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Radicals of soft intersectıonal ideals in semigroups

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Abstract

In this paper, we introduce IS-radical, IS-quasi radical, IS-interior radical and IS-nil radical in semigroups. We obtain radical structures that will contribute to the theoretical studies of soft sets. We consider the ideal structures of intersectional soft sets in semigroups and we define IS-radical, IS-quasi radical, IS-interior radical and IS-nil radical. We use two different methods to define the soft radicals and give the results. In our study, we also give several examples and propositions to see differences among these structures.

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1. Introduction

Since the modelling of uncertain data in many fields was very complex, it was difficult to successfully deal with them by classical methods. With the emergence of soft set theory in 1999, overcoming uncertain problems have been a major field of study for mathematicians. A lot of work has been done on soft set theory in applied and theoretical fields and it is continuing with an increasing speed. Molodtsov [1] proposed soft set theory for modelling vagueness and uncertainty. Soft set theory has provided various solutions to the information systems and decision-making methods in the literature [2- 8]. At the same time, the new algebraic operations in soft set theory was described by various researchers. In addition, by defining the soft group [9] in 2007, the soft set theory has experienced rapid growth in algebraic structures [10-16]. In 2014, Song et al. [17] introduced the notions of int-soft semigroups, int-soft ideal and int-soft quasi ideals. And in [18-19], Sezgin et al. made new approach to the classical semigroup theory via soft sets and defined soft intersection semigroups, soft intersection ideal, soft intersection quasi ideal and soft intersection interior ideal. Using these notions, we have produced radicals of intersectional soft ideals. We have used two different methods to define the soft radicals. Firstly, we have examined whether a soft set is a radical of own sub-soft set. Secondly, we have defined a radical of intersectional soft ideal. Also, we have gave several examples and propositions to see differences among these structures.

2. Materials and methods

Let $\mathcal{S}$ be a semigroup and $\mathcal{I}$ be nonempty ideal of semigroup $\mathcal{S}$. $\sqrt{\mathcal{I}} = \{x: x^n \in \mathcal{I}, \forall x \in \mathcal{S}, \exists n \in \mathbb{N}\}$ is called radical of $\mathcal{I}$. Let $\mathcal{S}^1$ be a monoid and $\mathcal{I} = \{e\}$ be the identity-ideal of $\mathcal{S}^1$. $\sqrt{\mathcal{I}} = \{s: s^n = e,\ \forall s \in \mathcal{S}^1, \exists n \in \mathbb{N}\}$ is called nil radical of $\mathcal{I}$ [20]. Avoiding the details, other definitions of semigroups are not given in this study. The reader can refer to Howie [21].

2.1. Definition A pair (Φ, Γ) is called a soft set over $\mathcal{U}$, where Φ is a mapping $\Phi: \Gamma \to \mathcal{P}(\mathcal{U})$ [1]. In other words, a soft set over $\mathcal{U}$ is a parameterized family of subsets of the universe $\mathcal{U}$. For $e \in \Gamma$, $\Phi(e)$ is considered as the set of e-elements of the soft set (Φ, Γ) or as the set of e-approximate elements of the soft set.

2.2. Definition Let $\mathcal{A}$ be a non-empty subset of $\mathcal{S}$ over $\mathcal{U}$. Then $(\chi_{\mathcal{A}}, \mathcal{S})$ is called the characteristic soft set over $\mathcal{U}$, defined as follows:

$$\chi_{\mathcal{A}}: \mathcal{S} \to \mathcal{P}(\mathcal{U}),\ \ a \to \begin{cases} \mathcal{U}, & a \in \mathcal{A} \\ \emptyset, & otherwise \end{cases} \quad \text{for all } a \in \mathcal{S} \text{ [17].}$$

