Cumhuriyet Science Journal

e-ISSN: 2587-246X
ISSN: 2587-2680

Cumhuriyet Sci. J., 41(4) (2020) 875-883
http://dx.doi.org/10.17776/csj.752182

On characterizations of fuzzy supra soft connected spaces

Alaa Mohamed Abd El-latif [1,*]

[1]*Northern Border University, Mathematics Department, Faculty of Arts and Science, , Rafha, P. O. Box, 840/ K. S. A.*

[1]*Ain Shams University, Mathematics Department, Faculty of Education, , Roxy, 11341, Cairo/ EGYPT*

Abstract

In this work, we initiate and study notion of connectedness for fuzzy supra soft topological spaces and present fundamentals properties. Moreover, we discuss its properties as well as with respect to fuzzy supra soft subspaces and fuzzy supra irresolute soft functions. Furthermore, we generalize C_i-fuzzy soft connectedness ($i = 1,2,3,4$), which plays an important role in fuzzy soft topological spaces, to fuzzy supra soft topological spaces. The relationship between the classes FSSCi-connected sets is discussed. Moreover, we introduce counterexamples to clarify that the reverse implications aren't satisfied.

Article info

History:
Received:12.06.2020
Accepted:18.11.2020

Keywords:
Fss-connected sets, Fuzzy supra irresolute soft functions, $FSSC_i$-connected spaces, $i = 1,2,3,4$.

1. Introduction

Fuzzy set theory initiated by Zadeh [1]. He introduced it to became an important mathematical tool in solving different types of complicated problems, which having uncertainties in real life problems. But, it has many difficulties, like constructing the membership function. Due to similar reasons, soft set theory was introduced [2] by Molodtsov. Recently, many papers published in soft set theory. Not only applied mathematicians, but also theoretic mathematicians are interested in this topic.

After introducing the soft set, fuzzy soft set (fs-set) theory was initiated by Maji et al. [3] and its basic properties were investigated by Ahmad and Kharal [4] and Çağman et al [5]. The notion of fuzzy soft topological spaces was introduced by Tanay and Kandemir [6] as a generalization to soft topological spaces [7] and fuzzy topological spaces [8]. More topological properties to fuzzy soft topological spaces were investigated in [9, 10]. As an extension to the notion of supra soft topological spaces [11] and fuzzy soft topological spaces [6,9], Abd El-latif [12] proposed the definition of fuzzy supra soft topological spaces. More topological properties, like fss-generalized closed (open) sets and fss-almost compact spaces, were investigated in [13].

Lin [14] introduced the notion of soft connectedness. Mahanta et al. [9] generalized this notion to fuzzy soft topological spaces . After that, Kandil et al. [15] in 2015, initiated the concept of fuzzy soft semi connected sets as a generalization to that's in [9]. Recently, Hussain [16], Karataş et al. [17] and Kandil et al. [18, 19, 20] investigated many properties of fuzzy soft connectedness. Our purpose of this paper is to present and study the concept of fss-connectedness and study its basic properties in detail.

2. Preliminaries

In this section, we recall the following notions and results for the development of fs-set theory and fssts's, which will be needed in this paper.

Definition 2.1 [1] A fuzzy set A on X is characterized by a membership function $\mu_A: X \longrightarrow I$ whose value $\mu_A(x)$ represents the "degree of membership" of x in A for $x \in X$.

*Corresponding author. *e-mail address: Alaa.ali@nbu.edu.sa*

Definition 2.2 [2] A pair (G, E), denoted by G_E, is called a soft set over X , where $G: E \rightarrow P(X)$. The family of all soft sets over X is denoted by $S(X)_E$.

Definition 2.3 [3] A pair (f, A), denoted by f_A, is called a fuzzy soft set (fs-set, for short) over X, where $A \subseteq E$ and $f: A \rightarrow I^X$ defined by $f_A(e) = \mu_{f_A}^e$ where $\mu_{f_A}^e = \overline{0}$ if $e \not\subseteq A$ and $\mu_{f_A}^e \neq \overline{0}$ if $e \in A$, where $\overline{0}(e) = 0 \ \forall \ x \in X$. The family of all fs-sets over X denoted by $FSS(X)_E$.

Definition 2.4 [10] The complement of a fuzzy soft set (f, A), denoted by $(f, A)^c$, is defined by $(f, A)^c = (f^c, A)$, $f_A^c: E \rightarrow I^X$ is a mapping given by $\mu_{f_A^c}^e = \overline{1} - \mu_{f_A}^e \ \ \forall \, e \in A$, where $\overline{1}(e) = 1 \ \forall \ x \in X$. Clearly $(f_A^c)^c = f_A$.

Definition 2.5 [21] A fuzzy soft set f_A over X is said to be a null fuzzy soft set, denoted by $\tilde{0}_A$, if for all $e \in A$, $f_A(e) = \overline{0}$.

Definition 2.6 [21] A fuzzy soft set f_A over X is said to be an absolute fuzzy soft set, denoted by $\tilde{1}_A$, if for all $e \in A, \ f_A(e) = \overline{1}$, where $\overline{1}$ is the membership function of absolute fuzzy set over X, which takes value 1 for all for all $x \in X$. Clearly, we have $(\tilde{1}_A)^c = \tilde{0}_A$ and $(\tilde{0}_A)^c = \tilde{1}_A$.

Definition 2.7 [10] Let $f_A, \ g_B \in FSS(X)_E$. Then, f_A is fuzzy soft subset of g_B, denoted by $f_A \sqsubseteq g_B$, if $\mu_{f_A}^e \subseteq \mu_{g_B}^e \ \ \forall \ e \in A$, i.e. $\mu_{f_A}^e(x) \leq \mu_{g_B}^e(x) \ \ \forall \ x \in X \ \ and \ \ \forall \ e \in A$.

Definition 2.8 [10]. The union of two fuzzy soft sets f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $C = A \cup B$ and for all $e \in C$,

$h_C(e) = \mu_{h_c}^e = \mu_{f_A}^e \vee \mu_{g_B}^e \ \ \forall e \in C$. Here, we write $h_C = f_A \sqcup g_B$.

