a,b,c\}$, $\mathcal{A}=\{\{a\},\{b\},\{a,c\}\}\subseteq\wp(X)$ and $\mathcal{I}=\{\phi,\{a\}\}$. Take $B=\{b,c\}$. Then $B^{c^*}=\{b,c\}$.

Theorem 2.3. Let $(X,\mathcal{A},\mathcal{I})$ be a space. Then following hold:

1. $(\varnothing)^{c^*}=\varnothing$.
2. If $A\subseteq B\subseteq X$, $A^{c^*}\subseteq B^{c^*}$.
3. If $I\in\mathcal{I}$, then $(I)^{c^*}=\varnothing$.
4. If $A\subseteq X$ and $I\in\mathcal{I}$, then $(A\setminus I)^{c^*}=(A\cup I)^{c^*}=A^{c^*}$.
5. For an ideal $\mathcal{H}$ on X with $\mathcal{H}\subseteq\mathcal{I}$, $(A)^{c^*}(\mathcal{I})\subseteq(A)^{c^*}(\mathcal{H})$.
6. For an ideal $\mathcal{J}$ on X, $A^{c^*}(\mathcal{I}\cap\mathcal{J})=A^{c^*}(\mathcal{I})\cup B^{c^*}(\mathcal{J})$.

Proof. 1. Obvious from the fact that $\varnothing\in\mathcal{I}$.

2. Let $x\in A^{c^*}$. Then for all $U\in\mathcal{A}(x)$, $U\cap A\notin\mathcal{I}$. Thus $U\cap B\notin\mathcal{I}$, otherwise $U\cap A\in\mathcal{I}$. Hence the result.

3. It follows from the fact that, for any $U\in\mathcal{A}$, $U\cap I\in\mathcal{I}$, since $I\in\mathcal{I}$.

4. Claim: $A^{c^*}\subseteq(A\setminus I)^{c^*}$.

Let $x\in(A)^{c^*}$. Then for all $U\in\mathcal{A}(x)$, $A\cap U\notin\mathcal{I}$. If possible, suppose that $U\cap(A\setminus I)\in\mathcal{I}$. Then for some $J\in\mathcal{I}$, $U\cap(A\setminus I)=J$. Then $(U\cap A)\setminus I=J$, and hence $U\cap A=I\cup J\in\mathcal{I}$, a contradiction.

Claim: $(A\cup I)^{c^*}\subseteq A^{c^*}$.

Let $x\in(A\cup I)^{c^*}$. Then for all $U_x\in\mathcal{A}(x), U_x\cap(A\cup I)\notin\mathcal{I}$. If possible, suppose that $A\cap U_x\in\mathcal{I}$.

Then for some $J\in\mathcal{I}$, $A\cap U_x=J$. Note that $U_x\cap(A\cup I)=(U_x\cap A)\cup(U_x\cap I)=J\cup(U_x\cap I)$. Since $U_x\cap I\subseteq I$, then $U_x\cap(A\cup I)\in\mathcal{I}$, a contradiction.

5. It follows from the fact that every member of $\mathcal{H}$ is also a member of $\mathcal{I}$.

6. From (5), $A^{c*}(\mathcal{I}) \cup A^{c*}(\mathcal{J}) \subseteq A^{c*}(\mathcal{I} \cap \mathcal{J})$. Let $x \in A^{c*}(\mathcal{I} \cap \mathcal{J})$. Then for all $U \in \mathcal{A}(x)$, $U \cap A \notin \mathcal{I} \cap \mathcal{J}$. Thus $U \cap A \notin \mathcal{I}$ or $U \cap A \notin \mathcal{J}$. This implies that $x \in A^{c*}(\mathcal{I})$ or $x \in A^{c*}(\mathcal{J})$, and hence $x \in A^{c*}(\mathcal{I}) \cup A^{c*}(\mathcal{J})$.

Lemma 2.4. Let $(X, \mathcal{A}, \mathcal{I})$ be a space. Then $\mathcal{A} \cap \mathcal{I} = \{\varnothing\}$ if and only if $X = X^{c*}$.

Proof. Suppose $\mathcal{A} \cap \mathcal{I} = \{\varnothing\}$. It is obvious that $X^{c*} \subseteq X$. For reverse inclusion, let $x \in X$. If possible suppose that $x \notin X^{c*}$. Then there exists $U_x \in \mathcal{A}(x)$ such that $U_x \cap X \in \mathcal{I}$. This implies that $U_x \in \mathcal{I}$, a contradiction.