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2.3. Definition Let $(\omega, \mathcal{S})$ and $(\varphi, \mathcal{S})$ be soft sets over $\mathcal{U}$. Then $(\omega \tilde{\circ} \varphi, \mathcal{S})$ is called intersectional soft product of $(\omega, \mathcal{S})$ and $(\varphi, \mathcal{S})$, defined as follows:

$$(\omega \tilde{\circ} \varphi, \mathcal{S})(\mathrm{a}) = \begin{cases} \bigcup_{\mathrm{a}=\mathrm{bc}}\{\omega\mathrm{f}(\mathrm{b}) \cap \varphi(\mathrm{c})\}, & \exists \mathrm{b}, \mathrm{c} \in \mathcal{S} \text{ için } \mathrm{a} = \mathrm{bc} \\ \emptyset, & \text{otherwise} \end{cases} \quad \text{for all } \mathrm{a} \in \mathcal{S} \text{ [17].}$$

2.4. Definition Let $\mathcal{S}$ be a semigroup and $(\varphi, \mathcal{S})$ be a soft set over $\mathcal{U}$. Then,

2.4.1. $(\varphi, \mathcal{S})$ is called an intersectional soft semigroup of $\mathcal{S}$ if it satisfies:

$\varphi(xy) \supseteq \varphi(x) \cap \varphi(y)$ for all $x, y \in \mathcal{S}$ [17].

2.4.2. $(\varphi, \mathcal{S})$ is called an intersectional soft ideal of $\mathcal{S}$ if it satisfies:

$\varphi(xy) \supseteq \varphi(x)$ and $\varphi(xy) \supseteq \varphi(y)$ for all $x, y \in \mathcal{S}$ [17].

2.4.3. $(\varphi, \mathcal{S})$ is called an intersectional soft quasi-ideal of $\mathcal{S}$ if it satisfies:

$(\varphi \tilde{\circ} \chi_{\mathcal{S}}, \mathcal{S}) \tilde{\cap} (\chi_{\mathcal{S}} \tilde{\circ} \varphi, \mathcal{S}) \subseteq (\varphi, \mathcal{S})$ [17].

2.4.4. $(\varphi, \mathcal{S})$ is called an intersectional soft interior-ideal of $\mathcal{S}$ if it satisfies:

$\varphi(xsy) \supseteq \varphi(s)$ for all $x, y, s \in \mathcal{S}$ [19].

3. Results

In this section, we examine whether a soft set is a radical of own sub-soft set. And also, we define radicals of intersectional soft ideals (ideal/quasi ideal/interior ideal). Next, we produce nil radicals of these int-ideals.

In what follows, $\mathcal{S}$ regarded as a semigroup and $\mathcal{A}$ regarded as a non-empty subset of $\mathcal{S}$. We briefly show intersectional-soft as IS to avoiding repetition.

3.1. Definition Let $(\varphi, \mathcal{A})$ be an IS- ideal over $\mathcal{U}$. A non-null soft set $(\varphi, \mathcal{S})$ is called IS-radical of $(\varphi, \mathcal{A})$ if it satisfies:

$\varphi(s^n x) \supseteq \varphi(x)$ and $\varphi(xs^n) \supseteq \varphi(x)$ for all $x \in \mathcal{A}$, $s \notin \mathcal{A}$, $s \in \mathcal{S}$ and $\exists n \in \mathbb{N}$.

3.2. Example Let $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5\}$ and $S = \{0, x, 1, 2, y\}$. Then, $(\mathcal{S}, *)$ is semigroup for the following Cayley table:

*	0	x	1	2	y
0	0	0	0	0	0
x	0	0	0	x	1
1	0	x	1	0	0
2	0	0	0	2	y
y	2	2	y	0	0

Let $(\varphi, \mathcal{S})$ be soft set over $\mathcal{U}$ defined as follows:

$\varphi: \mathcal{S} \to \mathcal{P}(\mathcal{U})$, $(\varphi, \mathcal{S}) = \{(0, \{u_1, u_2, u_3, u_4\}), (x, \{u_1, u_3\}), (1, \{u_2, u_3, u_4\}), (2, \mathcal{U}), (y, \mathcal{U})\}$.

Let $\mathcal{A} = \{0, x, 1\}$ be the subset of $\mathcal{S}$, then $(\varphi, \mathcal{A})$ is an IS- ideal over U. There exists $n \in \mathbb{N}$ such that $2^n = 2$ and $y^n = 0$.