Definition 2.9 [10]. The intersection of two fuzzy soft sets f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $C = A \cap B$ and for all $e \in C$,

$h_C(e) = \mu_{h_c}^e = \mu_{f_A}^e \wedge \mu_{g_B}^e \ \ \forall e \in C$. Here, we write $h_C = f_A \sqcap g_B$.

Definition 2.10 [22]. The family $\mathfrak{T} \subseteq FSS(X)_E$ is called a fuzzy soft topology (fst, for short) on X if

(1) $\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}$,

(2) If $\{f_{iC}; i \in I\} \subseteq \mathfrak{T}$, then $\sqcup_{i \in I} f_{iC} \in \mathfrak{T}$,

(3) If $f_A, g_B \in \mathfrak{T}$, then $f_A \sqcap g_B \in \mathfrak{T}$.

The triplet $(X, \mathfrak{T}, E)$ is called fuzzy soft topological space. Also, each member of $\mathfrak{T}$ is called a fuzzy open soft in $(X, \mathfrak{T}, E)$.

Definition 2.11 [23] The fs-set $f_A \in FSS(X)_E$ is called fuzzy soft point if there exist $x \in X$ and $e \in E$ such that $\mu_{f_A}^e(x) = \alpha \ (0 < \alpha \leq 1)$ and $\mu_{f_A}^e(y) = \overline{0}$ for each $y \in X - \{x\}$, and this fuzzy soft point is denoted by x_α^e or f_e.

Definition 2.12 [23] The soft fuzzy point f_A^e is said to be belonging to the fs-set g_B, denoted by $f_A^e \tilde{\in} g_B$, if for the element $e \in A \cap B$, $\mu_{f_A}^e(x) \leq \mu_{g_B}^e(x)$.

Definition 2.13 [9] Let $(X, \mathfrak{T}, E)$ be a fsts. A fuzzy soft separation (fs-separation, for short) of $\tilde{1}_E$ is a pair of non null proper fuzzy open soft sets g_B, h_C such that $g_B \sqcap h_C = \tilde{0}_E$ and $\tilde{1}_E = g_B \sqcup h_C$.

Definition 2.14 [9] A fsts $(X, \mathfrak{T}, E)$ is said to be fs-connected if and only if there is no fs-separations of $\tilde{1}_E$. Otherwise, $(X, \mathfrak{T}, E)$ is said to be fs-disconnected space.

Definition 2.15 [12] Let $\mathfrak{T}$ be a family of fs-sets over a universe X. Then $\mathfrak{T}$ is called fuzzy supra soft topology (briefey fsst) on X if

(1) $\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}$,

(2) If $\{f_{iC}; i \in I\} \subseteq \mathfrak{T}$, then $\sqcup_{i \in I} f_{iC} \in \mathfrak{T}$.

The triplet $(X, \mathfrak{T}, E)$ is called a fuzzy supra soft topological space (briefly fssts). Also, each member of $\mathfrak{T}$ is called a fuzzy supra open soft set (fsos-set for short) in $(X, \mathfrak{T}, E)$. A fs-set f_A over X is said to be fuzzy supra closed soft set ((fscs-set for short)) in X, if its relative complement f_A^c is a fsos-set. We denote the set of all fsos- (fscs-) sets by $FSOS(X)$ $(FSCS(X))$.

Definition 2.16 [12] Let $(X, \mathfrak{T}^*, E)$ be a fsts and $(X, \mathfrak{T}, E)$ be a fssts. We say that $\mathfrak{T}$ is a fsst associated with $\mathfrak{T}^*$ if $\mathfrak{T}^* \subset \mathfrak{T}$.

Definition 2.17 [12] Let $(X, \mathfrak{T}, E)$ be a fssts and $g_B \in FSS(X)_E$. Then the fuzzy supra soft interior of g_B, denoted by $Fint^s(g_B)$ is defined as

$$Fint^s(g_B) = \sqcup \{h_A: h_A \ is \ fsos - set \ and \ h_A \sqsubseteq g_B\}. \quad (1)$$

Also, the fuzzy supra soft closure of g_B, denoted by $Fcl^s(g_B)$ is defined as

$$Fcl^s(g_B) = \sqcap \{h_A: h_A \ is \ fscs - set \ and \ g_B \sqsubseteq h_A\}. \quad (2)$$

Definition 2.18 [24] Let $FSS(X)_E$ and $FSS(Y)_K$ be families of fs-sets over X and Y, respectively. Let $u: X \to Y$ and $p: E \to K$ be mappings. Then the map f_{pu} is called a fuzzy soft mapping from X to Y and denoted by $f_{pu}: FSS(X)_E \ \to \ FSS(Y)_K$ such that,

If $f_A \in FSS(X)_E$. Then the image of f_A under the fuzzy soft mapping f_{pu} is the fs-set over Y defined by $f_{pu}(f_A)$, where $\forall \ k \in p(E), \ \forall \ y \in Y$,

(1) $f_{pu}(f_A)(k)(y) = \begin{cases} \bigvee_{u(x)=y} \ [\vee_{p(e)=k} \ (f_A(e))](x) & if \ x \in u^{-1}(y), \\ 0 & otherwise. \end{cases}$

If $g_B \in FSS(Y)_K$, then the pre-image of g_B under the fuzzy soft mapping f_{pu} is the fs-set over X defined by $f_{pu}^{-1}(g_B)$, where $\forall \ e \in p^{-1}(K), \ \forall \ x \in X$,

(2) $f_{pu}^{-1}(g_B)(e)(x) = \begin{cases} g_B(p(e))(u(x)) & for \ p(e) \in B, \\ 0 & otherwise. \end{cases}$

The fuzzy soft mapping f_{pu} is called surjective (resp. injective) if p and u are surjective (resp. injective), also it is said to be constant if p and u are constant.