Conversely, suppose that $X = X^{c*}$ holds. If possible, suppose that $U \in \mathcal{A} \cap \mathcal{I}$, where $x \in U$. Then $U \cap X \in \mathcal{I}$, by hereditary property. Thus $x \notin X^{c*}$, a contradiction.

If an ideal $\mathcal{I}$ satisfies the property $\mathcal{A} \cap \mathcal{I} = \{\varnothing\}$, then the ideal $\mathcal{I}$ is called $\mathcal{A}$-codense ideal.

This property is similar to Dontchev, Ganster and Rose's [26] 'codense' ideal. An ideal $\mathcal{I}$ in an ideal topological space is called codense ideal if $\mathcal{I} \cap \tau = \{\varnothing\}$. Newcomb [23], Hamlett and Jankovic[18] called such ideal as 'τ-boundary' whereas Dontchev [27] called such spaces as 'Hayashi-Samuel' spaces. In fact such ideals play a very important role in the study of ideals (see: [6,19,28,29,30]).

We define an operator similar to Natkaniec's ψ-operator [17]:

Definition 2.5. Let $(X, \mathcal{A}, \mathcal{I})$ be a space. An operator $\psi_c : \wp(X) \to \wp(X)$ is defined as follows: for every $A \in \wp(X), \psi_c(A) = \{x \in X :$ there exists a $U \in \mathcal{A}(x)$ such that $U \setminus A \in \mathcal{I}\}$.

Equivalently $\psi_c(A) = X \setminus (X \setminus A)^{c*}$.

Proof. Let $x \in X \setminus (X \setminus A)^{c*}$. Then $x \notin (X \setminus A)^{c*}$, and thus there exists $U \in \mathcal{A}(x)$ such that $U \cap (X \setminus A) \in \mathcal{I}$. So $U \setminus A \in \mathcal{I}$. Hence $x \in \psi_c(A)$.

Let $x \in \psi_c(A)$. Then from definition, there exists $U \in \mathcal{A}(x)$ such that $U \setminus A \in \mathcal{I}$. This implies that $U \cap (X \setminus A) \in \mathcal{I}$. Thus $x \notin (X \setminus A)^{c*}$, and hence $x \in X \setminus (X \setminus A)^{c*}$.

Here we find out the value of ψ_c of a set in a space.

Example 2.6. Let $X = \{a,b,c\}$, $\mathcal{A} = \{\phi, \{a,b\}, \{a,c\}\}$ and $\mathcal{I} = \{\phi, \{b\}\}$. Take $B = \{a,b\}$. Then $\psi_c(B) = X \setminus (X \setminus B)^{c*} = X \setminus \{c\}^{c*} = X \setminus \{c\} = \{a,b\}$.

Theorem 2.7. Let $(X, \mathcal{A}, \mathcal{I})$ be a space.

1. If $A \subseteq B$, then $\psi_c(A) \subseteq \psi_c(B)$.
2. If $U \in \mathcal{A}$, then $U \subseteq \psi_c(U)$.
3. If $A, B \in \wp(X)$, then $\psi_c(A) \cup \psi_c(B) \subseteq \psi_c(A \cup B)$.
4. If $A \subseteq X$, then $\psi_c(A) = \psi_c(\psi_c(A))$ if and only if $(X \setminus A)^{c*} = ((X \setminus A)^{c*})^{c*}$.
5. If $I \in \mathcal{I}$, then $\psi_c(I) = X \setminus X^{c*}$.
6. If $A \subseteq X$, $I \in \mathcal{I}$, then $\psi_c(A \cup I) = \psi_c(A)$.

7. If $(A\setminus B)\cup(B\setminus A)\in\mathcal{I}$, then $\psi_c(A)=\psi_c(B)$.

Proof. 1. Obvious from Theorem 2.3 (2).

2. Let $x\in U$. Then $x\notin X\setminus U$ and $U\cap(X\setminus U)=\varnothing\in\mathcal{I}$. Thus $x\notin(X\setminus U)^{c*}$. Hence $x\in\psi_c(U)$.