$\varphi(2.0) \supseteq \varphi(0)$ and $\varphi(0.2) \supseteq \varphi(0)$.
$\varphi(2.x) \supseteq \varphi(x)$ and $\varphi(x.2) \supseteq \varphi(x)$.

$\varphi(2.1) \supseteq \varphi(1)$and $\varphi(1.2) \supseteq \varphi(1)$.
$\varphi(0.0) \supseteq \varphi(0)$.
$\varphi(0.x) \supseteq \varphi(x)$ and $\varphi(x.0) \supseteq \varphi(x)$.
$\varphi(0.1) \supseteq \varphi(1)$and $\varphi(1.0) \supseteq \varphi(1)$.

For all $x \in \mathcal{A}$, $\varphi(s^n x) \supseteq \varphi(x)$ and $\varphi(xs^n) \supseteq \varphi(x)$. Therefore, $(\varphi, \mathcal{S})$ is IS-radical of $(\varphi, \mathcal{A})$.

3.3. Proposition If $(\varphi, \mathcal{S})$ be an IS- ideal over $\mathcal{U}$, then $(\varphi, \mathcal{S})$ is IS-radical of $(\varphi, \mathcal{A})$ for a subset $\mathcal{A}$ of $\mathcal{S}$.

Proof. Let $(\varphi, \mathcal{S})$ be an IS- ideal over $\mathcal{U}$. It is clear that $\varphi(s^n x) \supseteq \varphi(x)$ and $\varphi(xs^n) \supseteq \varphi(x)$ for $n = 1$ by Definition 3.1. Thus, $(\varphi, \mathcal{S})$ is IS-radical of $(\varphi, \mathcal{A})$ for a subset $\mathcal{A}$ of $\mathcal{S}$.

3.4. Example Let $\mathcal{U} = \{i_1, i_2, i_3, i_4, i_5\}$ and $\mathcal{S} = \{r, s, t, k\}$. Then, $(\mathcal{S},*)$ is a semigroup for the following Cayley table:

*	r	s	t	k
r	r	r	r	r
s	r	r	r	r
t	r	r	s	r
k	r	r	s	s

Let $(\mu, \mathcal{S})$ be soft set over $\mathcal{U}$ defined as follows:

$$\mu: \mathcal{S} \to \mathcal{P}\,(\mathcal{U}), \quad \dot{x} \to \begin{cases} \{i_1, i_2, i_3, i_4\} & if\ \dot{x} = r, \\ \{i_2, i_3\} & if\ \dot{x} = s, \\ \{i_1, i_4\} & if\ \dot{x} = t, \\ \{i_1, i_2, i_5\} & if\ \dot{x} = k, \end{cases}$$

Consider the subset $\mathcal{A} = \{r, s, t\}$ of $\mathcal{S}$. Then, $(\mu, \mathcal{A})$ is an IS- ideal over $\mathcal{U}$. For $n = 2, k^2 = s$.

$\mu(r.s) \supseteq \mu(r)$ and $\mu(s.r) \supseteq \mu(r)$,
$\mu(s.s) \supseteq \mu(s)$,
$\mu(t.s) \supseteq \alpha(t)$and $\mu(r.t) \supseteq \mu(t)$.

Hence, $(\mu, \mathcal{S})$ is IS-radical of $(\mu, \mathcal{A})$. Since $\mu(r.k) \not\supseteq \mu(k)$ and $\mu(k.r) \not\supseteq \mu(k)$, $(\mu, \mathcal{S})$ is not IS-ideal over $\mathcal{U}$.

3.5. Definition Let $(\phi, \mathcal{A})$ be an IS-ideal of $\mathcal{S}$. Then, the $\left(\sqrt{\phi}, \mathcal{S}\right)$ is called an IS-radical of $(\phi, \mathcal{A})$, defined by

$$\sqrt{\phi}(s) = \begin{cases} \cap\, \phi(s^n); & s \notin \mathcal{A},\ s^n \in \mathcal{A}, \exists n \in \mathbb{N} \\ \emptyset; & s \notin \mathcal{A},\ s^n \notin \mathcal{A}, \forall\, n \in \mathbb{N} \\ \phi(s); & s \in \mathcal{A} \end{cases},$$

where $\sqrt{\phi}: \mathcal{S} \to \mathcal{P}(\mathcal{S})$. The IS-radical $\left(\sqrt{\phi}, \mathcal{S}\right)$ can be denoted by $\sqrt{(\phi, \mathcal{A})}$.