Definition 2.19 [12] Let $(X, \mathfrak{T}^*_1, E)$, $(Y, \mathfrak{T}^*_2, K)$ be two fssts's and $\mathfrak{T}_1$, $\mathfrak{T}_2$ be associated fsst's with $\mathfrak{T}^*_1$, $\mathfrak{T}^*_2$, respectively. Then, soft function $f_{pu}: FSS(X)_E \ \to \ FSS(Y)_K$ is called fuzzy supra soft continuous (resp. fuzzy supra irresolute soft) if $f_{pu}^{-1}(g_B) \in \mathfrak{T}_1 \ \forall \ g_B \in \mathfrak{T}^*_2$ (resp. $f_{pu}^{-1}(g_B) \in \mathfrak{T}_1 \ \forall \ g_B \in \mathfrak{T}_2$).

Definition 2.20 [12] Let $(X, \mathfrak{T}^*_1, E)$, $(Y, \mathfrak{T}^*_2, K)$ be two fsts's and $\mathfrak{T}_1$, $\mathfrak{T}_2$ be associated fsst's with $\mathfrak{T}^*_1$, $\mathfrak{T}^*_2$, respectively. Then, the soft function $f_{pu}: FSS(X)_E \ \to \ FSS(Y)_K$ is called

(1) Fuzzy supra open soft if $f_{pu}(g_E) \in \mathfrak{T}_2 \ \forall \ g_E \in \mathfrak{T}^*_1$.

(2) Fuzzy supra closed soft if $f_{pu}(g_E) \in \mathfrak{T}^c_2 \ \forall \ g_E \in \mathfrak{T}^{*c}_1$, where $\mathfrak{T}^c_2 and \mathfrak{T}^{*c}_1$ are the family of closed sets of $\mathfrak{T}_2 \ and \ \mathfrak{T}^*_1$, respectively.

(3) Fuzzy supra irresolute open soft if $f_{pu}(g_E) \in \mathfrak{T}_2 \ \forall \ g_E \in \mathfrak{T}_1$.

(4) Fuzzy supra irresolute closed soft if $f_{pu}(g_E) \in \mathfrak{T}^c_2 \ \forall \ g_E \in \mathfrak{T}^c_1$.

Theorem 2.21 [4, 23] Let $FSS(X)_E$ and $FSS(Y)_K$ be two families of fs-sets. For the fuzzy soft function $f_{pu}: FSS(X)_E \ \to \ FSS(Y)_K$, the following statements hold,

(a) $f_{pu}^{-1}((g,B)^c) = (f_{pu}^{-1}(g,B))^c \forall \ (g,B) \in FSS(Y)_K$.

(b) $f_{pu}(\tilde{0}_E) = \tilde{0}_K$, $f_{pu}(\tilde{1}_E) \sqsubseteq \tilde{1}_K$. If f_{pu} is surjective, then the equality holds.

(c) $f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$ and $f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$.

(d) If $(f,A) \sqsubseteq (g,A)$, then $f_{pu}(f,A) \sqsubseteq f_{pu}(g,A)$.

(e) If $(f,B) \sqsubseteq (g,B)$, then $f_{pu}^{-1}(f,B) \sqsubseteq f_{pu}^{-1}(g,B) \ \forall \ (f,B),(g,B) \in FSS(Y)_K$.

3. Fuzzy supra soft connected spaces

In this section, we initiate the concept of fss-connected sets. we study its properties in general with respect to fss-subspaces and fuzzy supra irresolute soft functions. We noticed that, the fss-disconnectedness property is hereditary property. Also, the image of fss-connected sets under fuzzy supra irresolute soft function is fss-connected.

Definition 3.1 Two fs-subsets f_A and g_B of a fssts $(X, \mathfrak{T}, E)$ are said to be disjoint, denoted by $f_A \sqcap g_B = \tilde{0}_E$, if $A \cap B = \phi$ or $\mu^e_{f_A} \wedge \mu^e_{g_B} = \overline{0}\ \forall e \in E$.

Definition 3.2 Let $(X, \mathfrak{T}, E)$ be a fssts. A fss-separation of $f_A \sqsubseteq \tilde{1}_E$ is a pair of non null disjoint proper fuzzy supra open soft sets h_B, g_C such that $f_A \sqsubseteq h_B \sqcup g_C$, $f_A \sqcap h_B \neq \tilde{0}_E$ and $f_A \sqcap g_C \neq \tilde{0}_E$. If there exist such two proper fuzzy supra open soft sets, then the fs-set f_A is said to be fss-disconnected set. Otherwise, f_A is called fss-connected. If we take $\tilde{1}_E$ instead of f_A, then the space $(X, \mathfrak{T}, E)$ is said to be fss-disconnected (connected) space.

Proposition 3.3 Let $(X, \mathfrak{T}, E)$ and (X, σ, E) be two fssts's such that $\mathfrak{T} \subseteq \sigma$. If (X, σ, E) is fss-connected space, then so $(X, \mathfrak{T}, E)$.

Proof. Immediate.

Remark 3.4 The converse of the above proposition is not true in general as shall shown in the following example.

Example 3.5 Let $X = \{x, y, z\}$, $E = \{e_1, e_2, e_3, e_4\}$ and $A, B, C \subseteq E$ where $A = \{e_1, e_2, e_3\}$, $B = \{e_3, e_4\}$ and $C = \{e_4\}$. Let f_A, g_B, k_C, h_B, s_E be fs-sets defined as follows:

$\mu^{e_1}_{f_A} = \{x_1, y_1, z_1\}$, $\mu^{e_2}_{f_A} = \{x_1, y_1, z_1\}$, $\mu^{e_3}_{f_A} = \{x_1, y_1, z_1\}$,

$\mu^{e_3}_{g_B} = \{x_1, y_1, z_1\}$, $\mu^{e_4}_{g_B} = \{x_{0.2}, y_{0.3}, z_{0.5}\}$,

$\mu^{e_4}_{k_C} = \{x_1, y_1, z_1\}$,

$\mu^{e_3}_{h_B} = \{x_1, y_1, z_1\}$, $\mu^{e_4}_{h_B} = \{x_1, y_1, z_1\}$,

$\mu^{e_1}_{s_E} = \{x_1, y_1, z_1\}$, $\mu^{e_2}_{s_E} = \{x_1, y_1, z_1\}$, $\mu^{e_3}_{s_E} = \{x_1, y_1, z_1\}$, $\mu^{e_4}_{s_E} = \{x_{0.2}, y_{0.3}, z_{0.5}\}$.