3. Obvious from monotonicity of ψ_c.

4. Suppose $\psi_c(A)=\psi_c(\psi_c(A))$. Then $X\setminus(X\setminus A)^{c*}=\psi_c[X\setminus(X\setminus A)^{c*}]$, and hence $X\setminus(X\setminus A)^{c*}=[X\setminus[(X\setminus A)^{c*}]^{c*}]$. This implies that $(X\setminus A)^{c*}=((X\setminus A)^{c*})^{c*}$.

5. Obvious from Theorem 2.3 (4).

6. Obvious from 5.

7. Given that $(A\setminus B)\cup(B\setminus A)\in\mathcal{I}$, and let $(A\setminus B)=I$, $(B\setminus A)=J$. Note that that $I,J\in\mathcal{I}$ by heredity. Also observe that $B=(A\setminus I)\cup J$. Thus $\psi_c(B)=\psi_c[(A\setminus I)\cup J]=\psi_{c*}(A\setminus I)=\psi_c(A)$.

If we take reverse implication of the above relations, then we get the converse part.

Corollary 2.8. Let $(X,\mathcal{A},\mathcal{I})$ be a space, then the following properties are equivalent:

1. $\mathcal{A}\cap\mathcal{I}=\{\varnothing\}$;
2. $\psi_c(\varnothing)=\varnothing$;
3. If $I\in\mathcal{I}$, then $\psi_c(I)=\varnothing$.

Proof. 1 implies 2:

$\psi_c(\varnothing)=X\setminus X^{c*}=\varnothing$, since $\mathcal{A}\cap\mathcal{I}=\varnothing$.

2 implies 3:

$\psi_c(I)=X\setminus(X\setminus I)^{c*}=X\setminus X^{c*}$, since $I\in\mathcal{I}$. Thus $\psi_c(I)=\varnothing$.

3 implies 1:

$\psi_c(I)=\varnothing$ gives $X=X^{c*}$. Thus $\mathcal{A}\cap\mathcal{I}=\{\varnothing\}$.

Definition 2.9. Let $(X,\mathcal{A},\mathcal{I})$ be a space. We say the $\mathcal{A}$-structure $\mathcal{A}$ is $\mathcal{A}$-compatible with the ideal $\mathcal{I}$, denoted $\mathcal{A}\sim\mathcal{I}$, if the following property holds: for every $A\subseteq X$, if for every $x\in A$ there exists $U\in\mathcal{A}(x)$ such that $U\cap A\in\mathcal{I}$, then $A\in\mathcal{I}$.

Lemma 2.10. Let $(X,\mathcal{A},\mathcal{I})$ be a space. Then $\mathcal{A}\sim\mathcal{I}$ if and only if $\psi_c(A)\setminus A\in\mathcal{I}$, for every $A\subseteq X$.

Proof. Suppose $\mathcal{A}\sim\mathcal{I}$ holds. Then for $U\setminus A\in\mathcal{I}$, $\psi_c(A)\cap(U\setminus A)\in\mathcal{I}$. This implies that $U\cap(\psi_c(A)\setminus A)\in\mathcal{I}$, then from $\mathcal{A}\sim\mathcal{I}$, $\psi_c(A)\setminus A\in\mathcal{I}$.

Conversely suppose that $\psi_c(A)\setminus A\in\mathcal{I}$, for every $A\subseteq X$. Let $x\in A$. Also there is $U_x\in\mathcal{A}(x)$ such that $U_x\cap A\in\mathcal{I}$ for every $x\in A$. Then $x\notin A^{c*}$, and hence $x\in X\setminus A^{c*}$. Thus $A\subseteq X\setminus A^{c*}$. Note that $\psi_c(X\setminus A)\setminus(X\setminus A)=[X\setminus(X\setminus(X\setminus A))^{c*}]\setminus(X\setminus A)=(X\setminus A^{c*})\setminus(X\setminus A)=(X\setminus A^{c*})\cap A$. Therefore $\psi_c(X\setminus A)\setminus(X\setminus A)=(X\setminus A^{c*})\cap A=A$ (as $A\subseteq X\setminus A^{c*}$). Since $\psi_c(A)\setminus A\in\mathcal{I}$ for every $A\subseteq X$, thus $A\in\mathcal{I}$. Therefore, $\mathcal{A}\sim I$.

Following example supports the Lemma 2.10.