3.6. Example Let consider the IS- ideal $(\varphi, \mathcal{A})$ in Example 3.2.

By Definition 3.5, $(0, \{u_1, u_2, u_3, u_4\}), (x, \{u_1, u_3\})$ and $(1, \{u_2, u_3, u_4\}) \in \left(\sqrt{\varphi}, \mathcal{S}\right)$. For all $n \in \mathbb{N}$, $2^n = 2$ and $y^n = 0$, So $(2, \emptyset)$ and $(y, \{u_1, u_2, u_3, u_4\}) \in \left(\sqrt{\varphi}, \mathcal{S}\right)$. Hence,

$$\left(\sqrt{\varphi}, \mathcal{S}\right) = \{(0, \{u_1, u_2, u_3, u_4\}), (x, \{u_1, u_3\}), (1, \{u_2, u_3, u_4\}), (2, \emptyset), (y, \{u_1, u_2, u_3, u_4\})\}.$$

One can see that the IS-radical $\left(\sqrt{\varphi}, \mathcal{S}\right)$ is not IS-ideal. Since, $\sqrt{\varphi}(xy) = \sqrt{\varphi}(1) \not\supseteq \sqrt{\varphi}(x)$.

3.7. Remark The radical of an IS-ideal may not be an IS-ideal over $\mathcal{U}$.

3.8. Definition Let $(\phi, \mathcal{A})$ be an IS-quasi ideal over $\mathcal{U}$. A nonempty soft set $(\phi, \mathcal{S})$ is called IS-quasi radical of $(\phi, \mathcal{A})$ if it satisfies:

$\phi(xs^n) \cap \phi(s^n x) \subseteq \phi(x)$ for all $x \in \mathcal{A}, s \in \mathcal{S}, s \notin \mathcal{A}, \exists n \in \mathbb{N}$.

3.9. Example Consider the semigroup $\mathcal{S} = \{1, a, b, c\}$. Then, $(\mathcal{S},*)$ is semigroup for the following Cayley table:

*	*1*	*a*	*b*	*c*
1	*1*	*1*	*1*	*1*
a	*1*	*1*	*1*	*1*
b	*1*	*1*	*1*	*a*
c	*1*	*1*	*a*	*b*

Let $(\psi, \mathcal{S})$ be soft set over $\mathbb{Z}_4$ defined as follows:

$$\psi\colon \mathcal{S} \to \mathcal{P}\,(\mathbb{Z}_4), \quad x \to \begin{cases} \{\bar{0}\} & , x = 1 \ ise, \\ \{\bar{0}, \bar{1}, \bar{2}\} & , x = a \ ise, \\ \{\bar{0}, \bar{2}, \bar{3}\} & , x = b \ ise, \\ \mathbb{Z}_4 & , x = c \ ise, \end{cases}$$

Consider $\mathcal{A} = \{a, b\}$ subset of $\mathcal{S}$. Since $\psi(a) \supseteq \psi(b.c) \cap \psi(c.b)$ and $\psi(b) \supseteq \psi(c.c) \cap \psi(c.c)$, $(\psi, \mathcal{A})$ is an IS-quasi ideal over $\mathbb{Z}_4$.

For all $n \in \mathbb{N}$, $1^n = 1$ and $c^4 = 1$. One can easily see that

$\psi(a.1) \cap \psi(1.a) \subseteq \psi(a)$; $\{\bar{0}\} \cap \{\bar{0}\} \subseteq \{\bar{0}, \bar{1}, \bar{2}\}$,
$\psi(b.1) \cap \psi(1.b) \subseteq \psi(b)$; $\{\bar{0}\} \cap \{\bar{0}\} \subseteq \{\bar{0}, \bar{2}, \bar{3}\}$.