Consider the collection $\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, f_A, g_B, s_E\}$. It follows, $\mathfrak{T}$ defines a fsst on X which has no fss-separation of $\tilde{1}_E$. Hence, $(X, \mathfrak{T}, E)$ is a fss-connected space. Also, consider $\sigma = \{\tilde{1}_E, \tilde{0}_E, f_A, g_B, k_C, h_B, s_E\}$. So, σ defines a fsst on X such that $\mathfrak{T} \subseteq \sigma$ and σ is a fss-disconnected space, since $f_A, k_C \in \sigma$ and form a fss-separation of $\tilde{1}_E$.

Definition 3.6 [12] Let $(X, \mathfrak{T}, E)$ be a fssts and $Y \subseteq X$. Let y_E be a fs-set over (Y, E) defined by:

$y_E : E \to I^Y$ such that $y_E(e) = \mu^e_{y_E}$, where $\mu^e_{y_E}(x) = \begin{cases} 1, & x \in Y, \\ 0, & x \not\in Y. \end{cases}$

Then, the fssts $\mathfrak{T}_{y_E} = \{y_E \sqcap g_B : g_B \in \mathfrak{T}\}$ is called the fss-subspace topology for y_E and $(Y, \mathfrak{T}_{y_E}, E)$ is called fss-subspace of $(X, \mathfrak{T}, E)$.

Remark 3.7 The fss-disconnectedness property isn't hereditary in general as shall shown in the following example.

Example 3.8 Let $X = \{x, y, z\}$, and $E = \{e_1, e_2\}$. Let f_A, g_B be fs-sets defined as follows:

$\mu^{e_1}_{f_E} = \{x_1, y_1\}$, $\mu^{e_2}_{f_E} = \{x_1, y_1\}$,

$\mu^{e_1}_{g_E} = \{z_1\}$, $\mu^{e_2}_{g_E} = \{z_1\}$.

Consider the collection $\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, f_E, g_E\}$. It follows, $\mathfrak{T}$ defines a fsst on X which is fss-disconnected.

Now, let $Y = \{x, y\}$, then $y_E = f_E$ and $\mathfrak{T}_{Y_E} = \{\tilde{1}_E, \tilde{0}_E, f_E\}$. Therefore, $\mathfrak{T}_{Y_E}$ is is fss-connected.

Theorem 3.9 Let $(X, \mathfrak{T}, E)$, (Y, σ, K) be two fsts's , μ, ν be two associated fssts with $\mathfrak{T}$ and σ, respectively. Let$f_{pu} : FSS(X)_E \ \to\ FSS(Y)_K$ be an bijective fuzzy supra irresolute soft function. If k_A is fss-connected subset of $\tilde{1}_E$, then $f_{pu}(k_A)$ is fss-connected subset of $\tilde{1}_K$.

Proof. Let $f_{pu}(k_A)$ is fss-disconnected subset of $\tilde{1}_K$. Then, there exists a fss-separation $h_C, g_B \in \nu$ of $f_{pu}(k_A)$. Since f_{pu} is surjective fuzzy supra irresolute soft function, $k_A \sqsubseteq f_{pu}^{-1}(f_{pu}(k_A)) \sqsubseteq f_{pu}^{-1}(h_C \sqcup g_B) = f_{pu}^{-1}(h_C) \sqcup f_{pu}^{-1}(g_B)$ and $f_{pu}^{-1}(h_C \sqcap g_B) = f_{pu}^{-1}(h_C) \sqcap f_{pu}^{-1}(g_B) = f^{-1}(\tilde{0}_K) = \tilde{0}_E$ from Theorem 2.21. Also, $f_{pu}^{-1}[g_B \sqcap f_{pu}(k_A)] = f_{pu}^{-1}(g_B) \sqcap k_A \neq f_{pu}^{-1}(\tilde{0}_E) = \tilde{0}_K$ and $f_{pu}^{-1}[h_C \sqcap f_{pu}(k_A)] = f_{pu}^{-1}(h_C) \sqcap k_A \neq \tilde{0}_K$. This means that, $f_{pu}^{-1}(h_C), f_{pu}^{-1}(g_B) \in \mu$ forms a fss-separation of k_A, which is a contradiction. Hence, $f_{pu}(k_A)$ is a is fss-connected.

Theorem 3.10 Let h_C be a fss-connected subspace of fssts $(X, \mathfrak{T}, E)$. If $\tilde{1}_E$ has a fss-separations f_A, g_B, then either $h_C \sqsubseteq f_A$, or $h_C \sqsubseteq g_B$.

Proof. Let f_A, g_B be fss-separation on $\tilde{1}_E$. Then, $f_A \sqcap h_C, g_B \sqcap h_C \in \mathfrak{T}_{h_C}$. Since $h_C \sqsubseteq \tilde{1}_E = f_A \sqcup g_B$, $h_C \sqcap (f_A \sqcup g_B) = h_C = (h_C \sqcap f_A) \sqcup (h_C \sqcap g_B)$ and $(f_A \sqcap h_C) \sqcap (g_B \sqcap h_C) = h_C \sqcap (f_A \sqcap g_B) = \tilde{0}_E$. Also, $(f_A \sqcap h_c) \sqcap h_c = f_A \sqcap h_c \neq \tilde{0}_E$ and $(g_B \sqcap h_c) \sqcap h_c = g_B \sqcap h_c \neq \tilde{0}_E$. Since h_C is fss-connected, either $h_C \sqcap f_A = \tilde{0}_E$ or $h_C \sqcap g_B = \tilde{0}_E$. It follows, either $h_C = h_C \sqcap f_A$ or $h_C = h_C \sqcap g_B$. Therefore, either $h_C \sqsubseteq f_A$, or $h_C \sqsubseteq g_B$.

Theorem 3.11 Let h_A be a fss-connected subspace of fssts $(X, \mathfrak{T}, E)$ and z_B be fs-subset of $(X, \mathfrak{T}, E)$ such that $h_A \sqsubseteq z_B \sqsubseteq Fcl^s(h_A)$.Then, z_B is fss-connected.

