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A defuzzification method on single-valued trapezoidal neutrosophic numbers and multiple attribute decision making

İrfan DELİ [1,*] ***Emel KIRMIZI ÖZTÜRK*** [2]

[1] *Muallim Rıfat Faculty of Education, Kilis 7 Aralık University, 79000, Kilis / TURKEY.*

[2] *Faculty of Arts and Sciences,, Kilis 7 Aralık University, 79000, Kilis / TURKEY.*

Abstract

In this paper, we give multiple attribute decision-making (MADM) method where both the attribute value and attribute weight of alternatives are single-valued trapezoidal neutrosophic numbers (SVTN-numbers). In spite of existing ranking methods, no one can rank SVTN-numbers with human intuition consistently in all cases. Therefore, we introduce a novel defuzzification method for ranking SVTN-numbers. To do this, some basic definitions and operations on the concepts of fuzzy set, fuzzy number, intuitionistic fuzzy set, intuitionistic fuzzy number, single-valued neutrosophic set, SVTN-number are presented. Then, concepts of $\boldsymbol{I}$. score function and $\boldsymbol{II}$. score function to reduce the SVTN-numbers to fuzzy numbers are defined. Finally, multiple criteria decision-making (MCDM) method for multiple criteria decision-making problems by using the concept of $\boldsymbol{I}$. score function and $\boldsymbol{II}$. score function of SVTN-numbers and defuzzification of fuzzy numbers are developed. Also, we have used a numerical example to verify the feasibility and the superiority of the proposed method compared to the existing methods.

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1. Introduction

Multi-criteria decision-making (MCDM) problems with uncertainty and fuzziness information are discussed by many authors such as fuzzy set [1], intuitionistic fuzzy set [2] and neutrosophic set [3]. To describe uncertainty and fuzziness information more flexibly and effectively in the real life, the theory of single-valued neutrosophic sets initially introduced by Wang et al. [4] by flexible degrees of truth-membership, degrees of indeterminacy membership and degrees of falsity-membership which is the generalization of the classical set, the fuzzy set, the intuitionistic fuzzy set and so on. Also, the academic community has witnessed growing research interests on the set theories in [5-9]. Because of the complexity and ambiguity involved in real-life situations, the theories are extended to fuzzy number [10-13], intuitionistic fuzzy number [14-19] and neutrosophic number [20-27] on $\in \mathbb{R}$. Concept of the single-valued neutrosophic number including single-valued trapezoidal neutrosophic number and single-valued triangular neutrosophic number that is a special neutrosophic set on $\in \mathbb{R}$ is first proposed by Şubaş [26].

However, few studies have focused on an extension of the neutrosophic numbers within the neutrosophic environment. Currently, there has been little research on decision-making methods on MCDM problems with neutrosophic numbers, and thus, it is necessary to pay attention to this issue. Many authors have developed some methods such as: on neutrosophic AHP-Delphi group decision-making model based on trapezoidal neutrosophic numbers [28], on aggregation of triangular fuzzy neutrosophic numbers [26,29], on value and ambiguity of single-valued trapezoidal neutrosophic numbers [20,22,26], on computation of shortest path problem in a network with of single-valued trapezoidal-triangular neutrosophic numbers [21-30], on operators with single-valued trapezoidal-triangular neutrosophic numbers [23,24,26,27,31], on multi-criteria group decision-making method

*Corresponding author. *Email address: irfandeli@kilis.edu.tr*

based on interdependent inputs of single-valued trapezoidal neutrosophic information [32], on single-valued trapezoidal neutrosophic preference relations with complete weight information [33], and so on.
Also, many useful defuzzification methods have been proposed to solve various MCDM problems on fuzzy numbers and intuitionistic fuzzy numbers in [6,11-15,19], but very few methods take into account the perspectives of both the defuzzification and the SVTN-numbers. Therefore, we proposed an MCDM method based on the concept of I. and II. score function of SVTN-numbers and defuzzification of fuzzy numbers. The remainder of this paper is organised as follows. Section 2 briefly introduces the basic definitions and operations on the concepts of fuzzy set, fuzzy number, intuitionistic fuzzy set, intuitionistic fuzzy number, single-valued neutrosophic set, single-valued neutrosophic number (SVN-number). Section 3 first develops the concept of I. score function and II. score function of SVTN-numbers to reduce the SVTN-numbers to fuzzy numbers and investigates some essential properties. Section 4 applies the proposed definitions and operations to an MCDM method for MCDM problems. Section 5 presents a numerical example to demonstrate how to apply the proposed method. Section 6 includes certain comparative discussions to verify the effectiveness and advantages of the developed approach with comparative analysis. The final section contains the conclusions. The present expository paper is a condensation of part of the dissertation [25].

2. Preliminary Definitions

This section reviews some basic concepts related to fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets that are used throughout this paper.

Definition 1. [1] A fuzzy set A on universe set E is given by

$$\mathrm{A} = \{\langle \mu_{\mathrm{A}}(\mathrm{x})/\mathrm{x}\rangle : \mathrm{x} \in \mathrm{E}\}$$

where the functions $\mu_A: E \to [0,1]$ define the degree of membership $x \in E$.

Definition 2. [13] Let $a \le b \le c \le d$ such that $a, b, c, d \in \mathbb{R}$. A generalized fuzzy number is a special fuzzy set on the real number set $\mathbb{R}$, whose membership function $\mu_{\hat{A}}: \mathbb{R} \to [0, w_{\hat{A}}]$ can generally be defined as

$$\mu_{\hat{A}}(x) = \begin{cases} f_{\mu l}(x) & a \le x < b \\ w_{\hat{A}} & b \le x < c \\ f_{\mu r}(x) & c \le x < d \\ 0 & otherwise \end{cases}$$

where $w_{\hat{A}} \in [0,1]$ is a constant, $f_{\mu l}(x): [a,b] \to [0, w_{\hat{A}}]$ and $f_{\mu r}(x): [c,d] \to [0, w_{\hat{A}}]$ are two strictly monotonical and continuous mappings from $\mathbb{R}$ to the closed interval $[0, w_{\hat{A}}]$. If the membership function $\mu_{\hat{A}}(x)$ is piecewise linear, then $\hat{A}$ is referred to as a trapezoidal generalized fuzzy number and is usually denoted by $\hat{A} = (a, b, c, d; w_{\hat{A}})$.

Also, Wang [12] introduce a new definition is called centroid point of the trapezoidal generalized fuzzy number $\hat{A} = (a, b, c, d; w_{\hat{A}})$ as $(\hat{A}) = \frac{\int_a^d x\mu_{\hat{A}}(x)dx}{\int_a^d \mu_{\hat{A}}(x)dx}$.

Definition 3. [2] An intuitionistic fuzzy set K on universe set E is given by $K = \{\langle x, \mu_K(x), \nu_K(x)\rangle : x \in E\}$ where $\mu_K(x): E \to [0,1]$ and $\nu_K(x): E \to [0,1]$ satisfy the condition $0 \le \mu_K(x) + \nu_K(x) \le 1$, for every $x \in E$. The values $\mu_K(x)$ and $\nu_K(x)$ define the degree of membership and degree of non-membership, respectively.

Definition 4. [7] Let $a_i \le b_i \le c_i \le d_i$ such that $a_i, b_i, c_i, d_i \in [0,1]$ for $i = 1,2$. An intuitionistic fuzzy number $\bar{A} = \langle (a_1, b_1, c_1, d_1; w_{\bar{A}}), (a_2, b_2, c_2, d_2; u_{\bar{A}})\rangle$ is a special intuitionistic set on the real number set $\mathbb{R}$, whose membership function $\mu_{\bar{A}}$ and non-membership function $\nu_{\bar{A}}$ are given as follows:

$$\mu_{\bar{A}}(x)=\begin{cases}\dfrac{(x-a_1)w_{\bar{A}}}{(b_1-a_1)}, & a_1\le x<b_1\\ w_{\bar{A}}, & b_1\le x<c_1\\ \dfrac{(d_1-x)w_{\bar{A}}}{(d_1-c_1)}, & c_1\le x<d_1\\ 0, otherwise\end{cases} \quad \text{and} \quad \nu_{\bar{A}}(x)=\begin{cases}\dfrac{(b_2-x)+(x-a_2)u_{\bar{A}}}{(b_2-a_2)}, & a_2\le x<b_2\\ u_{\bar{A}}, & b_2\le x<c_2\\ \dfrac{(x-c_2)+(d_2-x)u_{\bar{A}}}{(d_2-c_2)}, & c_2\le x<d_2\\ 1, & otherwise\end{cases}$$

If $(a_1,b_1,c_1,d_1)=(a_2,b_2,c_2,d_2)$ then the intuitionistic fuzzy number is reduced to trapezoidal intuitionistic fuzzy numbers $\bar{A}=\langle(a_1,b_1,c_1,d_1);w_{\bar{A}},u_{\bar{A}}\rangle$.