Therefore, $(\psi, \mathcal{S})$ is IS-quasi radical of $(\psi, \mathcal{A})$.

3.10. Proposition If $(\varphi, \mathcal{S})$ be an IS-quasi ideal over $\mathcal{U}$, then $(\varphi, \mathcal{S})$ is IS-quasi radical of $(\varphi, \mathcal{A})$ for a subset $\mathcal{A}$ of $\mathcal{S}$.

Proof. Let $(\varphi, \mathcal{S})$ be an IS-quasi ideal over $\mathcal{U}$. It is clear that $\phi(xs^n) \cap \phi(s^n x) \subseteq \phi(x)$ for $n = 1$ by Definition 3.8. Thus, $(\varphi, \mathcal{S})$ is IS-quasi radical of $(\varphi, \mathcal{A})$ for a subset $\mathcal{A}$ of $\mathcal{S}$.

3.11. Example Consider the semigroup $\mathcal{S} = \{0, 1, 2, 3\}$. Then, $(\mathcal{S},*)$ is semigroup for the following Cayley table:

*	*0*	*1*	*2*	*3*
0	*0*	*0*	*0*	*0*
1	*0*	*1*	*1*	*1*
2	*0*	*1*	*2*	*3*
3	*0*	*1*	*1*	*1*

Let β be non-empty subset of $\mathcal{U}$ and $(\sigma, \mathcal{S})$ be soft set over $\mathcal{U}$ defined as follows:

$$\sigma\colon \mathcal{S} \to \mathcal{P}\,(\mathcal{U}), \quad x \to \begin{cases} \beta, & x = 3 \\ \emptyset, & x = 0, 1, 2 \end{cases}$$

Consider $\mathcal{A} = \{0, 2, 3\}$ subset of $\mathcal{S}$, $(\sigma, \mathcal{A})$ is an IS-quasi ideal over $\mathcal{U}$. Since $1^n = 1$ for all $n \in \mathbb{N}$,

$\sigma(0) \supseteq \sigma(0.1) \cap \sigma(1.0)$; $\sigma(0) \supseteq \sigma(0) \cap \sigma(0) = \sigma(0)$,
$\sigma(2) \supseteq \sigma(2.1) \cap \alpha(1.2)$; $\sigma(2) \supseteq \sigma(1) \cap \sigma(1) = \sigma(1)$,
$\sigma(3) \supseteq \sigma(3.1) \cap \sigma(1.3)$; $\sigma(3) \supseteq \sigma(1) \cap \sigma(1) = \sigma(1)$.

Therefore, $(\sigma, \mathcal{S})$ is IS-quasi radical of $(\sigma, \mathcal{A})$, Since $\sigma(1) \nsupseteq \sigma(3.3)$, $(\sigma, \mathcal{S})$ is not IS-quasi ideal over $\mathcal{U}$.

3.12. Definition Let $(\phi, \mathcal{A})$ be an IS-quasi ideal of $\mathcal{S}$. Then, the $\left(\sqrt{\phi}, \mathcal{S}\right)$ is called an IS-quasi radical of $(\phi, \mathcal{A})$, defined by

$$\sqrt{\phi}(s) = \begin{cases} \cap\, \phi(s^n); & s \notin \mathcal{A},\ s^n \in \mathcal{A}, \exists n \in \mathbb{N} \\ \emptyset; & s \notin \mathcal{A},\ s^n \notin \mathcal{A}, \forall\, n \in \mathbb{N} \\ \phi(s); & s \in \mathcal{A} \end{cases},$$

where $\sqrt{\phi}: \mathcal{S} \rightarrow \mathcal{P}(\mathcal{S})$. The IS-quasi radical $\left(\sqrt{\phi}, \mathcal{S}\right)$ can be denoted by $\sqrt{(\phi, \mathcal{A})}$.