Example 2.11. Let $X=\{a,b\}$, $\mathcal{A}=\{\{a\},\{b\}\}$ and $\mathcal{I}=\{\phi,\{b\}\}$. Then $\phi^{c^*}=\phi, \{a\}^{c^*}=\{a\}, \{b\}^{c^*}=\phi$ and $X^{c^*}=\{a\}$. Then $\psi_c(\phi)=X\setminus X^{c^*}=\phi$, $\psi_c(\{a\})=X\setminus\{b\}^{c^*}=X$, $\psi_c(\{b\})=X\setminus\{a\}^{c^*}=\{b\}$ and $\psi_c(X)=X\setminus\phi^{c^*}=X$. Then we see that $\psi_c(\phi)\setminus\phi=\phi\in\mathcal{I}$, $\psi_c(\{a\})\setminus\{a\}=\{b\}\in\mathcal{I}$, $\psi_c(\{b\})\setminus\{b\}=\phi\in\mathcal{I}$ and $\psi_c(X)\setminus X=\phi\in\mathcal{I}$ and $\mathcal{A}\sim\mathcal{I}$.

Corollary 2.12. Let $(X,\mathcal{A},\mathcal{I})$ be a space with $\mathcal{A}\sim\mathcal{I}$. Then $\psi_c(\psi_c(A))\subseteq\psi_c(A)$, for every $A\subseteq X$.

Proof. From above theorem, for any $A\subseteq X$, $\psi_c(A)\setminus A\in\mathcal{I}$. Then $\psi_c(A)\subseteq A\cup I$, for some $I\in\mathcal{I}$. Then $\psi_c(\psi_c(A))\subseteq\psi_c(A\cup I)=\psi_c(A)$ (from Theorem 2.7 (6)).

We shall give an example against the Corollary 2.12.

Example 2.13. Consider the spsce $(X,\mathcal{A},\mathcal{I})$, where $X=\{a,b\}$, $\mathcal{A}=\{\{a\},\{b\}\}$ and $\mathcal{I}=\{\phi,\{b\}\}$. Then $\mathcal{A}\sim\mathcal{I}$, by Example 2.11.Now $\phi^{c^*}=\phi, \{a\}^{c^*}=\{a\}, \{b\}^{c^*}=\phi$ and $X^{c^*}=\{a\}$ and $\psi_c(\phi)=X\setminus X^{c^*}=\phi$, $\psi_c(\{a\})=X\setminus\{b\}^{c^*}=X$, $\psi_c(\{b\})=X\setminus\{a\}^{c^*}=\{b\}$ and $\psi_c(X)=X\setminus\phi^{c^*}=X$. Then $\psi_c(\psi_c(\phi))=\phi=\psi_c(\phi)$, $\psi_c(\psi_c(\{a\}))=X=\psi_c(\{a\})$, $\psi_c(\psi_c(\{b\}))=\{b\}=\psi_c(\{b\})$ and $\psi_c(\psi_c(X))=X=\psi_c(X)$.

Newcomb has defined $A=B[mod\ \mathcal{I}]$ [23] if $(A\setminus B)\cup(B\setminus A)\in\mathcal{I}$.

Definition 2.14. Let $(X,\mathcal{A},\mathcal{I})$ be a space. A subset B of X is called a Baire set with respect to $\mathcal{A}$ and $\mathcal{I}$, denoted $A\in\mathbf{B}_r(X,\mathcal{A},\mathcal{I})$, if there exists a $A\in\mathcal{A}$ such that $B=A[mod\ \mathcal{I}]$.

Following example is the existence of Baire set.

Example 2.15. Let $X=\mathbb{R}$, set of real numbers, $\mathcal{A}=\{\varnothing,\mathbb{R},\mathbb{Q}\cup\{i\},\mathbb{R}\setminus\mathbb{Q}\}$, and $\mathcal{I}=\wp(\mathbb{Q})$, where $\mathbb{Q}$ is the set of rational numbers and $i\in\mathbb{R}\setminus\mathbb{Q}$. Consider $A=\{i\}$. Then for $B=\mathbb{Q}\cup\{i\}$, $(A\setminus B)\cup(B\setminus A)=\varnothing\cup(\mathbb{Q}\cup\{i\}\setminus\{i\}=\mathbb{Q}\in\mathcal{I}$. Thus A is a Baire set in $(X,\mathcal{A},\mathcal{I})$.