Definition 5. [3] A single-valued neutrosophic set A on universe set E is given by $A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle : x\in E\}$ where $T_A:E\to[0,1]$, $I_A:E\to[0,1]$, and $F_A:E\to[0,1]$ satisfy the condition $0\le T_A(x)+I_A(x)+F_A(x)\le 3$, for every x ∈ E. The functions T_A, I_A, and F_A define the degree of truth-membership function, indeterminacy-membership function and falsity-membership function, respectively.

Definition 6. [26] Let $a_1\le b_1\le c_1\le d_1$ such that $a_1,b_1,c_1,d_1\in[0,1]$. A single-valued trapezoidal neutrosophic number (SVTN-number) $\tilde{A}=\langle(a_1,b_1,c_1,d_1);w_{\tilde{A}},u_{\tilde{A}},y_{\tilde{A}}\rangle$ is a special neutrosophic set on the real number set $\mathbb{R}$, whose truth-membership function $\mu_{\tilde{A}}:\mathbb{R}\to[0,w_{\tilde{A}}]$, indeterminacy-membership function $\nu_{\tilde{A}}:\mathbb{R}\to[u_{\tilde{A}},1]$ and falsity-membership function $\lambda_{\tilde{A}}:\mathbb{R}\to[y_{\tilde{A}},1]$ are given as follows; (An example of SVTN-number is given in Fig. 1)

$$\mu_{\tilde{A}}(x)=\begin{cases}(x-a_1)w_{\tilde{A}}/(b_1-a_1), & a_1\le x\le b_1\\ w_{\tilde{A}}, & b_1\le x\le c_1\\ (d_1-x)w_{\tilde{A}}/(d_1-c_1), & c_1\le x\le d_1\\ 0, & otherwise\end{cases}$$

$$\nu_{\tilde{A}}(x)=\begin{cases}b_1-x+u_{\tilde{A}}(x-a_1), & a_1\le x\le b_1\\ u_{\tilde{A}}, & b_1\le x\le c_1\\ x-c_1+u_{\tilde{A}}(d_1-x), & c_1\le x\le d_1\\ 1, & otherwise\end{cases}$$

$$\lambda_{\tilde{A}}(x)=\begin{cases}b_1-x+\lambda_{\tilde{A}}(x-a_1), & a_1\le x\le b_1\\ y_{\tilde{A}}, & b_1\le x\le c_1\\ x-c_1+\lambda_{\tilde{A}}(d_1-x), & c_1\le x\le d_1\\ 1, & otherwise\end{cases}$$

Note that the set of all SVTN-numbers on $\mathbb{R}$ will be denoted by $N_{\mathbb{R}}$.

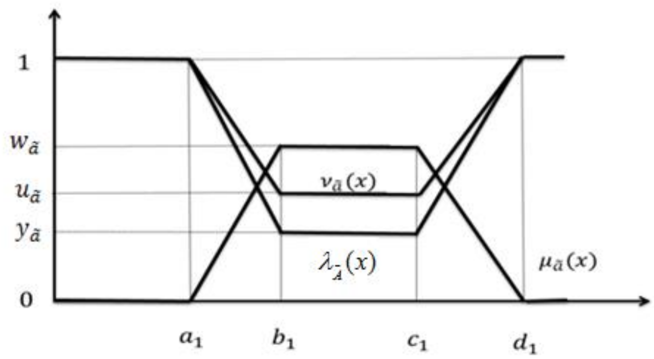

Fig. 1: Example of an SVTN-number

Definition 7. [26] Let $\tilde{A}=\langle(a_1,b_1,c_1,d_1);w_{\tilde{A}},u_{\tilde{A}},y_{\tilde{A}}\rangle$ and $\tilde{B}=\langle(a_2,b_2,c_2,d_2);\ w_{\tilde{B}},u_{\tilde{B}},\ y_{\tilde{B}}\rangle$ be two SVTN-numbers and $\gamma\ne 0$. Then,

i. $\tilde{A}+\tilde{B}=\langle(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2);w_{\tilde{A}}\wedge w_{\tilde{B}},u_{\tilde{A}}\vee u_{\tilde{B}},y_{\tilde{A}}\vee y_{\tilde{B}}\rangle$

ii. $\tilde{A}\tilde{B} = \begin{cases} \langle (a_1a_2, b_1b_2, c_1c_2, d_1d_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle, \ d_1 > 0 \ and \ d_2 > 0 \\ \langle (a_1d_2, b_1c_2, c_1b_2, d_1a_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle, \ d_1 < 0 \ and \ d_2 > 0 \\ \langle (d_1d_2, c_1c_2, b_1b_2, a_1a_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle, \ d_1 < 0 \ and \ d_2 < 0 \end{cases}$

iii. $\tilde{A}/\tilde{B} = \begin{cases} \langle (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle, \ d_1 > 0 \ and \ d_2 > 0 \\ \langle (d_1/d_2, c_1/c_2, b_1/b_2, a_1/a_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle, \ d_1 < 0 \ and \ d_2 > 0 \\ \langle (d_1/a_2, c_1/b_2, b_1/c_2, a_1/d_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle, \ d_1 < 0 \ and \ d_2 < 0 \end{cases}$

iv. $\gamma\tilde{A} = \begin{cases} \langle (\gamma a_1, \lambda b_1, \lambda c_1, \lambda d_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle, \ \gamma > 0 \\ \langle (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle, \ \gamma < 0 \end{cases}$

v. $\tilde{A}^{\gamma} = \begin{cases} \langle ({a_1}^{\gamma}, {b_1}^{\gamma}, {c_1}^{\gamma}, {d_1}^{\gamma}); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle, \ \gamma > 0 \\ \langle ({d_1}^{\gamma}, {c_1}^{\gamma}, {b_1}^{\gamma}, {a_1}^{\gamma}); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle, \ \gamma < 0 \end{cases}$

Definition 8. **[**26] Let $\tilde{A} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$ be an SVTN-number. Then,

i. score function of $\tilde{A}$, is denoted by $S_Y(\tilde{A})$, is defined as:

$$S_Y(\tilde{A}) = \frac{1}{16}[a + b + c + d] \times (2 + \mu_{\tilde{A}} - \nu_{\tilde{A}} - \gamma_{\tilde{A}})$$

ii. accuracy function of $\tilde{A}$, is denoted by $A_Y(\tilde{a})$, is defined as:

$$A_Y(\tilde{A}) = \frac{1}{16}[a + b + c + d] \times (2 + \mu_{\tilde{A}} - \nu_{\tilde{A}} + \gamma_{\tilde{A}})$$

Definition 9. [26] Let $\tilde{A}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{A}_j}, u_{\tilde{A}_j}, y_{\tilde{A}_j} \rangle \ (j = 1,2, \dots, n) \in N$. Then,

i. SVTN weighted arithmetic operator $\mathrm{N}_{ao} : N_{\mathbb{R}}^n \to N_{\mathbb{R}}$ is defined as;

$$\mathrm{N}_{ao}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \sum_{j=1}^{n} w_j \tilde{A}_j = \langle \left(\sum_{j=1}^{n} w_j a_j, \sum_{j=1}^{n} w_j b_j, \sum_{j=1}^{n} w_j c_j, \sum_{j=1}^{n} w_j d_j \right); \bigwedge_{j=1}^{n} w_{\tilde{a}_j}, \bigvee_{j=1}^{n} u_{\tilde{a}_j}, \bigvee_{j=1}^{n} y_{\tilde{a}_j} \rangle$$

where, $w = (w_1, w_2, \dots, w_n)^T$ is a weight vector associated with the N_{ao} operator, for every j ($j = 1,2, \dots, n$) $w_j \in [0,1]$ ve $\sum_{j=1}^{n} w_j = 1$.