3.13. Example Consider the IS-quasi ideal $(\psi, \mathcal{A})$ in Example 3.9. Since $c^2 = b$, $c^3 = a$ and $1^n = 1$ for all $n \in \mathbb{N}$, $(1, \emptyset)$ and $(c, \{\bar{0}, \bar{1}, \bar{2}\} \cap \{\bar{0}, \bar{2}, \bar{3}\}) \in \left(\sqrt{\psi}, \mathcal{S}\right)$. Hence,

$$\left(\sqrt{\psi}, \mathcal{S}\right) = \{(1, \emptyset), (a, \{\bar{0}, \bar{1}, \bar{2}\}), (b, \{\bar{0}, \bar{2}, \bar{3}\}), (c, \{\bar{0}, \bar{2}\})\}.$$

One can see that the IS-quasi radical $\left(\sqrt{\psi}, \mathcal{S}\right)$ is not IS-quasi ideal.

3.14. Remark The radical of an IS-quasi ideal may not be an IS-quasi ideal over $\mathcal{U}$.

3.15. Theorem Every IS-quasi radical is an IS-bi radical over $\mathcal{S}$.

Proof. Let $\left(\sqrt{\psi}, \mathcal{S}\right)$ be an IS-quasi radical over a semigroup $\mathcal{S}$. Then, $\sqrt{\psi}(x)$ is a quasi ideal over $\mathcal{S}$ for all $x \in \mathcal{S}$. Since every quasi-ideal is a bi-ideal of $\mathcal{S}$, $\left(\sqrt{\psi}, \mathcal{S}\right)$ is an IS-bi radical over $\mathcal{S}$.

3.16. Definition Let $(\phi, \mathcal{A})$ be an IS-interior ideal over $\mathcal{U}$. A non-null soft set $(\phi, \mathcal{S})$ is called IS-interior radical of $(\phi, \mathcal{A})$ if it satisfies:

$\phi(xs^n y) \supseteq \phi(s)$ for all $x, y \in \mathcal{A}, s \in \mathcal{S}, s \notin \mathcal{A}, \exists n \in \mathbb{N}$.

3.17. Example Let $\mathcal{U} = \{i_1, i_2, i_3, i_4, i_5, i_6\}$ and $\mathcal{S} = \{a, b, c, d\}$. Then, $(\mathcal{S}, *)$ is semigroup for the following Cayley table:

*	a	b	c	d
a	a	a	a	a
b	a	b	b	b
c	a	b	c	d
d	a	b	b	b

Let $(\mu, \mathcal{S})$ be soft set over $\mathcal{U}$ defined as follows:

$$\mu: \mathcal{S} \rightarrow \mathcal{P}\,(\mathcal{U}) \quad \dot{x} \rightarrow \begin{cases} \mathcal{U} & if\ \dot{x} = a, \\ \{i_1, i_2, i_5\} & if\ \dot{x} = b, \\ \{i_4, i_5, i_6\} & if\ \dot{x} = c, \\ \{i_5\} & if\ \dot{x} = d, \end{cases}$$

Consider the subset $\mathcal{A} = \{a, b, c\}$ of $\mathcal{S}$. Then, $(\mu, \mathcal{A})$ is an IS- interior ideal over $\mathcal{U}$. Since $d^2 = b$ and $\mu(x.b.y) \supseteq \mu(d)$ for all $x, y \in \mathcal{A}$, $(\mu, \mathcal{S})$ is IS-interior radical of $(\mu, \mathcal{A})$.

3.18. Proposition If $(\mu, \mathcal{S})$ be an IS-interior ideal over $\mathcal{U}$, then $(\mu, \mathcal{S})$ is IS-interior radical of $(\mu, \mathcal{A})$ for a subset $\mathcal{A}$ of $\mathcal{S}$.

Proof. Let $(\mu, \mathcal{S})$ be an IS-interior ideal over $\mathcal{U}$. It is clear that $\mu(xs^n y) \supseteq \mu(s)$ for $n = 1$ by Definition 3.16. Thus, $(\mu, \mathcal{S})$ is IS-interior radical of $(\mu, \mathcal{A})$ for a subset $\mathcal{A}$ of $\mathcal{S}$.

3.19. Example Consider the IS-interior radical $(\mu, \mathcal{S})$ in Example 3.17. Since $\mu(b.c.d) \not\supseteq \mu(c)$, the subset $(\mu, \mathcal{S})$ is not an IS- interior ideal over $\mathcal{U}$.