Proof. It is sufficient to prove that $Fcl^s(h_A)$ is fss-connected. Contrarily, assume that $Fcl^s(h_A)$ is fss-disconnected. Then, there exists a fss-separation $f_C, g_D \in \mathfrak{T}_{h_A}$ of $Fcl^s(h_A)$. It follows, $h_A \sqcap f_C$ and $h_A \sqcap g_D$ are fuzzy supra open soft sets in $\mathfrak{T}_{h_A}$ which forms a fss-separation of h_A, which is a contradiction. Therefore, z_B is fss-connected.

Corollary 3.12 Let f_A be a fss-connected subspace of fssts $(X, \mathfrak{T}, E)$. Then, $Fcl^s(f_A)$ is fss-connected.

Proof. Follows from Theorem 3.11.

Theorem 3.13 If for all pair of distinct fuzzy soft points f_e, g_e, there exists a fuzzy supra soft connected subset z_N of fssts $(X, \mathfrak{T}, E)$ such that $f_e, g_e \tilde{\in} z_N$, then $\tilde{1}_E$ is fuzzy supra soft connected.

Proof. Assume that $\tilde{1}_E$ is fss-disconnected space. Then, there exists a fss-separation $f_C, g_D \in \mathfrak{T}$ of $\tilde{1}_E$. Since $f_A \sqcap g_B = \tilde{0}_E$, there exist two distinct fuzzy soft points f_e, g_e such that $f_e \in f_A$ and $g_e \in g_B$. By hypothesis, there exists a fuzzy fuzzy supra soft connected set z_N such that $f_e, g_e \in z_N$. From Theorem 3.10, either $z_N \sqsubseteq f_A$ or $z_N \sqsubseteq g_B$ and both cases is a contradiction. Therefore, $\tilde{1}_E$ is fuzzy supra soft connected.

Theorem 3.14 Let $\{v_{A_\epsilon}: \epsilon \in \lambda\}$ be a collection of fss-connected subspaces of fssts $(X, \mathfrak{T}, E)$ such that $\sqcap_{\epsilon \in \lambda} v_{A_\epsilon} \neq \tilde{0}_E$. Then, $\sqcup_{\epsilon \in \lambda} v_{A_\epsilon}$ is fss-connected.

Proof. Suppose that $\sqcup_{\epsilon \in \lambda} v_{A_\epsilon}$ is fss-disconnected. Then, there exists a fss-separation $m_F, n_G \in \mathfrak{T}$. It follows, v_{A_ϵ} is fss-connected subspace of fss-disconnected space $\tilde{1}_A$ and $\sqcap_{\epsilon \in \lambda} v_{A_\epsilon} \neq \tilde{0}_E$. By Theorem 3.10, either $v_{A_\epsilon} \sqsubseteq m_F$ or $v_{A_\epsilon} \sqsubseteq n_G$ $\forall \epsilon \in \lambda$. If $v_{A_\epsilon} \sqsubseteq m_F$, then $n_G = \tilde{0}_E$ and if $v_{A_\epsilon} \sqsubseteq n_G$, then $m_F = \tilde{0}_E$ and both cases is a contradiction. Therefore, $\tilde{1}_A$ is fss-connected.

4. Fuzzy supra soft C_i-connected spaces, $i = 1, 2, 3, 4$

It is well known that, for fs-sets theory [3] and for any two fs-sets f_A and g_B, even though the following implication is valid: $f_A \sqcap g_B = \tilde{0}_E \Rightarrow f_A \sqsubseteq g_B^c$, the reverse implication isn't true in general. This deviation gave the opportunity to the authors [25] to introduce new types of fuzzy connectedness, which weren't exist in the classical set theory, named C_i-fuzzy connectedness, (i = 1,2,3,4). S. Karataş et al. [17] generalized these notions to fsts, which generalized in [18, 19]. Here, we introduce the notion of fuzzy supra soft C_i-connected spaces, named $FSSC_i$-connected sets (i = 1,2,3,4), as a generalization to such similar concepts in [17, 18, 19]. The relation between these classes is studied in details, supported by counter examples.

Definition 4.1 Let $(X, \mathfrak{T}, E)$ be a fssts and $f_E \in FSS(X)_E$. Then, f_E is called

(1) Fuzzy supra soft C_1-connected (or $FSSC_1$-connected) if doesn't exist two non null fuzzy supra soft open sets g_A and h_B such that $f_E \sqsubseteq g_A \sqcup h_B$, $g_A \sqcap h_B \sqsubseteq f_E^c$, $f_E \sqcap g_A \neq \tilde{0}_E$ and $f_E \sqcap h_B \neq \tilde{0}_E$.

(2) Fuzzy supra soft C_2-connected (or $FSSC_2$-connected) if doesn't exist two non null fuzzy supra soft open sets g_A and h_B such that $f_E \sqsubseteq g_A \sqcup h_B$, $f_E \sqcap g_A \sqcap h_B = \tilde{0}_E$, $f_E \sqcap g_A \neq \tilde{0}_E$ and $f_E \sqcap h_B \neq \tilde{0}_E$.
(3) Fuzzy supra soft C_3-connected (or $FSSC_3$-connected) if doesn't exist two non null fuzzy supra soft open sets g_A and h_B such that $f_E \sqsubseteq g_A \sqcup h_B$, $g_A \sqcap h_B \sqsubseteq f_E^c$, $g_A \not\sqsubseteq f_E^c$ and $h_B \not\sqsubseteq f_E^c$.
(4) Fuzzy supra soft C_4-connected (or $FSSC_4$-connected) if doesn't exist two non null fuzzy supra soft open sets g_A and h_B such that $f_E \sqsubseteq g_A \sqcup h_B$, $f_E \sqcap g_A \sqcap h_B = \tilde{0}_E$, $g_A \not\sqsubseteq f_E^c$ and $h_B \not\sqsubseteq f_E^c$.