ii. SVTN weighted geometric operator $\mathrm{N}_{go} : N_{\mathbb{R}}^n \to N_{\mathbb{R}}$, is defined as;

$$\mathrm{N}_{ao}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \prod_{j=1}^{n} {\tilde{A}_j}^{w_j} = \langle \left(\prod_{j=1}^{n} {a_j}^{w_j}, \prod_{j=1}^{n} {b_j}^{w_j}, \prod_{j=1}^{n} {c_j}^{w_j}, \prod_{j=1}^{n} {d_j}^{w_j} \right); \bigwedge_{j=1}^{n} w_{\tilde{a}_j}, \bigvee_{j=1}^{n} u_{\tilde{a}_j}, \bigvee_{j=1}^{n} y_{\tilde{a}_j} \rangle$$

where, $w = (w_1, w_2, \dots, w_n)^T$ is a weight vector associated with the N_{ao} operator, for every j ($j = 1,2, \dots, n$) $w_j \in [0,1]$ ve $\sum_{j=1}^{n} w_j = 1$.

3. I. Score Function And II. Score Function Of SVTN-Numbers

In this section, the concepts of I. score function and II. score function to reduce the SVTN-numbers to fuzzy numbers are defined. Some of the definitions are quoted or inspired by [18,19,25,26].

Definition 10. Let $\tilde{A} = \langle (a, b, c, d); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$ be an SVTN-number by a truth-membership function $T_{\tilde{A}}: \mathbb{R} \to [0, w_{\tilde{A}}]$, an indeterminacy-membership function $I_{\tilde{A}}: \mathbb{R} \to [u_{\tilde{A}}, 1]$ and a falsity-membership function $F_{\tilde{A}}: \mathbb{R} \to [y_{\tilde{A}}, 1]$. Then,

i. I. score function of $\tilde{A}$, denoted by $\Theta_I^{\tilde{A}}: \mathbb{R} \to [-1,1]$, defined by

$$\Theta_I^{\tilde{A}}(x) = \frac{T_{\tilde{A}}(x) - I_{\tilde{A}}(x) - F_{\tilde{A}}(x) + 1}{3}$$

ii. II. score function of $\tilde{A}$, denoted by $\Theta_{II}^{\tilde{A}}: \mathbb{R} \to [0,1]$, defined by

$$\Theta_{II}^{\tilde{A}}(x) = \frac{T_{\tilde{A}}(x) + I_{\tilde{A}}(x) - F_{\tilde{A}}(x) + 1}{3}$$

Theorem 11. $\tilde{A} = \langle (a,b,c,d); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$ be an SVTN-number by a truth-membership function $T_{\tilde{A}}: \mathbb{R} \to [0, w_{\tilde{A}}]$, an indeterminacy-membership function $I_{\tilde{A}}: \mathbb{R} \to [u_{\tilde{A}}, 1]$ and a falsity-membership function $F_{\tilde{A}}: \mathbb{R} \to [y_{\tilde{A}}, 1]$. Then,

$$\Theta_I^{\tilde{A}}(x) = \begin{cases} \dfrac{x.k_1 + k_2}{(b-a)} & (a \le x < b), \\ w_{\theta_1} & (b \le x \le c), \\ \dfrac{x.k_3 + k_4}{(d-c)} & (c < x \le d), \\ 0, otherwise \end{cases} \qquad \begin{cases} k_1 = \dfrac{w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} + 2}{3} \in \mathbb{R} \\ k_2 = \dfrac{a.(-w_{\tilde{A}} + u_{\tilde{A}} + y_{\tilde{A}} - 1) - b}{3} \in \mathbb{R} \end{cases} \quad w_{\theta_1} = \frac{w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} + 1}{3} \in \mathbb{R} \quad \begin{cases} k_3 = \dfrac{w_{\tilde{A}} + u_{\tilde{A}} + y_{\tilde{A}} - 2}{3} \in \mathbb{R} \\ k_4 = \dfrac{d.(w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} + 1) + c}{3} \in \mathbb{R} \end{cases}$$

and

$$\Theta_{II}^{\tilde{A}}(x) = \begin{cases} \dfrac{x.k_5 + k_6}{(b-a)} & (a \le x < b), \\ w_{\theta_2} & (b \le x \le c), \\ \dfrac{x.k_7 + k_8}{(d-c)} & (c < x \le d), \\ 0, otherwise \end{cases} \qquad \begin{cases} k_5 = \dfrac{w_{\tilde{A}} + u_{\tilde{A}} - y_{\tilde{A}}}{3} \in \mathbb{R} \\ k_6 = \dfrac{a.(w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} - 1) + b}{3} \in \mathbb{R} \end{cases} \quad w_{\theta_2} = \frac{w_{\tilde{A}} + u_{\tilde{A}} - y_{\tilde{A}} + 1}{3} \in \mathbb{R} \quad \begin{cases} k_7 = \dfrac{(-w_{\tilde{A}} - u_{\tilde{A}} + y_{\tilde{A}})}{3} \in \mathbb{R} \\ k_8 = \dfrac{d.(w_{\tilde{A}} + u_{\tilde{A}} - y_{\tilde{A}} + 1) - c}{3} \in \mathbb{R} \end{cases}$$

are fuzzy numbers.

Proof: $\tilde{A} = \langle (a,b,c,d); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$ be an SVTN-number. Then, from Definition 6 and Definition 10, we have

$$\Theta_I^{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)w_{\tilde{A}}}{3.(b-a)} - \dfrac{\left(b - x + u_{\tilde{A}}(x-a)\right)}{3.(b-a)} - \dfrac{\left(b - x + y_{\tilde{A}}(x-a)\right)}{3.(b-a)} + \dfrac{1}{3}, & (a \le x < b) \\ \dfrac{w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} + 1}{3}, & (b \le x \le c) \\ \dfrac{(d-x)w_{\tilde{A}}}{3.(d-c)} - \dfrac{\left(x - c + u_{\tilde{A}}(d-x)\right)}{3.(d-c)} - \dfrac{\left(x - c + y_{\tilde{A}}(d-x)\right)}{3.(d-c)} + \dfrac{1}{3}, & (c < x \le d) \\ 0, otherwise \end{cases}$$

$$= \begin{cases} \dfrac{(x-a)w_{\tilde{A}} - \left(b - x + u_{\tilde{A}}(x-a)\right) - \left(b - x + y_{\tilde{A}}(x-a)\right) + (b-a)}{3.(b-a)}, & (a \le x < b) \\ \dfrac{w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} + 1}{3}, & (b \le x \le c) \\ \dfrac{(d-x)w_{\tilde{A}} - \left(x - c + u_{\tilde{A}}(d-x)\right) - \left(x - c + y_{\tilde{A}}(d-x)\right) + (d-c)}{3.(d-c)}, & (c < x \le d) \\ 0, otherwise \end{cases}$$

$$= \begin{cases} \dfrac{x.(w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} + 2) + a.(-w_{\tilde{A}} + u_{\tilde{A}} + y_{\tilde{A}} - 1) - b}{3.(b-a)}, & (a \leq x < b) \\ \dfrac{w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} + 1}{3}, & (b \leq x \leq c) \\ \dfrac{x.(w_{\tilde{A}} + u_{\tilde{A}} + y_{\tilde{A}} - 2) + d.(w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} + 1) + c}{3.(d-c)}, & (c < x \leq d) \\ 0, otherwise \end{cases}$$