3.20. Definition Let $(\phi, \mathcal{A})$ be an IS-interior ideal of $\mathcal{S}$. Then, the $\left(\sqrt{\phi}, \mathcal{S}\right)$ is called an IS-interior radical of $(\phi, \mathcal{A})$, defined by

$$\sqrt{\phi}(s) = \begin{cases} \cap\, \phi(s^n); & s \notin \mathcal{A},\ s^n \in \mathcal{A}, \exists n \in \mathbb{N} \\ \emptyset; & s \notin \mathcal{A},\ s^n \notin \mathcal{A}, \forall\, n \in \mathbb{N} \\ \phi(s); & s \in \mathcal{A} \end{cases},$$

where $\sqrt{\phi}: \mathcal{S} \to \mathcal{P}(\mathcal{S})$. The IS-interior radical $\left(\sqrt{\phi}, \mathcal{S}\right)$ can be denoted by $\sqrt{(\phi, \mathcal{A})}$.

3.21. Example Consider the IS-interior ideal $(\mu, \mathcal{A})$ in Example 3.17. Since $d^n = b$ for all $n \in \mathbb{N}$, $(d, \{i_1, i_2, i_3\}) \in (\sqrt{\mu}, \mathcal{S})$. Therefore,

$$\left(\sqrt{\mu}, \mathcal{S}\right) = \{(a, \mathcal{U}), (b, \{i_1, i_2, i_3\}), (c, \{i_4, i_5, i_6\}), (d, \{i_1, i_2, i_3\})\}.$$

One can see that the IS-interior radical $(\sqrt{\mu}, \mathcal{S})$ is not IS-interior ideal.

3.22. Remark The radical of an IS-interior ideal may not be an IS-interior ideal over $\mathcal{U}$.

3.23. Theorem Let $(\phi, \mathcal{A})$ be an IS-ideal (quasi/interior) of semigroup $\mathcal{S}$. Then, there exists an IS-ideal (quasi/interior) $(\phi, \mathcal{B})$ such that

$$(\phi, \mathcal{A}) = \sqrt{(\phi, \mathcal{B})} \Leftrightarrow (\phi, \mathcal{A}) = \sqrt{(\phi, \mathcal{A})}\,.$$

Proof. Suppose that there exists a soft ideal $(\phi, \mathcal{A})$ such that $(\phi, \mathcal{A}) = \sqrt{(\phi, \mathcal{B})}$.

If for all $s \notin \mathcal{A}$ there exists $n \in \mathbb{N}$ such that $s^n \in \mathcal{A}$, then we have

$$\begin{aligned} & s^n \in \mathcal{B} \Leftrightarrow (s,\ \cap\, \phi(s^n)) \in \sqrt{(\phi, \mathcal{B})} \\ & \Leftrightarrow (s,\ \cap\, \phi(s^n)) \in (\phi, \mathcal{A}) \\ & \Leftrightarrow (s,\ \cap\, \phi(s^n)) \in \sqrt{(\phi, \mathcal{A})} \\ & \Leftrightarrow (\phi, \mathcal{A}) = \sqrt{(\phi, \mathcal{A})} \end{aligned} \tag{1}$$

Similarly, if for all $s \notin \mathcal{A}$ there exists $n \in \mathbb{N}$ such that $s^n \notin \mathcal{A}$, then we have

$$\begin{aligned} & s^n \notin \mathcal{B} \Leftrightarrow (s, \emptyset) \in \sqrt{(\phi, \mathcal{B})} \\ & \Leftrightarrow (s, \emptyset) \in (\phi, \mathcal{A}) \\ & \Leftrightarrow (s, \emptyset) \in \sqrt{(\phi, \mathcal{A})} \\ & \Leftrightarrow (\phi, \mathcal{A}) = \sqrt{(\phi, \mathcal{A})} \end{aligned} \tag{2}$$