Theorem 2.16. Let $(X,\mathcal{A},\mathcal{I})$ be a space with $\mathcal{A}\sim\mathcal{I}$. If $U,V\in\mathcal{A}$ and $\psi_c(U)=\psi_c(V)$, then $U=V[mod\ \mathcal{I}]$.

Proof. Since $U\in\mathcal{A}$, we have $U\subseteq\psi_c(U)$ (from Theorem 2.7 (2)), and hence $U\setminus V\subseteq\psi_c(U)\setminus V=\psi_c(V)\setminus V\in\mathcal{I}$ (by Lemma 2.10). Similarly $V\setminus U\in\mathcal{I}$. Now $(U\setminus V)\cup(V\setminus U)\in\mathcal{I}$ (by finite additivity). Hence $U=V[mod\ \mathcal{I}]$.

It is obvious that $A=B[mod\ \mathcal{I}]$ is an equivalence relation.

Theorem 2.17. Let $(X,\mathcal{A},\mathcal{I})$ be a space with $\mathcal{A}\sim\mathcal{I}$. If $A,B\in\mathbf{B}_r(X,\mathcal{A},\mathcal{I})$, and $\psi_c(A)=\psi_c(B)$, then $A=B[mod\ \mathcal{I}]$.

Proof. Let $U,V\in\mathcal{A}$ such that $A=U[mod\ \mathcal{I}]$ and $B=V[mod\ \mathcal{I}]$. Now $\psi_c(A)=\psi_c(B)$ and $\psi_c(B)=\psi_c(V)$ (by Theorem 2.7 (7)). Since $\psi_c(A)=\psi_c(U)$ implies that $\psi_c(U)=\psi_c(V)$, hence $U=V[mod\ \mathcal{I}]$ (by Theorem 2.16). Hence $A=B[mod\ \mathcal{I}]$ by transitivity.

Theorem 2.18. Let $(X,\mathcal{A},\mathcal{I})$ be a space.

1. If $B\in\mathbf{B}_r(X,\mathcal{A},\mathcal{I})\setminus\mathcal{I}$, then there exists $A\in\mathcal{A}\setminus\{\varnothing\}$ such that $B=A[mod\ \mathcal{I}]$.

2. Suppose $\mathcal{A}\cap\mathcal{I}=\{\varnothing\}$, then $B\in\mathbf{B}_r(X,\mathcal{A},\mathcal{I})\setminus\mathcal{I}$ if and only if there exist $A\in\mathcal{A}\setminus\{\varnothing\}$ such that $B=A[mod\ \mathcal{I}]$.

Proof. 1. Let $B\in\mathbf{B}_r(X,\mathcal{A},\mathcal{I})\setminus\mathcal{I}$. Then $B\in\mathbf{B}_r(X,\mathcal{A},\mathcal{I})$. If there does not exist $A\in\mathcal{A}\setminus\{\varnothing\}$ such that $B=A[mod\ \mathcal{I}]$, we have $B=\varnothing[mod\ \mathcal{I}]$. This implies that $B\in\mathcal{I}$, which is a contradiction.

2. Here we prove converse part only. Let $A\in\mathcal{A}\setminus\{\varnothing\}$ such that $B=A[mod\ \mathcal{I}]$. Then $A=(B\setminus J)\cup I$, where $J=B\setminus A, I=A\setminus B\in\mathcal{I}$. If $B\in\mathcal{I}$, then $A\in\mathcal{I}$ by heredity and additivity, which contradicts $\mathcal{A}\cap\mathcal{I}=\{\varnothing\}$.

3. Approximation

Approximation is a part of analysis, but in this section, we introduce a method for approximation of local function and set operator ψ with the help of generalized open sets of topological space.

We shall denote '*Int*' and '*Cl*'as the 'interior' and 'closure' operator respectively of topological spaces.

Definition 3.1. Let (X,τ) be a topological space. A subset A of X is called semi-open [31] (resp. preopen [32], semi-preopen [33] ($=\beta$ open [34]), b-open [35]) set, if $A\subseteq Cl(Int(A))$ (resp. $A\subseteq Int(Cl(A))$, $A\subseteq Cl(Int(Cl(A))), A\subseteq Cl(Int(A))\cup Int(Cl(A))$).