$$= \begin{cases} \dfrac{x.k_1 + k_2}{(b-a)}, \quad (a \leq x < b)\ such\ that \begin{cases} k_1 = \dfrac{w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} + 2}{3} \in \mathbb{R} \\ k_2 = \dfrac{a.(-w_{\tilde{A}} + u_{\tilde{A}} + y_{\tilde{A}} - 1) - b}{3} \in \mathbb{R} \end{cases} \\ w_{\theta_1}, \qquad (b \leq x \leq c)\ such\ that\ w_{\theta_1} = \dfrac{w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} + 1}{3} \in \mathbb{R} \\ \dfrac{x.k_3 + k_4}{(d-c)}, \quad (c < x \leq d)\ such\ that \begin{cases} k_3 = \dfrac{w_{\tilde{A}} + u_{\tilde{A}} + y_{\tilde{A}} - 2}{3} \in \mathbb{R} \\ k_4 = \dfrac{d.(w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} + 1) + c}{3} \in \mathbb{R} \end{cases} \\ 0, otherwise \end{cases}$$

and similarly,

$$\Theta_{II}^{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)w_{\tilde{A}}}{3.(b-a)} + \dfrac{\left(b - x + u_{\tilde{A}}(x-a)\right)}{3.(b-a)} - \dfrac{\left(b - x + y_{\tilde{A}}(x-a)\right)}{3.(b-a)} + \dfrac{1}{3}, & (a \leq x < b) \\ \dfrac{w_{\tilde{A}} + u_{\tilde{A}} - y_{\tilde{A}} + 1}{3}, & (b \leq x \leq c) \\ \dfrac{(d-x)w_{\tilde{A}}}{3.(d-c)} + \dfrac{\left(x - c + u_{\tilde{A}}(d-x)\right)}{3.(d-c)} - \dfrac{\left(x - c + y_{\tilde{A}}(d-x)\right)}{3.(d-c)} + \dfrac{1}{3}, & (c < x \leq d) \\ 0, otherwise \end{cases}$$

$$= \begin{cases} \dfrac{(x-a)w_{\tilde{A}} + \left(b - x + u_{\tilde{A}}(x-a)\right) - \left(b - x + y_{\tilde{A}}(x-a)\right) + (b-a)}{3.(b-a)}, & (a \leq x \\ \dfrac{w_{\tilde{A}} + u_{\tilde{A}} - y_{\tilde{A}} + 1}{3}, & (b \leq x \\ \dfrac{(d-x)w_{\tilde{A}} + \left(x - c + u_{\tilde{A}}(d-x)\right) - \left(x - c + y_{\tilde{A}}(d-x)\right) + (d-c)}{3.(d-c)}, & (c < x \\ 0, otherwise \end{cases}$$

$$= \begin{cases} \dfrac{x.(w_{\tilde{A}} + u_{\tilde{A}} - y_{\tilde{A}}) + a.(w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} - 1) + b}{3.(b-a)}, & (a \leq x < b) \\ \dfrac{w_{\tilde{A}} + u_{\tilde{A}} - y_{\tilde{A}} + 1}{3}, & (b \leq x \leq c) \\ \dfrac{x.(-w_{\tilde{A}} - u_{\tilde{A}} + y_{\tilde{A}}) + d.(w_{\tilde{A}} + u_{\tilde{A}} - y_{\tilde{A}} + 1) - c}{3.(d-c)} & (c < x \leq d) \\ 0, otherwise \end{cases}$$

$$= \begin{cases} \dfrac{x.k_5 + k_6}{(b-a)}, & (a \le x < b) \; such \; that \begin{cases} k_5 = \dfrac{w_{\tilde{A}} + u_{\tilde{A}} - y_{\tilde{A}}}{3} \in \mathbb{R} \\ k_6 = \dfrac{a.(w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}} - 1) + b}{3} \in \mathbb{R} \end{cases} \\ w_{\theta_2}, & (b \le x \le c) \; such \; that \; w_{\theta_2} = \dfrac{w_{\tilde{A}} + u_{\tilde{A}} - y_{\tilde{A}} + 1}{3} \in \mathbb{R} \\ \dfrac{x.k_7 + k_8}{(d-c)}, & (c < x \le d) \; such \; that \begin{cases} k_7 = \dfrac{(-w_{\tilde{A}} - u_{\tilde{A}} + y_{\tilde{A}})}{3} \in \mathbb{R} \\ k_8 = \dfrac{d.(w_{\tilde{A}} + u_{\tilde{A}} - y_{\tilde{A}} + 1) - c}{3} \in \mathbb{R} \end{cases} \\ 0, otherwise \end{cases}$$

Finally, I. and II. score of $\tilde{A}$ are fuzzy numbers.

Definition 12. Let $\tilde{A} = \langle (a,b,c,d); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$ be an SVTN-number and $\Theta_I^{\tilde{A}}$ and $\Theta_{II}^{\tilde{A}}$ be the I. score function and II. score function of $\tilde{A}$, respectively. Then,

i. The centroid of $\tilde{A}$ based on I. score function, denoted by $C(\Theta_I^{\tilde{A}})$, defined by;

$$C(\Theta_I^{\tilde{A}}) = \frac{\int_a^b \frac{x.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+2)+a.(-w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-1)-b}{3.(b-a)} x dx + \int_b^c \frac{w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1}{3} x dx}{\int_a^b \frac{x.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+2)+a.(-w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-1)-b}{3.(b-a)} dx + \int_b^c \frac{w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1}{3} dx}$$

$$\frac{+\int_c^d \frac{x.(w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-2)+d.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1)+c}{3.(d-c)} x dx}{+\int_c^d \frac{x.(w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-2)+d.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1)+c}{3.(d-c)} x dx}$$

ii. The centroid of $\tilde{A}$ based on II. score function, denoted by $C(\Theta_{II}^{\tilde{A}})$, defined by;

$$C(\Theta_{II}^{\tilde{A}}) = \frac{\int_a^b \frac{x.(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}})+a.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}-1)+b}{3.(b-a)} x dx + \int_b^c \frac{w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1}{3} x dx}{\int_a^b \frac{x.(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}})+a.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}-1)+b}{3.(b-a)} dx + \int_b^c \frac{w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1}{3} dx}$$

$$\frac{+\int_c^d \frac{x.(-w_{\tilde{A}}-u_{\tilde{A}}+y_{\tilde{A}})+d.(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1)-c}{3.(d-c)} x dx}{+\int_c^d \frac{x.(-w_{\tilde{A}}-u_{\tilde{A}}+y_{\tilde{A}})+d.(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1)-c}{3.(d-c)} x dx}$$

Theorem 13. Let $\tilde{A} = \langle (a,b,c,d); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$be an SVTN-number and $\Theta_I^{\tilde{A}}$ and $\Theta_{II}^{\tilde{A}}$ be the I. score function and II. score function of $\tilde{A}$, respectively. Then,

i. The centroid of $\tilde{A}$ based on I. score function, denoted by $C(\Theta_I^{\tilde{A}})$, calculated as

$$C(\Theta_I^{\tilde{A}}) = \frac{1}{3} \frac{[(a^3 - b^3).(w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}}) - 2b^3 - a^3 + 3ba^2]}{[(a^2 - b^2).(w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}}) - 2b^2 + 2ab]}$$

$$\frac{+[(d^3 - c^3).(5w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}}) - 2c^3 - d^3 + 3cd^2]}{+[(d^2 - c^2).(3w_{\tilde{A}} - u_{\tilde{A}} - y_{\tilde{A}}) - 2c^2 + 2cd]}$$

ii. The centroid of $\tilde{A}$ based on II. score function, denoted by $C(\Theta_{II}^{\tilde{A}})$, calculated as

$$C(\Theta_{II}^{\tilde{A}}) = \frac{1}{3} \frac{[a^3.(-5w_{\tilde{A}} + u_{\tilde{A}} + 5y_{\tilde{A}} + 3) + b^3.(-w_{\tilde{A}} - u_{\tilde{A}} + y_{\tilde{A}}) + 6b^2a.(w_{\tilde{A}} - y_{\tilde{A}}) - 3a^2]}{[a^2.(3w_{\tilde{A}} + u_{\tilde{A}} + 3y_{\tilde{A}} + 2) - b^2.(w_{\tilde{A}} + u_{\tilde{A}} + y_{\tilde{A}}) + 4ab.(w_{\tilde{A}} - y_{\tilde{A}}) - 2ab]}$$

$$\frac{+[(d^3-c^3).(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}})+3d^3-3cd^2]}{+[(d^2-c^2).(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}})+2d^2-2cd]}$$