Similarly, for all $s \in \mathcal{A}$, we have

$$\begin{aligned} & s \in \mathcal{B} \Leftrightarrow (s,\ \phi(s)) \in \sqrt{(\phi, \mathcal{B})} \\ & \Leftrightarrow (s,\ \phi(s)) \in (\phi, \mathcal{A}) \\ & \Leftrightarrow (s,\ \phi(s)) \in \sqrt{(\phi, \mathcal{A})} \\ & \Leftrightarrow (\phi, \mathcal{A}) = \sqrt{(\phi, \mathcal{A})}\,. \end{aligned} \tag{3}$$

From (1), (2), (3), we have $(\phi, \mathcal{A}) = \sqrt{(\phi, \mathcal{B})} \Leftrightarrow (\phi, \mathcal{A}) = \sqrt{(\phi, \mathcal{A})}\,.$

In what follows, $\mathcal{S}^1$ regarded as a monoid and $\mathcal{A}$ regarded as be the non-empty subset of $\mathcal{S}^1$ which is consisting identity element.

3.24. Definition Let $(\phi, \mathcal{A})$ be an IS-ideal (quasi/interior) of $\mathcal{S}^1$ over $\mathcal{U}$. A non-null soft set $(\phi, \mathcal{S}^1)$ is called IS-nil radical of $(\phi, \mathcal{A})$ if it satisfies:

$\phi(s) \subseteq \phi(e)$ for all $s \in \mathcal{S}^1$, $s^n \in \mathcal{A}, \exists n \in \mathbb{N}$.

3.25. Example Let $\mathcal{S}^1 = (\mathbb{Z}_6, .)$ be a monoid and $\mathcal{A} = \{\bar{1}\}$ be the subset of $\mathcal{S}^1$.

Consider the function $\phi: \mathcal{S}^1 \to \mathcal{P}\,(\mathbb{Z}_6)$ over $\mathcal{U} = \{i_1, i_2, i_3, i_4, i_5, i_6\}$ defined by $\phi(\bar{0}) = \{i_1, i_2\}$, $\phi(\bar{1}) = \mathcal{U}$, $\phi(\bar{2}) = \{i_2, i_3\}$, $\phi(\bar{3}) = \mathcal{U}$, $\phi(\bar{4}) = \{i_4, i_5, i_6\}$ and $\phi(5) = \{i_1, i_2, i_3, i_4\}$.

Since, there exists some $n \in \mathbb{N}$ such that $\bar{0}^n, \bar{2}^n, \bar{3}^n, \bar{4}^n \notin \mathcal{A}$, but $\bar{5}^n \in \mathcal{A}$. Hence, $(\phi, \mathcal{S}^1)$ is IS-nil radical of $(\phi, \mathcal{A})$.

3.26. Definition Let $(\phi, \mathcal{A})$ be an IS-ideal (quasi/interior) of $\mathcal{S}^1$ over $\mathcal{U}$. A non-null soft set $(\sqrt{\phi}, \mathcal{S}^1)$ is called IS-nil radical of $(\phi, \mathcal{A})$ defined by

$$\sqrt{\phi}(s) = \begin{cases} \phi(e), & s^n = e \\ \emptyset\ , & s^n \neq e \end{cases},$$

where $\sqrt{\phi} : \mathcal{S} \to \mathcal{P}(\mathcal{U})$ for all $s \in \mathcal{S}^1$ and some $n \in \mathbb{N}$.

3.27. Example Consider the IS- ideal (quasi/interior) $(\phi, \mathcal{A})$ in Example 3.25. Hence, we have

$(\sqrt{\phi}, \mathcal{S}^1) = \{(\bar{0}, \emptyset), (\bar{1}, \mathcal{U}), (\bar{2}, \emptyset), (\bar{3}, \emptyset), (\bar{4}, \emptyset), (\bar{5}, \mathcal{U})\}$.

4. Discussion

In this study, it is aimed to construct a new algebraic structure on soft set theory which has paved the way for many studies. Throughout this paper, IS-radical, IS-quasi radical, IS-interior radical and IS-nil radical in semigroups were obtained. We have used two different methods to define the soft radicals. We have gave several examples and propositions. Important results were specifically mentioned in the article. Based on these results, some further works can be developed on the properties of the soft radicals for ideals in regular semigroups.

Conflicts of interest

The authors state that did not have conflict of interests

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