Proposition 4.2 In a fssts $(X, \mathfrak{T}, E)$, the relation between the classes $FSSC_i$-connected sets, $i = 1,2,3,4$ shall shown in the following diagram.

$$\begin{array}{ccc} FSSC_1\text{-connected} & \Longrightarrow & FSSC_2\text{-connected} \\ \Downarrow & & \Downarrow \\ FSSC_3\text{-connected} & \Longrightarrow & FSSC_4\text{-connected} \end{array}$$

Proof. Follows from Definition 4.1.

Remark 4.3 The implications in Proposition 4.2 are not reversible, as shall shown in the following examples.

Example 4.4 $FSSC_4$-connected $\not\Rightarrow$ $FSSC_3$-connected.

Let $X = [0,1]$ and $E = \{e_1, e_2\}$. Let consider the fs-sets f_E, g_E, h_E defined as follows:

$$\mu_{g_E}^{e_1}(x) = \begin{cases} \frac{1}{4}, & \frac{1}{4} < x \leq 1 \\ 1, & 0 \leq x \leq \frac{1}{4} \end{cases} \quad \text{and} \quad \mu_{g_E}^{e_2}(x) = \begin{cases} 1, & \frac{1}{4} < x \leq 1 \\ \frac{1}{4}, & 0 \leq x \leq \frac{1}{4} \end{cases}$$

$$\mu_{h_E}^{e_1}(x) = \begin{cases} 1, & \frac{1}{4} < x \leq 1 \\ \frac{1}{4}, & 0 \leq x \leq \frac{1}{4} \end{cases} \quad \text{and} \quad \mu_{h_E}^{e_2}(x) = \begin{cases} \frac{1}{4}, & \frac{1}{4} < x \leq 1 \\ 1, & 0 \leq x \leq \frac{1}{4} \end{cases}$$

$\mu_{f_E}^{e_1}(x) = \frac{1}{2} = \mu_{f_E}^{e_2}(x), for\ each\ x \in [0,1]$.

$\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, g_E, h_E\}$ defines a fsst on X. It is easy to see that f_E is $FSSC_4$-connected but not $FSSC_3$-connected.

Example 4.5 $FSSC_4$-connected $\not\Rightarrow$ $FSSC_2$-connected.

Let $X = [0,1]$ and $E = \{e_1, e_2\}$. Let consider the fs-sets $f_E, g_E, h_E, k_E, m_E, n_E$ defined as follows:

$$\mu_{g_E}^{e_1}(x) = \begin{cases} 0, & \frac{1}{2} < x \leq 1 \\ \frac{1}{2}, & 0 \leq x \leq \frac{1}{2} \end{cases} \quad \text{and} \quad \mu_{g_E}^{e_2}(x) = \begin{cases} \frac{1}{2}, & \frac{1}{2} < x \leq 1 \\ 0, & 0 \leq x \leq \frac{1}{2} \end{cases}$$

$$\mu_{h_E}^{e_1}(x) = \begin{cases} \frac{1}{2}, & \frac{1}{2} < x \leq 1 \\ 0, & 0 \leq x \leq \frac{1}{2} \end{cases} \quad \text{and} \quad \mu_{h_E}^{e_2}(x) = \begin{cases} 0, & \frac{1}{2} < x \leq 1 \\ \frac{1}{2}, & 0 \leq x \leq \frac{1}{2} \end{cases}$$

$$\mu_{f_E}^{e_1}(x) = \begin{cases} \frac{1}{2}, & \frac{1}{2} < x \leq 1 \\ \frac{1}{2}, & 0 \leq x \leq \frac{1}{2} \end{cases} \quad \text{and} \quad \mu_{f_E}^{e_2}(x) = \begin{cases} \frac{1}{2}, & \frac{1}{2} < x \leq 1 \\ \frac{1}{2}, & 0 \leq x \leq \frac{1}{2} \end{cases}$$

$\mu_{k_E}^{e_1}(x) = 0, for\ \ each\ \ x \in [0,1]$ and $\mu_{k_E}^{e_2}(x) = \begin{cases} \frac{1}{4}, & \frac{1}{2} < x \leq 1 \\ \frac{1}{4}, & 0 \leq x \leq \frac{1}{2} \end{cases}$

$\mu_{m_E}^{e_1}(x) = \begin{cases} \frac{1}{2}, & \frac{1}{2} < x \leq 1 \\ 0, & 0 \leq x \leq \frac{1}{2} \end{cases}$ and $\mu_{m_E}^{e_2}(x) = \begin{cases} \frac{1}{4}, & \frac{1}{2} < x \leq 1 \\ \frac{1}{2}, & 0 \leq x \leq \frac{1}{2} \end{cases}$

$\mu_{n_E}^{e_1}(x) = \begin{cases} 0, & \frac{1}{2} < x \leq 1 \\ \frac{1}{2}, & 0 \leq x \leq \frac{1}{2} \end{cases}$ and $\mu_{n_E}^{e_2}(x) = \begin{cases} \frac{1}{2}, & \frac{1}{2} < x \leq 1 \\ \frac{1}{4}, & 0 \leq x \leq \frac{1}{2} \end{cases}$

$\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, g_E, h_E, f_E, k_E, m_E, n_E\}$ defines a fsst on X. It is easy to see that f_E is $FSSC_4$-connected but not $FSSC_2$-connected.

Example 4.6 $FSSC_3$-connected $\not\Rightarrow$ $FSSC_1$-connected and $FSSC_2$-connected $\not\Rightarrow$ $FSSC_1$-connected.

Let $X = [0,1]$ and $E = \{e_1, e_2\}$. Let consider the fs-sets f_E, g_E, h_E defined as follows:

$\mu_{g_E}^{e_1}(x) = \begin{cases} \frac{1}{4}, & \frac{1}{4} < x \leq 1 \\ \frac{1}{2}, & 0 \leq x \leq \frac{1}{4} \end{cases}$ and $\mu_{g_E}^{e_2}(x) = \begin{cases} 1, & \frac{1}{4} < x \leq 1 \\ \frac{1}{4}, & 0 \leq x \leq \frac{1}{4} \end{cases}$

$\mu_{h_E}^{e_1}(x) = \begin{cases} \frac{1}{4}, & \frac{1}{4} < x \leq 1 \\ \frac{1}{2}, & 0 \leq x \leq \frac{1}{4} \end{cases}$ and $\mu_{h_E}^{e_2}(x) = \begin{cases} \frac{1}{2}, & \frac{1}{4} < x \leq 1 \\ \frac{1}{4}, & 0 \leq x \leq \frac{1}{4} \end{cases}$

$\mu_{f_E}^{e_1}(x) = \frac{1}{4} = \mu_{f_E}^{e_2}(x), for\ \ each\ \ x \in [0,1].$

$\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, g_E, h_E, g_E \sqcup h_E\}$ defines a fsst on X. Clearly, it is can be shown that f_E is $FSSC_3$-connected and $FSSC_2$-connected but not $FSSC_1$-connected.