Proof: Assume that $\tilde{A}=\langle(a,b,c,d);w_{\tilde{A}},u_{\tilde{A}},y_{\tilde{A}}\rangle$ be an SVTN-number and $\Theta_I^{\tilde{A}}$ and $\Theta_{II}^{\tilde{A}}$ be the I. score function and II. score function of $\tilde{A}$, respectively. Then,

i. The centroid of $\tilde{A}$ based on I. score function, denoted by $C(\Theta_I^{\tilde{A}})$, calculated as

$$C(\Theta_I^{\tilde{A}}) = \frac{\int_a^b \frac{x.k_1+k_2}{(b-a)}xdx+\int_b^c w_{\theta_1}.xdx+\int_c^d \frac{x.k_3+k_4}{(d-c)}xdx}{\int_a^b \frac{x.k_1+k_2}{(b-a)}dx+\int_b^c w_{\theta_1}.dx+\int_c^d \frac{x.k_3+k_4}{(d-c)}dx}$$

$$=\frac{\int_a^b \frac{x.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+2)+a.(-w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-1)-b}{3.(b-a)}xdx+\int_b^c \frac{w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1}{3}xdx}{\int_a^b \frac{x.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+2)+a.(-w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-1)-b}{3.(b-a)}dx+\int_b^c \frac{w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1}{3}dx}$$

$$\frac{+\int_c^d \frac{x.(w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-2)+d.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1)+c}{3.(d-c)}xdx}{+\int_c^d \frac{x.(w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-2)+d.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1)+c}{3.(d-c)}xdx}$$

$$=\frac{\left(\frac{(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+2)}{9.(b-a)}x^3+\frac{a.(-w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-1)-b}{6.(b-a)}x^2\right)\Big|_a^b+\left(\frac{w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1}{6}\right)x^2\Big|_b^c}{\left(\frac{(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+2)}{6.(b-a)}x^2+\frac{a.(-w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-1)-b}{3.(b-a)}x\right)\Big|_a^b+\left(\frac{w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1}{3}\right)x\Big|_b^c}$$

$$\frac{+\left(\frac{(w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-2)}{9.(d-c)}x^3+\frac{d.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1)+c}{6.(d-c)}x^2\right)\Big|_c^d}{+\left(\frac{(w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}}-2)}{6.(d-c)}x^2+\frac{d.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}+1)+c}{3.(d-c)}x\right)\Big|_c^d}$$

$$=\frac{\left(\frac{-b^3w_{\tilde{A}}+b^3u_{\tilde{A}}+b^3y_{\tilde{A}}-2b^3}{18.(b-a)}\right)+\left(\frac{a^3w_{\tilde{A}}-a^3u_{\tilde{A}}-a^3y_{\tilde{A}}-a^3+3ba^2}{18.(b-a)}\right)}{\left(\frac{-b^2w_{\tilde{A}}+b^2u_{\tilde{A}}+b^2y_{\tilde{A}}-2b^2}{6.(b-a)}\right)+\left(\frac{a^2w_{\tilde{A}}-a^2u_{\tilde{A}}-a^2y_{\tilde{A}}+2ab}{6.(b-a)}\right)}$$

$$\frac{+\left(\frac{-5c^3w_{\tilde{A}}+c^3u_{\tilde{A}}+c^3y_{\tilde{A}}-2c^3}{18.(d-c)}\right)+\left(\frac{5d^3w_{\tilde{A}}-d^3u_{\tilde{A}}-d^3y_{\tilde{A}}-d^3+3cd^2}{18.(d-c)}\right)}{+\left(\frac{-3c^2w_{\tilde{A}}+c^2u_{\tilde{A}}+c^2y_{\tilde{A}}-2c^2}{6.(d-c)}\right)+\left(\frac{3d^2w_{\tilde{A}}-d^2u_{\tilde{A}}-d^2y_{\tilde{A}}-d^2+2cd}{6.(d-c)}\right)}$$

$$=\frac{1}{3}\frac{[(a^3-b^3).(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}})-2b^3-a^3+3ba^2]}{[(a^2-b^2).(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}})-2b^2+2ab]}$$

$$\frac{+[(d^3-c^3).(5w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}})-2c^3-d^3+3cd^2]}{+[(d^2-c^2).(3w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}})-2c^2+2cd]}$$

Similarly,

ii. The centroid of $\tilde{A}$ based on II. score function, denoted by $C(\Theta_{II}^{\tilde{A}})$, calculated as

$$C(\Theta_{II}^{\tilde{A}}) = \frac{\int_a^b \frac{x.k_5+k_6}{(b-a)}xdx+\int_b^c w_{\theta_2}.xdx+\int_c^d \frac{x.k_7+k_8}{(d-c)}xdx}{\int_a^b \frac{x.k_5+k_6}{(b-a)}dx+\int_b^c w_{\theta_2}.dx+\int_c^d \frac{x.k_7+k_8}{(d-c)}dx}$$

$$= \frac{\int_a^b \frac{x.(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}})+a.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}-1)+b}{3.(b-a)} xdx + \int_b^c \frac{w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1}{3} xdx}{\int_a^b \frac{x.(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}})+a.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}-1)+b}{3.(b-a)} dx + \int_b^c \frac{w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1}{3} dx}$$

$$\frac{+\int_c^d \frac{x.(-w_{\tilde{A}}-u_{\tilde{A}}+y_{\tilde{A}})+d.(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1)-c}{3.(d-c)} xdx}{+\int_c^d \frac{x.(-w_{\tilde{A}}-u_{\tilde{A}}+y_{\tilde{A}})+d.(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1)-c}{3.(d-c)} xdx}$$

$$= \frac{\left(\frac{(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}})}{9.(b-a)} x^3 + \frac{a.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}-1)+b}{6.(b-a)} x^2\right)\Big|_a^b + \left(\frac{w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1}{6}\right) x^2\Big|_b^c}{\left(\frac{(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}})}{6.(b-a)} x^2 + \frac{a.(w_{\tilde{A}}-u_{\tilde{A}}-y_{\tilde{A}}-1)+b}{3.(b-a)} x\right)\Big|_a^b + \left(\frac{w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1}{3}\right) x\Big|_b^c}$$

$$\frac{+\left(\frac{(-w_{\tilde{A}}-u_{\tilde{A}}+y_{\tilde{A}})}{9.(d-c)} x^3 + \frac{d.(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1)-c}{6.(d-c)} x^2\right)\Big|_c^d}{+\left(\frac{(-w_{\tilde{A}}-u_{\tilde{A}}+y_{\tilde{A}})}{6.(d-c)} x^2 + \frac{d.(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}}+1)-c}{3.(d-c)} x\right)\Big|_c^d}$$

$$= \frac{\left(\frac{-b^3w_{\tilde{A}}-b^3u_{\tilde{A}}+b^3y_{\tilde{A}}+6b^2aw_{\tilde{A}}-6b^2ay_{\tilde{A}}}{18.(b-a)}\right) + \left(\frac{-5a^3w_{\tilde{A}}+a^3u_{\tilde{A}}+5a^3y_{\tilde{A}}+3a^3-3a^2}{18.(b-a)}\right)}{\left(\frac{-b^2w_{\tilde{A}}-b^2u_{\tilde{A}}+b^2y_{\tilde{A}}+4aby_{\tilde{A}}}{6.(b-a)}\right) + \left(\frac{3a^2w_{\tilde{A}}+a^2u_{\tilde{A}}+3a^2y_{\tilde{A}}+2a^2-2ab}{6.(b-a)}\right)}$$