The collection of all semi-open (resp. preopen, semi-preopen, b-open) in a topological space (X,τ) is denoted as $SO(X,\tau)$ (resp. $PO(X,\tau)$, $\beta O(X,\tau)$, $BO(X,\tau)$).

Further $SO(X,x)$ (resp. $PO(X,x)$, $\beta O(X,x)$, $BO(X,x)$) is the collection of all semi-open (resp. preopen, semi-preopen, b-open) sets containing x in a topological space (X,τ).

From definition, for a topological space (X,τ), we have $\tau\subseteq SO(X,\tau)\subseteq\beta O(X,\tau)\subseteq BO(X,\tau)$, and $\tau\subseteq PO(X,\tau)\subseteq\beta O(X,\tau)\subseteq BO(X,\tau)$.

Definition 3.2. Let $(X,\tau,\mathcal{I})$ be an ideal topological space. For a subset A of X, we define the following operator: A^{*p}(resp. $A^{*s},A^{*\beta},A^{*b}$) $=\{x\in X: A\cap U_x\notin\mathcal{I}$ for every $U_x\in PO(X,x)$ (resp. $SO(X,x)$, $\beta O(X,x), BO(X,x)$).

Theorem 3.3. Let $(X,\tau,\mathcal{I})$ be an ideal topological space. Then

1. for $A\subseteq X$, $A^{*b}\subseteq A^{*\beta}\subseteq A^{*s}\subseteq A^{*}$.
2. for $A\subseteq X$, $A^{*b}\subseteq A^{*\beta}\subseteq A^{*p}\subseteq A^{*}$.

Let (X,τ) be a topological space. Then for $A\subseteq X$, we define $sCl(A)$ (resp. $pCl(A)$, $\beta Cl(A)$, $bCl(A)$) $=\bigcap\{F\supseteq A: X\setminus F\in SO(X,\tau) \text{ (resp. } PO(X,\tau), \beta O(X,\tau), BO(X,\tau))\}$.

Proposition 3.4. Let (X,τ) be a topological space. Then for any $A\subseteq X$,

(1) $bCl(A)\subseteq \beta Cl(A)\subseteq sCl(A)\subseteq Cl(A)$,

(2) $bCl(A)\subseteq \beta Cl(A)\subseteq pCl(A)\subseteq Cl(A)$.

Definition 3.5. Let $(X,\tau,\mathcal{I})$ be an ideal topological space. For a subset A of X, we define the following operator: $\gamma_c(A)$ [3] (resp. $\gamma_{pc}(A),\gamma_{sc}(A),\gamma_{\beta c}(A),\gamma_{bc}(A))=\{x\in X: A\cap Cl(U_x)$ (resp. $A\cap pCl(U_x)$, $A\cap sCl(U_x)$, $A\cap \beta Cl(U_x)$, $A\cap bCl(U_x))\notin\mathcal{I}$ for every $U_x\in\tau(x)\}$, where $\tau(x)=\{U\in\tau: x\in U\}$.

Theorem 3.6. Let $(X,\tau,\mathcal{I})$ be an ideal topological space. Then for any $A\subseteq X$,

1. $A^{*}\subseteq\gamma_c(A)\subseteq\gamma_{sc}(A)\subseteq\gamma_{\beta c}(A)\subseteq\gamma_{bc}(A)$,
2. $A^{*}\subseteq\gamma_c(A)\subseteq\gamma_{pc}(A)\subseteq\gamma_{\beta c}(A)\subseteq\gamma_{bc}(A)$.

Next we consider following operators:

$\psi_s(A)$ (resp., $\psi_p(A),\psi_\beta(A),\psi_b(A))=X\setminus(X\setminus A)^{*s}$ (resp., $(X\setminus A)^{*p}$, $(X\setminus A)^{*\beta}$, $(X\setminus A)^{*b}$).

Theorem 3.7. Let $(X,\tau,\mathcal{I})$ be an ideal topological space. Then

1. for $A\subseteq X$, $\psi(A)\subseteq\psi_s(A)\subseteq\psi_\beta(A)\subseteq\psi_b(A)$.
2. for $A\subseteq X$, $\psi(A)\subseteq\psi_p(A)\subseteq\psi_\beta(A)\subseteq\psi_b(A)$.