Theorem 4.7 Let $(X, \mathfrak{T}, E)$, (Y, σ, K) be two fsts's , μ, ν be two associated fssts's with $\mathfrak{T}$ and σ, respectively. Let$f_{pu}: FSS(X)_E \rightarrow FSS(Y)_K$ be a bijective fuzzy supra irresolute soft function. If v_E is $FSSC_i$-connected subset of $\tilde{1}_E$, then $f_{pu}(v_E)$ is $FSSC_i$-connected subset of $\tilde{1}_K$, $(i = 1,2)$.

Proof. We prove the case when $i = 1$, the other case $(i = 2)$ can be proved by a similar way.

Suppose that $f_{pu}(v_E)$ isn't $FSSC_1$-connected. Then, there exist ν-fuzzy supra open soft sets g_A and h_B such that $f_{pu}(v_E) \sqsubseteq g_A \sqcup h_B$, $g_A \sqcap h_B \sqsubseteq [f_{pu}(v_E)]^c = f_{pu}(v_E^c)$, $f_{pu}(v_E) \sqcap g_A \neq \tilde{0}_E$ and $f_{pu}(v_E) \sqcap h_B \neq \tilde{0}_E$. From Theorem 2.21,

$v_E \sqsubseteq f_{pu}^{-1}[f_{pu}(v_E)] \sqsubseteq f_{pu}^{-1}[g_A \sqcup h_B] = f_{pu}^{-1}(g_A) \sqcup f_{pu}^{-1}(h_B)$,

$f_{pu}^{-1}[g_A \sqcap h_B] = f_{pu}^{-1}(g_A) \sqcap f_{pu}^{-1}(h_B) \sqsubseteq f_{pu}^{-1}[f_{pu}(v_E^c)] = v_E^c$,

$f_{pu}^{-1}[f_{pu}(v_E) \sqcap g_A] = f_{pu}^{-1}[f_{pu}(v_E)] \sqcap f_{pu}^{-1}(g_A) = v_E \sqcap f_{pu}^{-1}(g_A) \neq f_{pu}^{-1}[\tilde{0}_K] = \tilde{0}_E$ and

$f_{pu}^{-1}[f_{pu}(v_E) \sqcap h_B] = f_{pu}^{-1}[f_{pu}(v_E)] \sqcap f_{pu}^{-1}(h_B) = v_E \sqcap f_{pu}^{-1}(h_B) \neq f_{pu}^{-1}[\tilde{0}_K] = \tilde{0}_E$.

Since f_{pu} is fuzzy supra irresolute soft function, $f_{pu}^{-1}(g_A), f_{pu}^{-1}(h_B)$ are μ-fuzzy supra open soft sets. This means that, v_E isn't $FSSC_1$-connected, which is a contradiction with the hypothesis.

Theorem 4.8 Let $(X, \mathfrak{T}, E)$, (Y, σ, K) be two fsts's , μ, ν be two associated fssts's with $\mathfrak{T}$ and σ, respectively. Let$f_{pu}: FSS(X)_E \rightarrow FSS(Y)_K$ be a bijective fuzzy supra irresolute soft function. If v_E is $FSSC_i$-connected subset of $\tilde{1}_E$, then $f_{pu}(v_E)$ is $FSSC_i$-connected subset of $\tilde{1}_K$, $(i = 3,4)$.

Proof. We prove the case when $i = 3$, the other case ($i = 4$) can be proved by a similar way.

Suppose that $f_{pu}(v_E)$ isn't $FSSC_3$-connected. Then, there exist ν-fuzzy supra open soft sets g_A and h_B such that $f_{pu}(v_E) \sqsubseteq g_A \sqcup h_B$, $g_A \sqcap h_B \sqsubseteq [f_{pu}(v_E)]^c = f_{pu}(v_E^c), g_A \not\sqsubseteq f_{pu}(v_E^c)$ and $h_B \not\sqsubseteq f_{pu}(v_E^c)$. From Theorem 2.21,

$$v_E \sqsubseteq f_{pu}^{-1}[f_{pu}(v_E)] \sqsubseteq f_{pu}^{-1}[g_A \sqcup h_B] = f_{pu}^{-1}(g_A) \sqcup f_{pu}^{-1}(h_B) \quad (3)$$

$$f_{pu}^{-1}[g_A \sqcap h_B] = f_{pu}^{-1}(g_A) \sqcap f_{pu}^{-1}(h_B) \sqsubseteq f_{pu}^{-1}[f_{pu}(v_E^c)] = v_E^c \quad (4)$$

Since f_{pu} is fuzzy supra irresolute soft function, $f_{pu}^{-1}(g_A), f_{pu}^{-1}(h_B)$ are μ-fuzzy supra open soft sets. Since f_{pu} is surjective, there exist $y_1, y_2 \in Y$ such that

$$\mu_{g_A}^e(y_1) \geq 1 - f_{pu}(v_E)(k)(y_1) \quad (i) \quad and \qquad \mu_{h_B}^e(y_2) \geq 1 - f_{pu}(v_E)(k)(y_2) \quad (ii)$$

Now, if $f_{pu}^{-1}(g_A) \sqsubseteq v_E^c$, then this claim contradicts with (i). Thus,

$$f_{pu}^{-1}(g_A) \not\sqsubseteq v_E^c \quad (5)$$

Also, if $f_{pu}^{-1}(h_B) \sqsubseteq v_E^c$, then this claim contradicts with (ii). Thus,

$$f_{pu}^{-1}(h_B) \not\sqsubseteq v_E^c \quad (6)$$

Equations (3), (4), (5) and (6) prove that v_E isn't $FSSC_3$-connected, which is a contradiction. Hence, $f_{pu}(v_E)$ is $FSSC_3$-connected.