$$\frac{+\left(\frac{-c^3w_{\tilde{A}}-c^3u_{\tilde{A}}+c^3y_{\tilde{A}}}{18.(d-c)}\right) + \left(\frac{d^3w_{\tilde{A}}+d^3u_{\tilde{A}}-d^3y_{\tilde{A}}+3d^3-3d^2c}{18.(d-c)}\right)}{+\left(\frac{-c^2w_{\tilde{A}}-c^2u_{\tilde{A}}+c^2y_{\tilde{A}}}{6.(d-c)}\right) + \left(\frac{d^2w_{\tilde{A}}+d^2u_{\tilde{A}}-d^2y_{\tilde{A}}+2d^2-2cd}{6.(d-c)}\right)}$$

$$= \frac{1}{3} \frac{[a^3.(-5w_{\tilde{A}}+u_{\tilde{A}}+5y_{\tilde{A}}+3)+b^3.(-w_{\tilde{A}}-u_{\tilde{A}}+y_{\tilde{A}})+6b^2a.(w_{\tilde{A}}-y_{\tilde{A}})-3a^2]}{[a^2.(3w_{\tilde{A}}+u_{\tilde{A}}+3y_{\tilde{A}}+2)-b^2.(w_{\tilde{A}}+u_{\tilde{A}}+y_{\tilde{A}})+4ab.(w_{\tilde{A}}-y_{\tilde{A}})-2ab]}$$

$$\frac{+[(d^3-c^3).(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}})+3d^3-3cd^2]}{+[(d^2-c^2).(w_{\tilde{A}}+u_{\tilde{A}}-y_{\tilde{A}})+2d^2-2cd]}$$

Therefore, the proof is valid.

Definition 14. Let $\tilde{A} = \langle(a_1, b_1, c_1, d_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}}\rangle$ and $\tilde{B} = \langle(a_2, b_2, c_2, d_2); w_{\tilde{B}}, u_{\tilde{B}}, y_{\tilde{B}}\rangle$ be two SVTN-numbers. Then,

1. $If\ C(\Theta_I^{\tilde{A}}) > C(\Theta_I^{\tilde{B}}), then\ \tilde{A} < \tilde{B}$
2. $If\ C(\Theta_I^{\tilde{A}}) < C(\Theta_I^{\tilde{B}}), then\ \tilde{A} > \tilde{B}$
3. $If\ C(\Theta_I^{\tilde{A}}) = C(\Theta_I^{\tilde{B}}), then$
   - $i.\ If\ C(\Theta_{II}^{\tilde{A}}) > C(\Theta_{II}^{\tilde{B}}), then\ \tilde{A} < \tilde{B}$
   - $ii.\ If\ C(\Theta_{II}^{\tilde{A}}) < C(\Theta_{II}^{\tilde{B}}), then\ \tilde{A} > \tilde{B}$
   - $iii.\ If\ C(\Theta_{II}^{\tilde{A}}) = C(\Theta_{II}^{\tilde{B}}), then\ \tilde{A} = \tilde{B}.$

Example 15. $\tilde{A} = \langle(3,5,6,8); 1.0,0.0,0.0\rangle$ and $\tilde{B} = \langle(1,4,6,9); 1.0,0.0,0.0\rangle$ be two SVTN-numbers. Then we have

$$\begin{aligned} C(\Theta_I^{\tilde{A}}) &= \frac{1}{3}\left(\frac{(-3.5^3+3.5.3^2)+(4.8^3-7.6^3+3.6.8^2)}{(3^2-3.5^2+2.3.5)+(3.8^2-5.6^2+2.6.8)}\right) \\ &= \frac{1}{3}\left(\frac{(-375+135)+(2048-1512+1152)}{(9-75+30)+(192-180+96)}\right) \\ &= 6{,}703 \end{aligned}$$

and

$$C(\Theta_I^{\tilde{B}}) = \frac{1}{3}\left(\frac{(-3.4^3+3.4)+(4.9^3-7.6^3+3.6.9^2)}{(1-3.4^2+2.4)+(3.9^2-5.6^2+2.6.9)}\right) = \frac{1}{3}\left(\frac{(-192+12)+(2916-1512+1458)}{(1-48+8)+(243-180+108)}\right) = 6{,}7727$$

Therefore, $(\Theta_I^{\tilde{A}}) < C(\Theta_I^{\tilde{B}}) \rightarrow \tilde{A} > \tilde{B}$.

Remark 16. $\tilde{A} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$,$\tilde{B} = \langle (a_2, b_2, c_2, d_2); w_{\tilde{B}}, u_{\tilde{B}}, y_{\tilde{B}} \rangle$ be two SVTN-numbers. Then, the following equations do not generally hold.

i. $C(\Theta_I^{\tilde{A}}) + C(\Theta_I^{\tilde{B}}) = C(\Theta_I^{\tilde{A}+\tilde{B}})$

ii. $C(\Theta_{II}^{\tilde{A}}) + C(\Theta_{II}^{\tilde{B}}) = C(\Theta_{II}^{\tilde{A}+\tilde{B}})$

iii. $C(\Theta_I^{\tilde{A}}).C(\Theta_I^{\tilde{B}}) = C(\Theta_I^{\tilde{A}.\tilde{B}})$

iv. $C(\Theta_{II}^{\tilde{A}}).C(\Theta_{II}^{\tilde{B}}) = C(\Theta_{II}^{\tilde{A}.\tilde{B}})$

Example 17. $\tilde{A} = \langle (3,5,6,8); 1.0,0.0,0.0 \rangle$ and $\tilde{B} = \langle (1,4,6,9); 1.0,0.0,0.0 \rangle$ be two SVTN-numbers. Then we have,

i.

$$C(\Theta_I^{\tilde{A}}) = \frac{339{,}5}{34{,}5} = 9{,}84 \quad (1)$$

$$C(\Theta_I^{\tilde{B}}) = \frac{1226{,}5}{162{,}9} = 7{,}52 \quad (2)$$

From (1) and (2)

$$C(\Theta_I^{\tilde{A}}) + C(\Theta_I^{\tilde{B}}) = 17{,}36 \quad (3)$$

Also, for $\tilde{A} + \tilde{B} = \langle (3,8,13,16); 0.5,0.7,0.3 \rangle$

$$C(\Theta_I^{\tilde{A}+\tilde{B}}) = \frac{3265}{42} = 77{,}73 \quad (4)$$

Therefore, from (3) and (4) $C(\Theta_I^{\tilde{A}}) + C(\Theta_I^{\tilde{B}}) = C(\Theta_I^{\tilde{A}+\tilde{B}})$ does not hold.

ii.

$$C(\Theta_{II}^{\tilde{A}}) = 6{,}84 \quad (5)$$

$$C(\Theta_{II}^{\tilde{B}}) = 4{,}91 \quad (6)$$

From (5) and (6)

$$C(\Theta_{II}^{\tilde{A}}) + C(\Theta_{II}^{\tilde{B}}) = 11{,}75 \quad (7)$$

Also, for $\tilde{A} + \tilde{B} = \langle (3,8,13,16); 0.5,0.7,0.3 \rangle$, $C(\Theta_{II}^{\tilde{A}+\tilde{B}}) = 13{,}37$ (8)

Therefore, from (7) and (8) $C(\Theta_{II}^{\tilde{A}}) + C(\Theta_{II}^{\tilde{B}}) = C(\Theta_{II}^{\tilde{A}+\tilde{B}})$ does't hold.

iii.

$$C(\Theta_I^{\tilde{A}}) = \frac{339{,}5}{34{,}5} = 9{,}84 \quad (9)$$

$$C(\Theta_I^{\tilde{B}}) = \frac{1226{,}5}{162{,}9} = 7{,}52 \quad (10)$$

From (9) and (10)

$$C(\Theta_I^{\tilde{A}}).C(\Theta_I^{\tilde{B}}) = 74{,}09 \quad (11)$$

Also, for $\tilde{A}.\tilde{B} = \langle(2,16,42,64); 0.5,0.7,0.3\rangle, C(\Theta_I^{\tilde{A}.\tilde{B}}) = 47{,}28$ (12)

Therefore, from (11) and (12) $C(\Theta_I^{\tilde{A}}).C(\Theta_I^{\tilde{B}}) = C(\Theta_I^{\tilde{A}.\tilde{B}})$ does't hold.

iv.

$$C(\Theta_{II}^{\tilde{A}}) = 6{,}84 \quad (13)$$

$$C(\Theta_{II}^{\tilde{B}}) = 4{,}91 \quad (14)$$

From (13) and (14) we have $C(\Theta_{II}^{\tilde{A}}).C(\Theta_{II}^{\tilde{B}}) = 33{,}61$

Also, for $\tilde{A}.\tilde{B} = \langle(2,16,42,64); 0.5,0.7,0.3\rangle$ (15)

$$C(\Theta_{II}^{\tilde{A}.\tilde{B}}) = 31{,}85 \quad (16)$$

Therefore, from (15) and (16) $C(\Theta_{II}^{\tilde{A}}).C(\Theta_{II}^{\tilde{B}}) = C(\Theta_{II}^{\tilde{A}.\tilde{B}})$ does't hold.