Definition 3.8. Let $(X,\tau,\mathcal{I})$ be an ideal topological space. For a subset A of X, we define the following operator: $\Gamma_c(A)$ (resp. $\Gamma_{pc}(A),\Gamma_{sc}(A),\Gamma_{\beta c}(A),\Gamma_{bc}(A))=X\setminus\gamma_c(X\setminus A)$ (resp. $\gamma_{pc}(X\setminus A)$, $\gamma_{sc}(X\setminus A)$, $\gamma_{\beta c}(X\setminus A),\gamma_{bc}(X\setminus A)$).

Theorem 3.9. Let $(X,\tau,\mathcal{I})$ be an ideal topological space. Then for any $A\subseteq X$,

1. $\psi(A)\supseteq\Gamma_c(A)\supseteq\Gamma_{sc}(A)\supseteq\Gamma_{\beta c}(A)\supseteq\Gamma_{bc}(A)$,
2. $\psi(A)\supseteq\Gamma_c(A)\supseteq\Gamma_{pc}(A)\supseteq\Gamma_{\beta c}(A)\supseteq\Gamma_{bc}(A)$.

Expansion of local function and set operator ψ have been shown by the following corollaries:

Corollary 3.10. Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then

1. for $A \subseteq X$, $A^{*b} \subseteq A^{*\beta} \subseteq A^{*s} \subseteq A^{*} \subseteq \gamma_c(A) \subseteq \gamma_{sc}(A) \subseteq \gamma_{\beta c}(A) \subseteq \gamma_{bc}(A)$.
2. for $A \subseteq X$, $A^{*b} \subseteq A^{*\beta} \subseteq A^{*p} \subseteq A^{*} \subseteq \gamma_c(A) \subseteq \gamma_{pc}(A) \subseteq \gamma_{\beta c}(A) \subseteq \gamma_{bc}(A)$.

Corollary 3.11. Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then

1. for $A \subseteq X$, $\Gamma_{bc}(A) \subseteq \Gamma_{\beta c}(A) \subseteq \Gamma_{sc}(A) \subseteq \Gamma_c(A) \subseteq \psi(A) \subseteq \psi_s(A) \subseteq \psi_\beta(A) \subseteq \psi_b(A)$.
2. for $A \subseteq X$, $\Gamma_{bc}(A) \subseteq \Gamma_{\beta c}(A) \subseteq \Gamma_{pc}(A) \subseteq \Gamma_c(A) \subseteq \psi(A) \subseteq \psi_p(A) \subseteq \psi_\beta(A) \subseteq \psi_b(A)$.

4. Conclusion

Anyone can introduce a new type of generalized open set in the topological space and if this collection lies in between the collection of semi-open sets and semi-preopen sets (resp. preopen sets and semi-pre open sets), then we get an another local function whose value lies in between $()^{*\beta}$ and $()^{*s}$ (resp. $()^{*\beta}$ and $()^{*p}$). Similarly we can split the values of ψ_s and ψ_β, ψ_p and ψ_β, γ_{sc} and $\gamma_{\beta c}$, γ_{pc} and $\gamma_{\beta c}$, $\Gamma_{\beta c}$ and Γ_{sc} and $\Gamma_{\beta c}$ and Γ_{pc}.

Conflicts of interest

There is no conflict of interest.

References

[1] Kuratowski, K., Topology I. Warszawa, 1933.

[2] Al-omari, A. and Noiri, T., A topology via $\mathcal{M}$-local functions in ideal m-spaces. *Questions Answers Gen. Topology*, 30 (2012) 105-114.

[3] Al-omari, A. and Noiri, T., Local closure functions in ideal topological spaces. *Novi Sad J. Math.*, 12(2) (2013) 139-149.

[4] Modak, S., Grill-filter space. *J. Indian Math. Soc.*, 80 (2012) 313-320.

[5] Modak, S., Ideal on generalized topological spaces. *Sci. Magna*, 11(2) (2016) 14-20.

[6] Modak, S., Minimal spaces with a mathematical structure. *J. Assoc. Arab Univ. Basic Appl. Sci.*, 22 (2017) 98-101.

[7] Modak, S. and Noiri, T., Remarks on locally closed sets. *Acta Comment.Univ.Tartu. Math.*, 22(1) (2018) 57-64.

[8] Modak, S. and Islam, Md. M., On * and Ψ operators in topological spaces with ideals. *Trans. A. Razmadze Math. Inst.*, 172 (2018) 491-497.