Theorem 4.9 Let $(X, \mathfrak{T}, E)$, (Y, σ, K) be two fsts's , μ, ν be two associated fssts's with $\mathfrak{T}$ and σ, respectively. Let$f_{pu}: FSS(X)_E \to FSS(Y)_K$ be an injective fuzzy supra irresolute open soft function. If s_K is $FSSC_i$-connected subset of $\tilde{1}_K$, then $f_{pu}^{-1}(s_K)$ is $FSSC_i$-connected subset of $\tilde{1}_E$, ($i = 1,2,3,4$).

Proof. It similar to the proof of Theorem 4.7 and Theorem 4.8.

5. Conclusion

In this paper, we introduced fss-connected sets, as a generalization to that's in [13, 13]. We discussed its basic properties such as the hereditary property and protecting image of fss-connected sets. Besides this, we introduced four types of fuzzy connectedness for a fs-set,named $FSSC_i$- connected sets. we discussed the relations between them. For future works, we consider to investigate more types of fss-connectedness, like fss-locally connectedness and fss-hyperconnected spaces by using the soft ideal notion as a generalization to that's in [11].

Acknowledgements

The author expresses his sincere thanks to the reviewers for their valuable suggestions which helped to improve the presentation of the paper.

Conflict of interest

No conflict of interest was declared by the author.

References

[1] Zadeh L. A., Fuzzy sets, Information and Control, 8 (1965) 338-353.

[2] Molodtsov D., Soft set theory-first results, *Comput. Math. Appl.*, (37) (1999) 19-31.

[3] Maji P. K., Biswas R. and Roy A. R., Fuzzy soft sets, *Journal of Fuzzy Mathematics*, 9 (3) (2001) 589-602.

[4] Ahmad B., Kharal, A., Mappings on fuzzy soft classes, *Adv. Fuzzy Syst*. (2009), Art. ID 407890, 1-6.

[5] Çağman N., Enginoğlu S.and Çitak F., Fuzzy soft set theory and its applications, *Iranian Journal of Fuzzy Systems*, 8(3) (2011) 137-147.

[6] Tanay B., Kandemir M. B., Topological structure of fuzzy soft sets, *Comput. Math. Appl.*, (61) (2011) 412-418.

[7] Shabir M., Naz M., On soft topological spaces, *Comput. Math. Appl.,* (61) (2011) 1786-1799.

[8] Chang C. L., Fuzzy topological spaces, *J. Math. Anal. Appl.,* (24) (1968) 182-190.

[9] Mahanta J., Das P.K., Results on fuzzy soft topological spaces, https://arxiv.org/abs/1203.0634.

[10] Roy S., Samanta T. K., A note on fuzzy soft topological spaces, *Ann. Fuzzy Math. Inform*., 3 (2) (2012) 305-311.

[11] El-Sheikh S. A., Abd El-latif A. M., Decompositions of some types of supra soft sets and soft continuity, *International Journal of Mathematics Trends and Technology*, 9 (1) (2014) 37-56.

[12] Abd El-latif A. M., Some properties of fuzzy supra soft topological spaces, *European Journal of Pure And Applied Mathematics*, (12) 3 (2019) 999-1017.

[13] Abd El-latif A. M., Results on Fuzzy Supra Soft Topological Spaces, *Journal of Interdisciplinary Mathematics*, 22 (8) (2019) 1311-1323.

[14] Lin F., Soft connected spaces and soft paracompact spaces, *International Journal of Mathematical Science and Engineering*, 7 (2) (2013) 1-7.

[15] Kandil A., Tantawy O. A. E., El-Sheikh S. A. and Abd El-latif A. M., Fuzzy soft semi connected properties in fuzzy soft topological spaces, *Math. Sci. Lett.,* 4 (2) (2015) 171-179.

[16] Hussain S., On properties of fuzzy soft locally connected spaces, *Hacettepe Journal of Mathematics and Statistics*, 47 (3) (2018) 589-599.

[17] Karataş S., Kihcç B. and Tellioğlu M., On fuzzy softconnected topological spaces, *J. Linear Topol. Algebra*, 3 (4) (2015) 229-240.

[18] Kandil A., Tantawy O. A. E., El-Sheikh S. A. and El-Sayed S.S.S., Fuzzy soft connected sets in fuzzy soft topological spaces II, *Journal of the Egyptian Mathematical Society*, 25 (2017) 171-177.

[19] Kandil A., Tantawy O. A. E., El-Sheikh S. A. and El-Sayed S.S.S., Fuzzy soft connected sets in fuzzy soft topological spaces I, *J. Adv. Math*. 8 (12) (2016) 6473-6488.

[20] Kandil A., Tantawy O. A. E., El-Sheikh S. A. and El-Sayed S.S.S., Fuzzy soft hyperconnected spaces, *Ann. Fuzzy Math. Inform.,* 13 (6) (2017) 689-708.

[21] Maji P. K., Biswas R. and Roy A. R., Soft set theory, *Comput. Math. Appl.,* 45 (2003) 555-562.

[22] Bakir T., Burcu Kandemir, M., Topological structure of fuzzy soft sets, *Comput. Math. Appl.,* 61 (2011) 2952-2957.

[23] Atmaca S. and Zorlutuna I., On fuzzy soft topological spaces, *Ann. Fuzzy Math. Inform*. 5 (2) (2013) 377-386.

[24] Pazar Varol B., Aygun H., Fuzzy soft topology, *Hacettepe Journal of Mathematics and Statistics*, 41 (3) (2012) 407-419.

[25] Ajmal N., Kohli J.K., Connectedness in fuzzy topological spaces, *Fuzzy Sets and Systems*, 31 (1989) 369-388.

[26] Kandil A., Tantawy O. A. E., El-Sheikh S. A. and Abd El-latif A. M., Soft connectedness via soft ideals, *Journal of New Results in Science*, 4 (2014) 90-108.