4. A New MCDM Method Based On The Concept Of Score Functions And Defuzzification

In the section, we propose a new MCDM method based on the proposed I. score function and II. score function of SVTN-numbers and defuzzification of fuzzy numbers to deal with MCDM problems in SVTN-number environments. Let $X = \{x_1, x_2, \ldots, x_m\}$ be a set of alternatives and $U = \{u_1, u_2, \ldots, u_n\}$ be a set of criteria. Assume that the evaluating value of criteria u_j $(j = 1,2, \ldots, n)$ with respect to alternative x_i $(i = 1,2, \ldots, m)$ be represented by an SVTN-number $(x_{ij})_{mxn} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}; w_{ij}, u_{ij}, y_{ij})$, and be the normalized decision matrix given by expert based on Table 1, let $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ be weighting vector of the criteria set $U = \{u_1, u_2, \ldots, u_n\}$ given by expert based on Table 2.

Note that if $a_{ij}, b_{ij}, c_{ij}, d_{ij} \in [0,1]$, then $(x_{ij})_{mxn} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}; w_{ij}, u_{ij}, y_{ij})_{mxn}$ is normal. Note that if $(x_{ij})_{mxn} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}; w_{ij}, u_{ij}, y_{ij})_{mxn}$ is not normal, we can normalize the matrix such as $\tilde{x}_{ij} = \frac{x_{ij}}{\max_{i \in I_m, j \in I_n}(a_{ij}+b_{ij}+c_{ij}+d_{ij})}$.

The proposed multiple criteria decision-making method based on proposed I. score function and II. score function of SVTN-numbers and defuzzification of fuzzy numbers is now presented as follows:

Table 1. The linguistic values of the SVTN-number for the evaluation matrix

Linguistic values	SVTN-number values
Very Poor (VP)	$\langle(0.0,0.0,0.1,0.2); 0.1,0.0,0.2\rangle$
Weak (W)	$\langle(0.0,0.1,0.2,0.3); 0.3,0.5,0.4\rangle$
Medium Weak (MW)	$\langle(0.2,0.3,0.3,0.4); 0.3,0.6,0.7\rangle$
Weak(W)	$\langle(0.4,0.4,0.5,0.6); 0.4,0.5,0.6\rangle$
Medium Good (MG)	$\langle(0.5,0.6,0.6,0.7); 0.7,0.4,0.5\rangle$
Good (G)	$\langle(0.6,0.7,0.7,0.8); 0.7,0.5,0.4\rangle$
Very Good (VG)	$\langle(0.8,0.8,0.9,1.0); 0.9,0.8,0.4\rangle$

Step 1: Give the normalized decision-making matrix $(x_{ij})_{mxn}$

Step 2: Give the weighting vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ of the criteria set

Step 3: Find of the normalized weights of the criteria set as; $w_j = \frac{C(\Theta_I^{\omega_j})}{\sum_{j=1}^{n} C(\Theta_I^{\omega_j})}$ for $j = 1,2,\dots,n$ $\left(or \quad w_j = \frac{C(\Theta_{II}^{\omega_j})}{\sum_{j=1}^{n} C(\Theta_{II}^{\omega_j})}\right)$

Table 2. The linguistic values of the SVTN-number for the criteria weights

Linguistic values	SVTN-number values
Very Low (VL)	$\langle(0.0,0.0,0.1,0.2); 0.1,0.1,0.0\rangle$
Low (L)	$\langle(0.1,0.1,0.2,0.3); 0.3,0.1,0.2\rangle$
Medium Low (ML)	$\langle(0.2,0.2,0.3,0.4); 0.4,0.3,0.2\rangle$
Medium (M)	$\langle(0.4,0.4,0.5,0.6); 0.5,0.4,0.3\rangle$
Medium High (MH)	$\langle(0.5,0.6,0.6,0.7); 0.7,0.5,0.4\rangle$
High (H)	$\langle(0.7,0.8,0.8,0.9); 0.9,0.8,0.7\rangle$
Very High (VH)	$\langle(0.8,0.9,0.9,1.0); 1.0,0.9,0.8\rangle$

Step 4: Compute the matrix $M_{ij} = w_j \times x_{ij}$, $(i = 1,2,\dots,m \; ; \; j = 1,2,\dots,n)$
Step 5: Compute the $S_i = \sum_{j=1}^{n} M_{ij}$ $(i = 1,2,\dots,m)$ for x_i $i = 1,2,\dots,m$
Step 6: Find $C(\Theta_I^{S_i})$ $\left(\text{or } C(\Theta_{II}^{S_i})\right)$ for the values $S_i(i = 1,2,\dots,m)$

Step 7: Rank all alternatives x_i for all $i = 1,2,\dots,m$, by using $C(\Theta_I^{S_i})$ $\left(\text{or } C(\Theta_{II}^{S_i})\right)$ and determine the best alternative

5. Application

Example 18. Assume that the next five years' strategic plan of a country will be prepared by authorities in the country. For this, there are five areas called x_1 = "defence area", x_2 ="social area", x_3 = "health area", x_4 = "agriculture area" and x_5 = "education area" to be evaluated with four criteria; u_1 ="Reduction of production and external dependence", u_2 ="finance", u_3 ="Marketing" and u_4 ="To have a voice in the world". The authorities use the linguistic terms shown in Table 1 to represent the characteristics of the potential areas with respect to different criteria. Also, the authorities use the linguistic terms shown in Table 2 to represent criteria weights.

Using the information, the authorities of the country will make the most advantageous investment order with the following algorithm.

Step 1: The normalized decision-making matrix $(x_{ij})_{5x4}$ is given by an expert in Table 3 based on Table 1

Table 3. The normalized decision-making matrix $(x_{ij})_{5x4}$

	u_1	u_2
x_1	$\langle(0.2,0.3,0.3,0.4); 0.3,0.6,0.7\rangle$	$\langle(0.5,0.6,0.7,0.7); 0.7,0.4,0.5\rangle$
x_2	$\langle(0.5,0.6,0.7,0.7); 0.7,0.4,0.5\rangle$	$\langle(0.2,0.3,0.3,0.4); 0.3,0.6,0.7\rangle$
x_3	$\langle(0.7,0.8,0.8,0.9); 0.8,0.3,0.4\rangle$	$\langle(0.0,0.1,0.2,0.2); 0.3,0.5,0.4\rangle$
x_4	$\langle(0.4,0.4,0.5,0.6); 0.4,0.5,0.6\rangle$	$\langle(0.7,0.8,0.8,0.9); 0.8,0.3,0.4\rangle$
x_5	$\langle(0.0,0.1,0.2,0.2); 0.3,0.5,0.4\rangle$	$\langle(0.8,0.9,1.0,1.0); 0.9,0.2,0.3\rangle$
	u_3	u_4
x_1	$\langle(0.0,0.0,0.1,0.1); 0.1,0.0,0.2\rangle$	$\langle(0.4,0.4,0.5,0.6); 0.4,0.5,0.6\rangle$
x_2	$\langle(0.2,0.3,0.3,0.4); 0.3,0.6,0.7\rangle$	$\langle(0.8,0.9,1.0,1.0); 0.9,0.2,0.3\rangle$
x_3	$\langle(0.4,0.4,0.5,0.6); 0.4,0.5,0.6\rangle$	$\langle(0.2,0.3,0.3,0.4); 0.3,0.6,0.7\rangle$
x_4	$\langle(0.7,0.8,0.8,0.9); 0.8,0.3,0.4\rangle$	$\langle(0.0,0.0,0.1,0.1); 0.1,0.0,0.2\rangle$
x_5	$\langle(0.0,0.1,0.2,0.2); 0.3,0.5,0.4\rangle$	$\langle(0.5,0.6,0.7,0.7); 0.7,0.4,0.5\rangle$