[9] Islam, Md. M. and Modak, S., Operator associated with the * and Ψ operators. *J. Taibah Univ. Sci.*, 12(4) (2018) 444-449.

[10] Islam, Md. M. and Modak, S., Second approximation of local functions in ideal topological spaces. *Acta Comment.Univ.Tartu.Math.*, 22(2) (2018) 245-256.

[11] Ekici, E. and Elmali, O., On Decompositions via Generalized Closedness in Ideal Spaces. *Filomat*, 29(4) (2015) 879-886.

[12] Modak, S. and Mistry, S., Ideal on supra topological space. *Int. Journal of Math. Analysis*, 6(1) (2012) 1-10.

[13] Khan, M., and Noiri, T., Semi-local functions in ideal topological spaces. *J. Adv. Res. Pure Math.*, 2(1) (2010) 36-42.

[14] Csaszar, A., Modification of generalized topologies via hereditary classes. *Acta Math. Hungar.*, 115(1-2) (2007) 29-36.

[15] $\ddot{O}$zbakir, O.B. and Yildirim, E.D., On some closed sets in ideal minimal spaces. *Acta Math. Hungar.*, 125(3) (2009) 227-235.

[16] Hayashi, E., Topologies defined by local properties. *Math. Ann.*, 156 (1964) 205-215.

[17] Natkaniec, T., On I-continuity and I-semicontinuity points. *Math. Slovaca*, 36(3) (1986) 297-312

[18] Hamlett, T.R. and Jankovic, D., Ideals in topological spaces and the set operator ψ. *Bull. U.M.I.*, 7(4-B) (1990) 863-874.

[19] Modak, S. and Bandyopadhyay, C., A note on ψ - operator. *Bull. Malyas. Math. Sci. Soc*., 30(1) (2007) 43-48.

[20] Csaszar, A., Generalized open sets. Acta *Math. Hungar.*, 75(1-2) (1997) 65-87.

[21] Popa V. and Noiri T., On M-continuous functions. *Anal. Univ. ``Dunarea de Jos" Galati, Ser. Mat. Fiz. Mec. Teor. Fasc.II*, 18(23) (2000) 31-41.

[22] Mashhour, A.S., Allam, A.A., Mahmoud, F.S. and Khedr, F.H., On supra topological spaces. *Indian J. Pure and Appl. Math.*, 14(4) (1983) 502-510.

[23] Newcomb, R.L., Topologies which are compact modulo an ideal. Ph. D. Dissertation, U*niv. of Cal. at Santa Barbara*, (1967).

[24] Njastad, O., Remarks on topologies defined by local properties. *Norske Vid-Akad. Oslo (N.S)*, 8 (1966) 1-16.

[25] Pavlovic, A., Local function versus local closure function in ideal topological spaces. *Filomat*, 30(14) (2016) 3725-3731.

[26] Dontchev, J., Ganster M., Rose D., Ideal resolvability. *Topology Appl.*, 93 (1999) 1-16.

[27] Dontchev, J., Idealization of Ganster-Reilly decomposition theorems. *arXIV, Math. Gn/9901017VI*, (1999).

[28] Modak, S., Some new topologies on ideal topological spaces. *Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci.*, 82(3) (2012) 233-243.

[29] Bandyopadhyay, C. and Modak, S., A new topology via ψ - operator. *Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci*., 76(2006) 17-20.

[30] Jankovic, D. and Hamlett, T.R., New topologies from old via ideals.*Amer. Math. Monthly*, 97(1990) 295-310.

[31] Levine, N., Semi-open sets and semi-continuity in topological spaces. *Amer. Math. Monthly*, 70 (1963) 36-41.

[32] Mashhour, A.S., El-Monsef, M.E.A. and El-Deeb, S.N., On precontinuous and week precontinuous mappings. *Proc. Math. Phys. Soc. Egypt*., 53 (1982) 47-53.

[33] Andrijevic, D., Semi-preopen sets. *Math. Vesnik*, 38 (1986) 24-32.

[34] El-Monsef, M.E.A., El-Deeb, S.N. and Mahmoud, R.A., β-open sets and β-continuous mappings. *Bull. Fac. Sci., Assiut Univ*. 12 (1983) 77-90.

[35] Andrijevic, D., On b-open sets. *Mat. Vesnik*., 48 (1996) 59-64.