Step 2: The weighting vector $\omega = (\omega_1 = ML, \omega_2 = M, \omega_3 = MH, \omega_4 = H)$ of the criteria set is given by expert based on Table 2, according to j^{th} attribute u_j $(j = 1,2,\ldots,4)$;

Step 3: The normalized weights of the criteria set are found as

$$w = (0.1145,\ 0.1937,\ 0.2998,\ 0.3920)$$

Step 4: The weighted matrix $M_{ij} = \omega_j \times k_{ij}, (i = 1,2,\ldots,5;\ j = 1,2,3,4)$ is computed as Table 4

Table 4. $M_{ij} = \omega_j \times k_{ij},\ (i = 1,2,\ldots,5\ ;\ j = 1,2,3,4)$

	u_1	u_2
x_1	$\langle(0.0229,0.0343,0.0343,0.0458); 0.3,0.6,0.7\rangle$	$\langle(0.0968,0.1162,0.1356,0.1356); 0.7,0.4,0.5\rangle$
x_2	$\langle(0.0572,0.0687,0.0801,0.0801); 0.7,0.4,0.5\rangle$	$\langle(0.0387,0.0581,0.0581,0.0774); 0.3,0.6,0.7\rangle$
x_3	$\langle(0.0801,0.0916,0.0916,0.1030); 0.8,0.3,0.4\rangle$	$\langle(0.0000,0.0193,0.0387,0.0387); 0.3,0.5,0.4\rangle$
x_4	$\langle(0.0458,0.0458,0.0572,0.0687); 0.4,0.5,0.6\rangle$	$\langle(0.1356,0.1549,0.15490.1743); 0.8,0.3,0.4\rangle$
x_5	$\langle(0.0000,0.0114,0.0229,0.0229); 0.3,0.5,0.4\rangle$	$\langle(0.1549,0.1743,0.1937,0.1937); 0.9,0.2,0.3\rangle$

	u_3	u_4
x_1	$\langle(0.0000,0.0000,0.0299,0.0299); 0.1,0.0,0.2\rangle$	$\langle(0.1567,0.1567,0.1959,0.2351); 0.4,0.5,0.6\rangle$
x_2	$\langle(0.0599,0.0899,0.0899,0.1199); 0.3,0.6,0.7\rangle$	$\langle(0.3135,0.3527,0.3919,0.3919); 0.9,0.2,0.3\rangle$
x_3	$\langle(0.1199,0.1199,0.1498,0.1798); 0.4,0.5,0.6\rangle$	$\langle(0.0783,0.1175,0.1175,0.1567); 0.3,0.6,0.7\rangle$
x_4	$\langle(0.2098,0.2398,0.2398,0.2698); 0.8,0.3,0.4\rangle$	$\langle(0.0000,0.0000,0.0391,0.0391); 0.8,0.3,0.4\rangle$
x_5	$\langle(0.0000,0.0299,0.0599,0.0599); 0.3,0.5,0.4\rangle$	$\langle(0.1959,0.2351,0.2743,0.2743); 0.7,0.4,0.5\rangle$

Step 5: The $S_i = \sum_{j=1}^{n} M_{ij}$ $(i = 1,2,\ldots,5)$ for x_i is computed as

$$S_1 = \langle(0.2765,0.3073,0.3959,0.4465); 0.1,0.6,0.7\rangle$$
$$S_2 = \langle(0.4695,0.5695,0.6201,0.6695); 0.3,0.6,0.7\rangle$$
$$S_3 = \langle(0.2784,0.3484,0.3978,0.4784); 0.3,0.6,0.7\rangle$$
$$S_4 = \langle(0.3912,0.4406,0.4912,0.5520); 0.4,0.5,0.6\rangle$$
$$S_5 = \langle(0.3509,0.4509,0.5509,0.5509); 0.3,0.5,0.5\rangle$$

Step 6: The $C(\Theta_I^{S_i})$ for the values $S_i\,(i = 1,2,\ldots,5)$is found as

$$C(\Theta_I^{S_1}) = 11.25007926$$
$$C(\Theta_I^{S_2}) = 0.699682108$$
$$C(\Theta_I^{S_3}) = 0.458601346$$
$$C(\Theta_I^{S_4}) = 0.505543469$$
$$C(\Theta_I^{S_5}) = 0.255190279$$

Step 7: All alternatives x_i for all $i = 1,2,\ldots,5$, by using the $C((\Theta_1)_{S_i})$ is ranked as

$$C(\Theta_I^{S_1}) > C(\Theta_I^{S_2}) > C(\Theta_I^{S_4}) > C(\Theta_I^{S_3}) > C(\Theta_I^{S_5})$$

Since we have

$$x_5 > x_3 > x_4 > x_2 > x_1$$

the best alternative is x_5.

In that case, primarily investment must be made in the education area.

6. Comparative Analysis

In order to further elucidate the advantages of the proposed MCDM method, we use the proposed MCDM method based on the defuzzification and concepts of 1. and 2. score function, method [20] based on the Value and ambiguity index, method [22] based on the cut sets and values and ambiguities value, method [32] based on the normalized weighted Bonferroni mean operator, method [33] based on preference relations and method [27] based on weighted aggregation operators to deal with the Example 18 and compare the ranking results in Table 5 for different decision-making methods.

Table 5. Comparative Analysis with the existing methods of Example 20

Methods	Ranking Order	Primarily investment	Not primarily investment
1. Method in [20]	$x_5 > x_4 > x_2 > x_3 > x_1$	x_5	x_1
2. Method in [22] $(\lambda, \mu, \nu) = (0.5,0.5,0.5)$	$x_2 > x_5 > x_4 > x_3 > x_1$	x_2	x_1
3. Method in [32]	$x_5 > x_4 > x_2 > x_3 > x_1$	x_5	x_1
4. Method in [33]	$x_5 > x_4 > x_2 > x_3 > x_1$	x_5	x_1
6. Proposed method	$x_5 > x_3 > x_4 > x_2 > x_1$	x_5	x_1

In this case, the proposed MCDM method is generally similar to the decision-making methods presented in [20,22,32,33]. From Table 5, we have a significant impact on the ranking results of the five alternatives for the decision-making method presented in [20,22,32,33]. The ranking result of the five alternatives obtained by the MCDM method in [22] the primary investment alternative is changed from the alternative x_5 to the alternative x_2. But, the not primarily investment alternative is the same as all the methods. Obviously, these ranking results of the alternatives obtained by the method presented in [22] based on the cut sets and values and ambiguities value is unreasonable. Therefore, the proposed MCDM method based on the defuzzification and concepts of I. score function and II. score function, method [20] based on the Value and ambiguity index, method [32] based on the normalized weighted Bonferroni mean operator, method [33] based on preference relations are more suitable to deal with practical decision-making problems.

7. Conclusions

In recent years, many useful defuzzification methods have been proposed to solve various MCDM problems in fuzzy numbers and intuitionistic fuzzy numbers, but very few methods take into account the perspectives of both the defuzzification and the SVTN-numbers. Therefore, we presented concepts of I. score function and II. score function to reduce the SVTN-numbers to fuzzy numbers. Finally, we developed MCDM method for MCDM problems by using the defined concepts. Also, we introduced a numerical example to demonstrate how to apply the proposed MCDM method and the superiority of the proposed MCDM method compared to the existing methods. In further research, we will develop new defuzzification methods of the SVTN-numbers based on fuzzy numbers.

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